COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 3, No. 2, 2013

ENERGETICAL PROPERTIES OF FRACTAL BROWNIAN SIGNAL WITH DIFFERENT HURST INDEXES

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Abstract. Power spectrum densities, probability density functions and correlation functions of the fractal Brownian signal (FBS) are studied with the aim of MATLAB simulation. The influence of a time-scaling factor on these properties is investigated. Particularly, the increase of this factor leads to increasing the influence of the Hurst index on power spectrum density. The results are conformed to Mandelbrot's investigations of FBS properties.

Key words: FBS, Hurst index, time-scaling factor.

1. Introduction

An interest in the study of the FBS is conditioned by its application in various branches of modern science and technology – economics, medicine, information technologies and telecommunications [1].

One of the aspects of applying the FBS refers to the self-similarity of data traffic in the Internet [2], because the capacity of a data flow is a predictable value. It enables data flows to be controlled by means of peak load regulation.

A promising area of research that has emerged recently is the use of fractal signals in communication systems to ensure the secrecy of information transfer [3, 4].

A useful feature of the FBS is its wide frequency band, which makes it possible to transfer information through multipath channels and under the conditions of a high noise level [5].

There are few publications on the subject, since the generation of the FBS is difficult, but the development of modern computational technologies in general and the increase of data processing speed in particular leads to finding the solution of this problem [6].

There are several methods of information transfer using different kinds of signals: fractal wavelet signals [6], FBS [3, 7], and a method using broad-band random noise generators described by the Ressler equations [8].

The implementation of the method using the synchronization between the generators on the transmitter and receiver sides is possible if their elementary components meet high demands for the precision of the parameters [9].

Systems described in [6, 7, 8, 9] are capable of providing data transfer in channels with a signal to noise ratio -10 dB.

The mathematical model of the FBS [10] used in [3,7] is represented in the following form (1):

$$S(t,H,n) \approx \frac{n^{-1/2}}{\Gamma(H+1/2)} \cdot \left[\sum_{i=0}^{n\cdot(t+1)-1} \left(t+1-\frac{i}{n} \right)^{H-1/2} \cdot \xi_i - \sum_{i=0}^{n\cdot t-1} \left(t-\frac{i}{n} \right)^{H-1/2} \cdot \xi_i \right], \quad (1)$$

where $\Gamma(x)$ is a gamma-function, H is the Hurst index, n is a time-scaling factor, t is a point of time when the FBS sample is taken (a positive integer number greater than 0), i is the set of integers indexing summands and designating the instant of time before t, $\{\xi_i\}$ is the set of random Gaussian numbers, n is the time-scaling factor (an integer number greater than zero); S is the value of the FBS sample.

The aim of this article is the investigation of the dependence of spectral, statistical and correlating properties of the FBS on the time-scaling factor.

2. Spectral properties of FBS

The FBS investigated in this work consists of 500 samples. Such an amount of FBS samples is sufficient for working at the signal to noise ratio equal to -10 dB.

The power spectrum density of the discrete FBS is obtained by averaging the squared DFT [7] according to the equation:

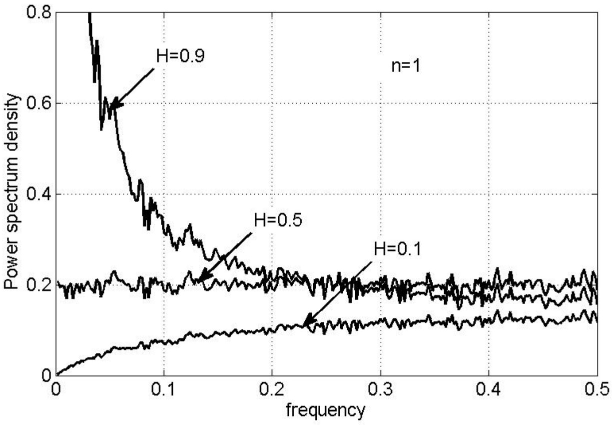
$$W(w) = \lim_{m\to\infty} \frac{1}{2\cdot m+1} \cdot \overline{\left| \sum_{k=-m}^{m} S(k) \cdot e^{-j\cdot w\cdot k\cdot T} \right|^2}, \quad (2)$$

where $S(k)$ is the k-th FBS sample; T is a period of sampling.

The power spectrum density of the FBS with different time-scaling factors and Hurst indexes are shown in Fig. 1 and Fig. 2.

It is evident, that the FBS is a wideband signal by all used values of the Hurst index and time-scaling factor.

From the results obtained it follows that the power spectrum density of the FBS with Hurst index H=0.5 is identical to the power spectrum density (PDS) of the white Gaussian noise in the whole frequency range. A low-frequency part of the spectrum is a dominant one for the FBS with the Hurst index H=0.9. On the contrary, for the FBS with the Hurst index H=0.1 the weak dependence of the PDS on the frequency is observed. The FBS with the Hurst index H=0.5 is the classical Brownian motion. These results conform to Mandelbrot's research [10, 11] and the changes of the time-scaling factor don't transform their correlation.



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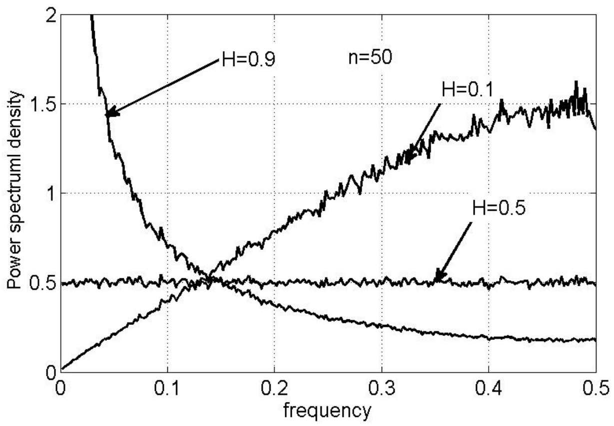


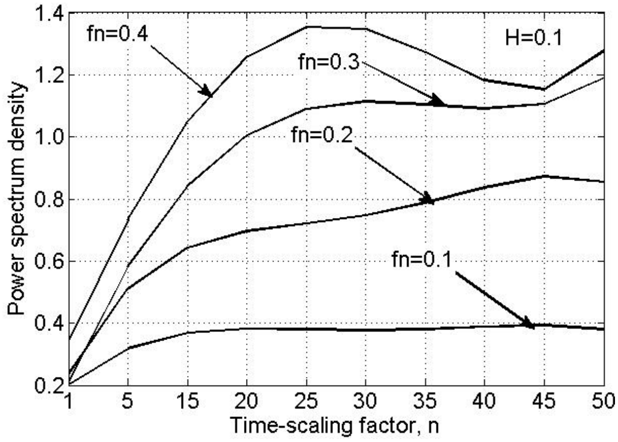
b)

Fig. 1. Power spectrum density of the FBS with different Hurst indexes (0.1; 0.5; 0.9) and time-scaling factors (a) n=1, b) n=50).

As the time-scaling factor increases, the dependence of the PSD on the frequency becomes much stronger and transforms to the linear one if time-scaling factor is equal to 50.

The dependences of the PSD on time-scaling factor suggest similar conclusions. Such dependences are depicted in Fig. 2. Four curves shown there were obtained at the different values of normal frequency: $f_н = 0.1, f_н = 0.2; f_н = 0.3; f_н = 0.4$.

As we can conclude from Fig. 2a, the high-frequency part of the spectrum becomes greater, as the time-scaling factor increases. If the value of the Hurst index is 0.5 or 0.9, such regularity is not revealed.



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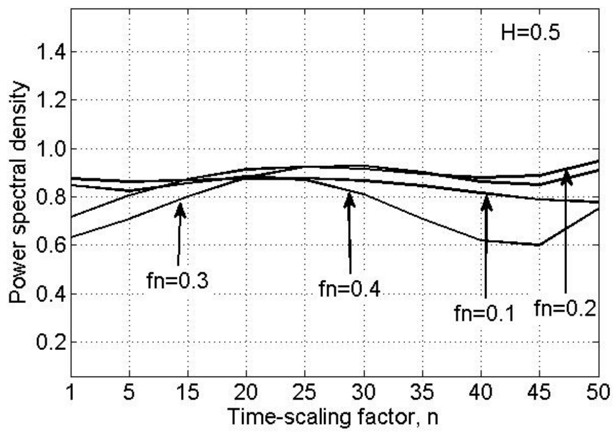

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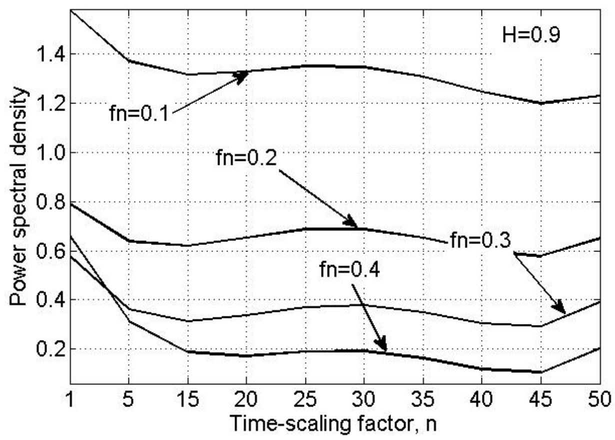

c)

Fig. 2. Power spectrum density at different values of the Hurst index and time-scaling factor (a) H=0.1; b) H=0.5; c) H=0.9).

3. Statistical properties and correlation functions of the FBS

The statistical properties of the FBS with the Hurst indexes H=0.1; 0.5; 0.9 and different time-scaling factors are shown in Fig. 3. As one can see, the FBS has the Gaussian distribution. If the Hurst index equals 0.1, variance increases when the time-scaling factor increases, and it decreases if the Hurst index equals 0.9.

The autocorrelation functions of the FBS with different Hurst indexes and time-scaling factors are calculated as well (H=0.1; H=0.5; H=0.9 and n=1; 50).

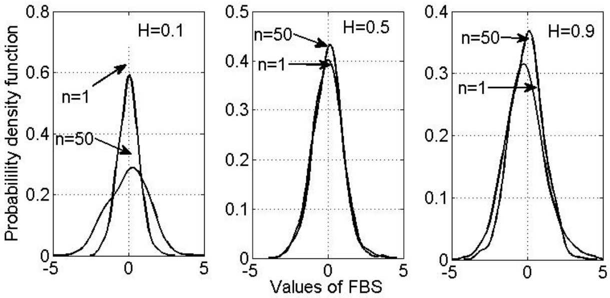

Fig. 3. Probability density functions for the FBS with the Hurst index H=0.1; H=0.5; H=0.9 and different values of the time-scaling factor.

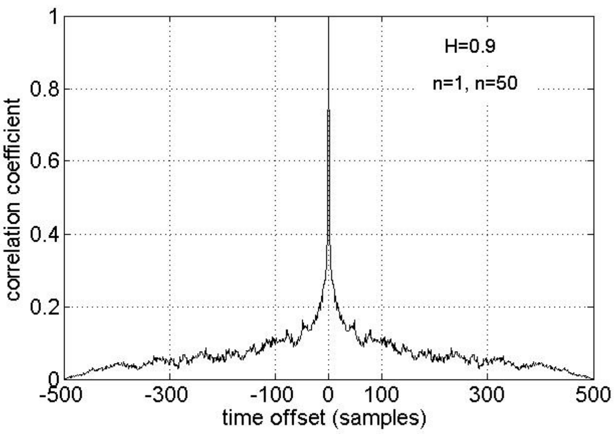

Fig. 4. Autocorrelation functions.

If the Hurst index equals 0.1 or 0.5, the FBS is slightly correlated; if the Hurst indexes equal 0.9, the correlation of the FBS becomes greater and does not depend on the time-scaling factor.

3. Conclusions

The FBSs with the different Hurst indexes and time scaling factors are chaotic and have the Gaussian distribution. If the Hurst index equals 0.1, variance increases as the time-scaling factor increases, and it decreases if the Hurst index equals 0.9.

The low-frequency spectrum part of the FBS is dominant if the Hurst index is equal to 0.9, while if Hurst index equals to 0.1, this dependence is an inverse one, being conformed to computer simulation provided by Mandelbrot.

As the time-scaling factor increases, the frequency dependence of the PSD becomes much stronger and turns into the linear dependence if the time-scaling factor equals 50.

The FBSs with the Hurst indexes H=0.1 and H=0.5 are weakly correlated with their copies in the frequency shift band [-500;0] and [0;500], while the FBS with the Hurst index H=0.9 are correlated with their copies in the frequency ranges mentioned above.

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ЕНЕРГЕТИЧНІ ВЛАСТИВОСТІ ФРАКТАЛЬНИХ БРОУНІВСЬКИХ СИГНАЛІВ З РІЗНИМИ ПОКАЗНИКАМИ ХЕРСТА

Юрій Бобало, Михайло Климаш, Руслан Політанський

Роботу присвячено вивченню властивостей сигналів типу фрактальний гауссів шум (далі – ФГШ). Автори дослідили спектральну густину потужності, числові характеристики та автокореляційну функцію сигналів типу ФГШ. Встановлено їх залежність від коефіцієнта часового масштабування. Зокрема показано, що збільшення коефіцієнта часового масштабування підсилює вплив параметра Херста на спектральну густину потужності, що узгоджується із результатами, отриманими раніше Мандельбротом.

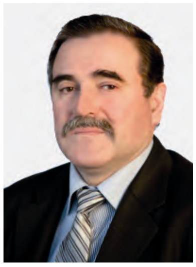
Yuriy Bobalo – Ph.D., DSc., Professor, graduated from of Lviv Polytechnic Institute, Department of Radio Engineering, Ukraine, in 1973. In 1984 after completing his post-graduate studies at S. Ordzhonikidze Moscow Aircraft Institute, Russia, he received his Ph.D. degree in Radio-engineering and Television Systems.

His futher scientific research is focused on ensuring quality and reliability of radio electronic devices using complex optimization of their design, production and operation. In 2000 he started working as Associate Professor and in 2004 as Professor at the Department of Theoretical Radio Engineeringand Measuring of Lviv Polytechnic National University, Ukraine and became a distinguished lecturer in the discipline “Control and diagnostics of radio electronic devices”.

In 2005 he was appointed Vice-Rector for Research and in 2007 Rector of Lviv Polytechnic National University, Ukraine.

In 2010 he successfully defended his Doctor of Sciences thesis on processes automation at National Technical University “Kharkiv Polytechnic Institute”, Ukraine. He is the author of more than 150 scientific publications; among them three monographs, seven text-books and numerous papers in proceedings of international conferences and symposiums. Under his supervision 4 Ph.D. dissertations have been defended.

Professor Yuriy Bobalo is the author of different scientific, didactic and teaching materials designed for higher schools. His activity has been distinquished with several awards, nationally and internationally.



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