COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 9 , No.2 , 2019

MODEL OF ELECTRICAL ACTIVITY OF THE BIOLOGIC NEURAL NETWORK AREAS

Vsevolod Horyachko, Oksana Hoholyuk, Taras Ryzyi, Serhiy Rendzinyak
Lviv Polytechnic National University, Lviv, Ukraine
vsevolod.i.horiachko@lpnu.ua , oksana.p.hoholyuk@lpnu.ua , taras.i.ryzyi@lpnu.ua , serhii.y.rendziniak@lpnu.ua

Abstract: In the paper, the mathematical model describing the generation of action potential and propagation of an impulse in the neuron's filaments on the basis of the analysis of parametric electric circuits with distributed parameters and the mathematical model of synaptic interneuron connections are proposed.

Developed models allow taking into account the influence of such factors as geometric, physical and chemical parameters of the neuron's filaments and the presence of different neurotransmitters in chemical synapses on transmitting a neural impulse. Further, such models can be used for investigating the conditions of neuron firing at spatial and time integration of input signals, as well as for the simulation of neuromuscular junctions.

Key words: mathematical model, neuron, axon, dendrite, synapse, neural impulse.

1. Introduction

The main function of the nervous system is the fast and accurate transfer of information. A signal from receptors to sensor centers in the spinal cord or brain, then from these centers to the motor centers and from them to effector organs – muscles and glands – should be transmitted quickly and accurately. Survival of organisms depends on their ability to catch environmental changes and react quickly to them [1, 6].

Although until now, such complex psychophysiological processes as memory and thinking occurring in the human brain and involving billions of neurons have not been understood quite well, the physiology of the nervous system and, in particular, its main structural element – a nervous cell – is a subject of research of many scientists for a long time.

Mathematical modeling of electrophysiological processes in biological objects is an actual and promising direction of their research. First of all, it concerns neurophysiology.

Nowadays, the Hodgkin–Huxley mathematical theory, awarded by Nobel Prize, is recognized. This theory is based on the electrophysiological experimental data [1]. Based on this theory, the new model of impulse propagation in a nervous fiber can be developed.

2. Simulation of a nerve impulse in the neuron's filaments

The nervous cell – neuron (Fig. 1) – is a structural unit of the central and peripheral nervous system of living organisms. Each neuron consists of the main part (body) and two types of filaments extruding from it. The latter group includes numerous dendrites accepting impulses from other cells and the single axon, by which neuron transmits the impulse to other cells. All processes of transmission of irritation from receptors, thinking processes and action control are implemented in living organisms as transmission of nerve impulses between neurons. The connections between neurons are made through special formations – synapses, which affect the formation of impulse in a nervous fiber.

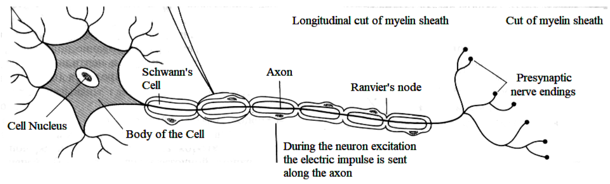

Fig. 1. Neuron structure:
1 – nucleus; 2 – cell body (soma); 3 – Schwann cell; 4 – dendrites; 5 – axon; 6 – node of Ranvier; 7 – presynaptic nerve endings.

A nervous impulse is fast and short depolarization that propagates through the nerve in order to transmit electrical signal to another neural cell, muscle fibre or gland cells. The emergence of a nerve pulse, called the potential of action, is caused by the opening and closing of ion channels in the neuron membranes dependent on potential under the influence of supraliminal stimuli (above threshold stimulation signals). The movement of ions during this process takes place both in the axial direction along the membrane and in the radial direction through the membrane.

The neuron's filaments form nerve fibers which due to their structure and properties can be considered as coaxial conductors. Electrical properties of such conductors are determined by their geometric dimensions and physical parameters of the environment.

Electrical properties of axoplasm (which is the inner medium of the axon) and external environment per unit length is equivalented using resistances R_i and R_o, which are determined by the specific conductances of the relevant media and their geometric dimensions.

The equivalent circuit of the length unit of an exciting membrane is presented in the form of four parallel branches (Fig. 2a).

One of them contains electrical capacity, others represent sodium and potassium conductivity of the membrane, as well as leakage conductivity. The electromotive forces are included into the last three branches. Electromotive force E_l is considered equal to the resting potential, and values E_{Na} and E_K are calculated using the Nernst equation:

$$E = \frac{RT}{F} \ln \frac{[C^+]_o}{[C^+]_i}, \qquad (1)$$

where E is equilibrium potential; $[C^+]_o$, $[C^+]_{in}$ are ion concentrations inside and outside the cell respectively; R is a gas constant, T is absolute temperature, F is the Faraday constant.

For example, for giant squid axon we have E_{Na} = +45 V, E_K = -73 V, E_i = -60 V. For mammal neurons, the EMF will be as follows:
E_{Na} = +66 V, E_K = -86 V, E_i = -80 V.

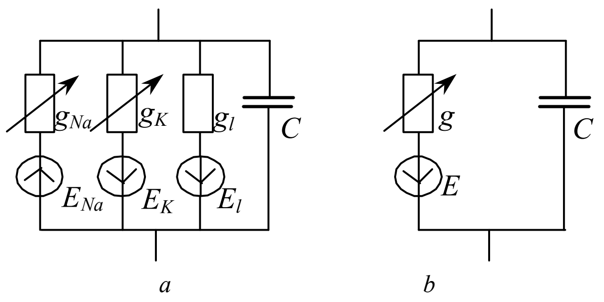

Fig. 2. Equivalent electrical circuit of a cell membrane.

Parameters g_K, g_{Na} are complex dependencies on membrane potential and time (Fig. 3) [1].

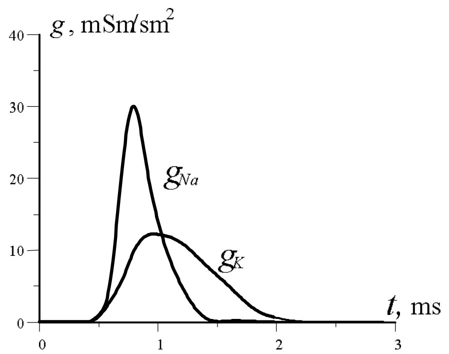

Fig. 3. Ion conductions of neuron membrane during excitation.

The Hodgkin-Huxley differential equation is used to define sodium and potassium conductances [1]:

$$g_{Na} = g_{0Na} m^3 h; \qquad (2)$$

$$\frac{dm}{dt} = \alpha_m (1-m) - \beta_m m; \qquad (3)$$

$$g_{Na} = g_{0Na} m^3 h; \qquad (2)$$

$$\frac{dh}{dt} = \alpha_h (1-h) - \beta_h h; \qquad (4)$$

$$g_K = g_{0K} n^4; \qquad (5)$$

$$\frac{dn}{dt} = \alpha_n (1-n) - \beta_n n, \qquad (6)$$

where g_{Na} and g_K are sodium and potassium membrane conductivities calculated per unit of the membrane area; g_{0Na} and g_{0K} are constants equal to the maximums of conductivities of a unit area; m, h and n are activation variables of sodium and potassium channels dependent on the magnitude of membrane potential, respectively; α_m, β_m, α_n, β_n,, α_h, β_h are constant velocities depending on the membrane potential, temperature and concentration of ions in the external solution, but not depending on time. Variables and constants in equations (2)-(6) are combined functions of membrane potential, temperature and ione concentration [2].

That is why we have proposed much more efficient approximation of nonlinearity by cubic splines under the condition when the value of membrane potential reaches the supraliminal level. Then the conductivity of ion channels can be represented as a function of time in the following form:

$$g_i = a_{i,j}(t-t_j)^3 + b_{i,j}(t-t_j)^2 + c_{i,j}(t-t_j) + d_{i,j}, \qquad (7)$$

where $a_{i,j}$, $b_{i,j}$, $c_{i,j}$, $d_{i,j}$ are polynomial coefficients of i-th conductivity and j-th interval.

After simple calculations, four parallel branches in the electric circuit (Fig. 3a) can be replaced by two equivalent branches (Fig. 3b):

$$g = g_{Na} + g_K + g_l; \qquad (8a)$$

$$E = \frac{E_{Na} g_{Na} + E_K g_K + E_l g_l}{g_{Na} + g_K + g_l}. \qquad (8b)$$

Transferring the action potential in axon or dendrites as an electrophysiological process can be described with the help of such electrical quantities as voltages and currents, which vary along the whole filament. Therefore, nervous fiber is represented as the electric circuit with distributed parameters [4]. The schematic interpretation of this circuit is shown in Fig. 4. In such a circuit voltages and currents are functions of two variables: time t and distance x.

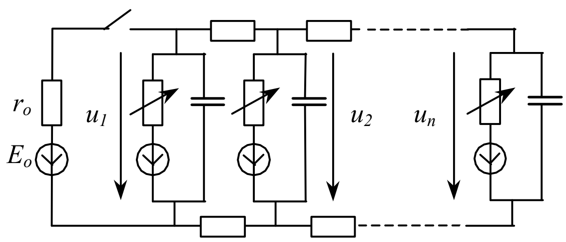

Fig. 4. Equivalent electric circuit of a nerve fibre with distributed parameters

The mathematical model of the electric circuit with distributed parameters can be described with the help of the well-known telegraph equation. In our case they will look like as follows:

$$\begin{aligned} &-\partial u / \partial x = r_0 i; \\ &-\partial i / \partial x = g_0 (u - E) + C_0 \partial u / \partial t, \end{aligned} \tag{9}$$

where u is a voltage between the internal and external parts of a nerve fibre (membrane potential); i is a current along neuron's filament; r_0, g_0, C_0 are per unit length parameters of the electric circuit.

The system of equations (9) consists of the partial differential equation. Besides, taking into account the peculiarities of ionic conductances g_K, g_{Na}, these equations have nonlinear and parametric character. For the numerical solution of these equations the method of lines was used [5].

The method of lines is a finite difference method with respect to one of the arguments. Such substitution of derivatives by one argument converts the partial differential equations with two independent variables to the form of ordinary differential equations. While solving the telegraph equations, this method can be applied to the argument x that is much more accurate than applying it to the argument t, because relative change of the mode coordinates (u, i) along the nerve fiber is smaller than by the time.

After the integration of equations (4) based on the method of lines [4] using discretization by linear coordinate, they will take the following form:

$$\begin{aligned} &u_m - u_{m-1} = r_0 \Delta x i_m; \\ &i_m - i_{m-1} = g_{0,m-1} \Delta x (u_{m-1} - E_{m-1}) + \\ &+ C_0 \Delta x \frac{du_{m-1}}{dt}, \quad m = 1, 2, \ldots, n, \end{aligned} \tag{10}$$

where m is a number of the section on which the entire length of the neuron's filament; $\Delta x = l/n$ is a linear integration step.

After substituting currents from the first equation into the second one (10), we will obtain parametric differential equations with one variable, that is, membrane potential u:

$$\begin{aligned} &(u_{m-1} - E_{m-1}) g_{0,m-1} \Delta x + C_0 \Delta x \frac{du_{m-1}}{dt} = \\ &= \frac{u_m - 2u_{m-1} + u_{m+1}}{r_0 \Delta x}, \quad m = 1, 2, \ldots, n. \end{aligned} \tag{11}$$

To integrate the equation (11) let us use the implicit backward differentiation method where the derivative is approximated by a discrete analogue in the following form:

$$(du/dt)_{k+1} = a_0 h^{-1} u_{k+1} + h^{-1} \sum_{s=1}^{p} a_s u_{k+1-s}\ , \tag{12}$$

where a_0, a_s are coefficients of the method; k is a number of the time integration step, h is the time integration step, p is an order of backward differentiation method.

Taking into account the equation (12), finally we get the mathematical model of the impulse transfer in the neuron's filaments:

$$\begin{aligned} &-\frac{u_{m-1,k}}{r} + u_{m,k} \left(g_{m,k} + \frac{C\alpha_0}{h} + \frac{2}{r} \right) - \frac{u_{m+1,k}}{r} = \\ &= E_{m,k} g_{m,k} + C h^{-1} \sum_{s=1}^{p} a_s u_{m,k-s}; \quad m = 1, 2, \ldots, n, \end{aligned} \tag{13}$$

where $g = g_0 \Delta x$; $r = r_0 \Delta x$; $C = C_0 \Delta x$.

A system of algebraic equations (13) has a string-diagonal form. Therefore, to solve such a system of equations, the modified Gauss method was applied. In this method operations were performed only with nonzero matrix elements.

On the basis of the mathematical model represented by equation (13) the digital model was developed and mathematical experiments were carried out. Some results of computer experiment are shown in Fig. 5 and Fig. 6.

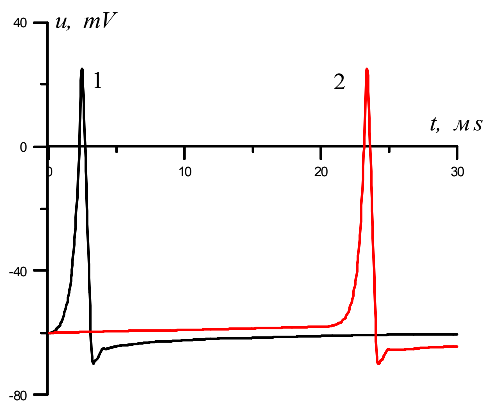

Fig. 5. Variation of the membrane voltage in time in two parts of axon during neuron firing (1 – at distance 2,7 cm; 2 – at distance 37,3 cm far from the firing point).

Mathematical experiments were conducted for the axon of the squid neuron with a diameter of 0.5 mm and a length of 40 cm. This material was chosen due to the fact that in the literature sources devoted to the problem under research [1–3] parameters, characteristics and results of the numerous physical experiments for such a research object can be found.

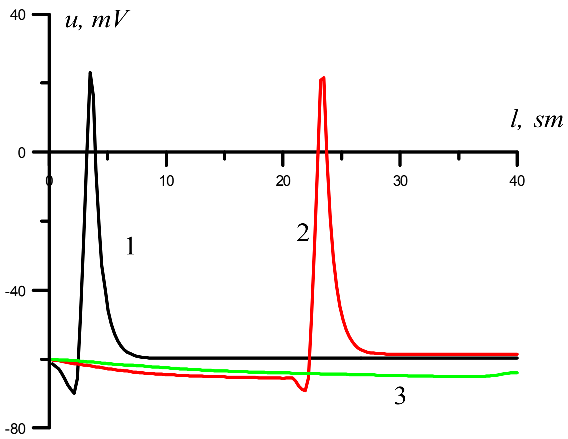

Fig. 6. Distribution of the membran voltage along axon during neuron firing (1 – 3 ms after; 2 – 15 ms after; 3 – 27 ms after neuron firing).

3. Simulation of synaptic transmission

Synapse is the functional connection between the presynaptic ending of the axon or dendrite and the membrane of the postsynaptic cell (Fig. 5). Synapse is an area where nerve impulses can affect the activities of postsynaptic cells, inhibiting or exciting it. There are two types of synapses: in the presynaptic synapse end of the chemical synapses a neurotransmitter is distinguished which generates synaptic potential in postsynaptic cell, while in the electric synapse the excitation of postsynaptic cells is carried out through the electric connection. In Fig. 7 schematic structures of different types of synapse are shown.

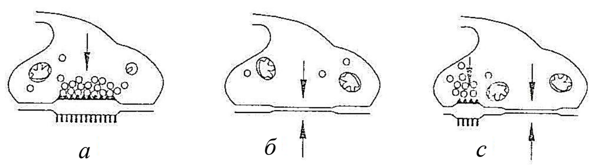

Fig. 7. Chemical and electrical synapse: a – Chemical synapse; b – electric synapse; c – combined type synapse.

In chemical synapse, intercellular connection is conditioned by events occurring in the presynaptic and postsynaptic structures. The presynaptic neuron produces a signal through the controlled release of the neurotransmitter, the postsynaptic cell percepts this signal by receptors localized in the postsynaptic structure. The low molecular organic matters serve as neurotransmitters. Among them there are excitation neurotransmitters (serotonin, aspartate) and inhibiting neurotransmitters (glycine).

In electrical synapse, a slit contact is so small that intercellular communication is carried out by depolarizing and hyperpolarization currents. For the chemical synapse one-way pulse transmission is typical, in opposite to the electric synapse which can be two-way. In addition, chemical synapse is characterized by a synaptic delay, which is the time interval between the emergence of action potential in a nervous end and postsynaptic potential. The total absence of such a delay is typical for electrical synapses.

The cascade connection of two electric circuits with distributed parameters is used to simulate nerve impulse on neurons, taking into account synaptic connections. In the case of the electric synapse, the first circuit output is connected to the second circuit input.

In the case of the chemical synapse an impulse with the delay which is modeled by the *RC* link is applied to the input of the second circuit. Input voltage of the second circuit is determined by the following expression:

$$u_2 = \pm U_1 (1 - e^{1/(RC)t}), \qquad (14)$$

where u_2 is the voltage of the second neuron input; U_1 is action potential at the end of the first neuron; C is a capacity of a second neuron membrane.

The parameter of the resistor R is calculated using the magnitude of a synaptic delay. Sign +/- is determined by a character of the neurotransmitter (exciting or inhibiting).

4. Conclusions

Comparison of the previously obtained simulation results of nerve impulse of neuron's axon with the results of physical experiments carried out by well-known neurophysiologists, allows us to state the adequacy of the proposed models.

The developed models allow taking into account the influence of such factors as geometric, physical and chemical parameters of neuronal processes on the neural impulse transfer and the presence of different neurotransmitters in the chemical synapse. These models can be further used to investigate the conditions of neuron firing in the spatial and time integration of input signals, as well as for the simulation of neuromuscular connections.

References

[1] A. Hodgkin and A. Huxley "A quantitative description of membrane current and its application to conduction and excitation in nerve", *Journal of Physiology*, vol. 117, pp. 500–544, 1952.

[2] I. Tasaki, "Physiology and Electrochemistry of Nerve Fibers", *Academic Press*, 1982.

[3] P. Kostiuk, V. Zyma, V. Mahura et al., *Biophysics*. Kyiv, Oberegy, 2001. (Ukrainian)

[4] V. Horyachko, Kh. Drohomyretska, and M. Kotsyuba, "Mathematical model of action potential propogation in neuron axon", in *Proc. VI International Workshop Computational Problems of Electrical Engineering*, Zakopane, Poland, September 1–4, pp. 137–138, 2004.

[5] V. Horyachko "Mathematical model of the neural impulse propagation in the neuron", *Teoretychna elektrotechnika*, Issue 58, pp. 20–26, Lviv, 2005. (Ukrainian)

[6] E. N. Marieb, Essentials of Human Anatomy and Physiology. 10th Edition, Benjamin Cummings, 2012.

МАТЕМАТИЧНА МОДЕЛЬ ЕЛЕКТРИЧНОЇ АКТИВНОСТІ ДІЛЯНОК БІЛОГІЧНОЇ НЕЙРОННОЇ МЕРЕЖІ

Всеволод Горячко, Оксана Гоголюк, Тарас Рижий, Сергій Рендзіняк

У статті запропоновано математичну модель генерування потенціалу дії та поширення імпульсу у відростках нейрона на підставі аналізу параметричних електричних кіл із розподіленими параметрами та математичну модель синаптичних міжнейронних зв'язків.

Розроблені моделі дозволяють враховувати вплив на проведення нервового імпульсу таких чинників, як геометричні, фізичні та хімічні параметри відростків нейронів та наявність різних медіаторів у хімічних синапсах. Такі моделі можна надалі використати для дослідження умов збудження нейрона при просторовому і часовому інтегруванні вхідних сигналів, а також для моделювання нервово-м'язових з'єднань.

Vsevolod Horyachko – Ph. D., Assoc. Professor graduated from biological faculty of Ivan Franko State University of Lviv and electric power engineering faculty of Lviv Polytechnic Institute, Ukraine. He received his Ph. D. degree in the field of theoretical electrical engineering. Since 1983 he has been working at the Department of Theoretical Electrical Engineering at Lviv Polytechnic National University.

Oksana Hoholyuk – D. Sc., Ass. Professor, graduated from Department of Radiohysics and electronics of Ivan Franko State University of Lviv, Ukraine. She received her Ph. D. and D. Sc. degrees in the field of theoretical electrical engineering. Since 2004 she has been working at the Department of Theoretical Electrical Engineering at Lviv Polytechnic National University, Ukraine. Her scientific interests are focused on mathematical modelling of electric circuits and systems.

Taras Ryzyi – Senior Lecturer graduated from electric power engineering faculty of Lviv Polytechnic Institute, Ukraine. Since 1983 he has been working at the Department of Theoretical Electrical Engineering at Lviv Polytechnic National University.

Serhiy Rendzinyak – Ph. D., D. Sc., Professor graduated from Ivan Franko State University, Lviv, Ukraine as a specialist in the field of Radiophysics and Electronics. He defended his Ph. D. thesis at Ivan Franko State University, Ukraine, and D. Sc. thesis in Theoretical Electrical Engineering at Lviv Polytechnic National University. He worked at Lviv Polytechnic National University as Associate Professor from 1997 to 2008; since 2008 he has held the position of Professor at the Department of Theoretical and General Electrical Engineering, Lviv Polytechnic National University.