

## STEP-CONTINUOUS PHASE POWER CONTROL OF MULTI-SECTION RESONANT INVERTER

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**Abstract:** The method of step-continuous phase power control of a multi-section resonant inverter on the basis of parallel connection of half-bridge inverter sections is proposed. Step power change consists of the commutation of synchronized step-controlled sections and continuous power control at power intervals between commutations is realized by the phase shift of pulses of a continuous-controlled section relative to the pulses of step-controlled sections. Such control provides the increase in inverter efficiency at the medium and low power levels, a wide power control range and constant inverter operation frequency.

**Key words:** multi-section resonant inverter, efficiency, power, step control, continuous control.

### 1. Introduction

Resonant inverters play a significant role in modern power conversion technology of electrical energy. They are widely used in renewable energy sources [1], induction heating systems [2], charging stations for electric vehicles [3], electronic ballasts of discharge light sources and drivers of LED light sources [4, 5], as well as in voltage converters of telecommunication systems, avionics, computer systems, measuring equipment [6], etc. The advantages of resonant inverters over traditional PWM converters are reduced switching losses due to the soft commutation of switches and reduced level of both radiating and conductive high-frequency interference which causes their high efficiency, reliability and electromagnetic compatibility. They have small volume, weight and cost.

### 2. Statement of the problem

Modern high-frequency resonant inverters are mainly realised on transistors using the circuitry of bridge or high-bridge switching circuits. However, the use of a bridge and, what is more, a half-bridge inverter in high-power resonant inverters becomes problematic due to the power limitations of the component basis of power electronics, primarily transistors. Therefore, in powerful inverters, it is advisable to use the parallel connection of half-bridge inverters as separate sections that work on a mutual load and form a multi-section resonant inverter. During in-phase operation of its sections the output power of the resonant inverter is proportional to the number of its

sections. Such multi-section inverters make it possible to create high-power resonant inverters on cheaper and less power transistors and inductors with lower saturation currents and smaller dimensions, because the inverter currents are evenly distributed between its sections.

Relatively few works are devoted to considering of multi-section resonant inverters with continuous power control. Such inverters are based on half-bridge resonant inverter sections. The outputs of each section are connected to a common resonant capacitor via section resonant inductors and coupling capacitors, forming a series resonant circuit. The load of inverter is connected in parallel to the resonant capacitor. In [7] a two-section inverter is considered, in [4] – a four-section inverter, and in [8] – a five-section inverter. The general case of construction of inverters on the basis of arbitrary number of sections and their analysis for the purpose of research of its characteristics is considered in [9]. In these works it is shown that by changing the phase shifts between the output pulses of the half-bridge sections of resonant inverter it is possible to implement continuous phase control of the inverter power in the range from 0 to 100 % ensuring its operation at a constant frequency.

However, although the phase shifts between sections may change in a wide range (from 0 to  $\pi$ ), the section currents change slightly in the inverter with continuous phase power control. This leads to increasing the ratio of reactive power to its active power at medium and low power levels and, therefore, to the decrease in efficiency at these power levels. This is a disadvantage of such continuous control.

The purpose of this work is to develop a method of step-continuous power control of a multi-section resonant inverter made on the basis of half-bridge inverter sections which increases its efficiency at medium and low power while providing a constant operating frequency.

### 3. Theoretical part

The structure of a N-section inverter with step-continuous control is the same as that of the inverter with continuous control [9], but as opposed to [9], this inverter has only one continuously-controlled section and (N-1) identical step-controlled sections (Fig. 1).

The proposed method of step-continuous power control of multi-stages inverter combines step power control through the ON/OFF switch of the step-controlled sections of the inverter, providing discrete power levels and continuous power control by means of the continuous phase shift between this neighboring discrete levels using a continuously-controlled section.

However, when OFF/ON section switching takes place, the inverter resonant frequency changes abruptly, because it depends on the number of its sections [9]. Such a step change in the resonant frequency requires a step change in the operating frequency of the inverter, which impairs its electromagnetic compatibility and complicates the control procedure of the inverter.

To maintain the invariance of inverter operating frequency, it is proposed to include the semi-OFF state of step-controlled half-bridge sections, in which the "upper" transistor of the section (Fig. 1) is in open state but the "lower" transistor is in closed state. As a result, the "upper" transistor will block the energy transmission through this section and "lower" transistor will support including the section inductance into resonant system of the multi-section inverter. Therefore, a step change in the resonant frequency will be eliminated. The power of the inverter is determined by the number of ON-switched sections and the phase shift of the continuously-controlled section. So the currents and switching losses in OFF-switched section at medium and low power will be essentially reduced.

The procedure of inverter control when reducing its power is as follows: 1) if instantaneous power decreases to the level which can be provided by a smaller number of sections, the "excessive" section is turned off; 2) power jump is compensated by corresponding phase jump to zero in continuously-controlled section; 3) further control is provided by the phase shift in the continuously-controlled section. When the inverter power increases, the opposite procedure is applied: the step-controlled section is turned on with corresponding phase jump in the continuously-controlled section.

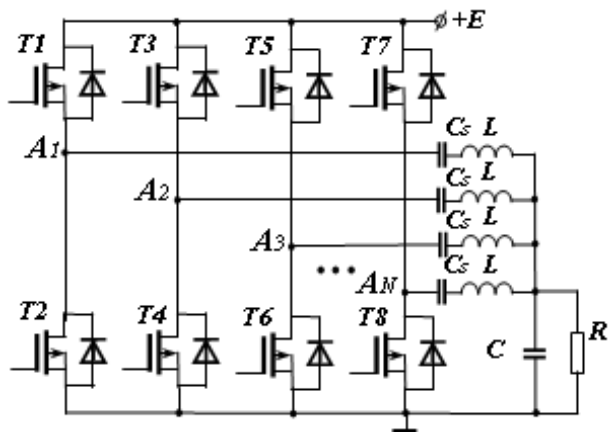


Fig. 1. *N*-section resonant inverter.

Each sections of an *N*-section resonant inverter (Fig. 1) consists of a half-bridge inverter the output of which is connected through a capacitor  $C_s$  and inductance  $L$  to a resonant capacitor  $C$  and load  $R$  connected in parallel. Voltages at points  $A_1, A_2, \dots, A_N$  are periodic rectangular pulses (Fig. 2) with an amplitude equal to the supply voltage  $E$  and a duty-cycle close to 0.5;  $\varphi$  is the phase shift of continuously-controlled section. The number  $k$  of ON step-controlled sections can vary from 1 to  $(N-1)$ , and the number  $z$  of OFF-sections is  $z = N-k-1$  respectively.

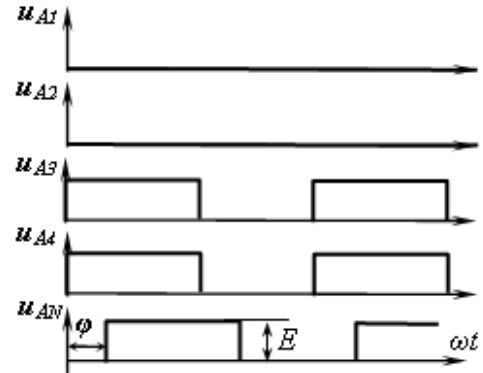


Fig. 2. Example of output section voltages.

Due to the filtering properties of the resonant system, the analysis of the inverter is performed by a fundamental approximation method. An equivalent circuit of the *N*-section resonant inverter is shown in Fig. 3. In this circuit, the ON sections are represented by the sources of sinusoidal voltage.

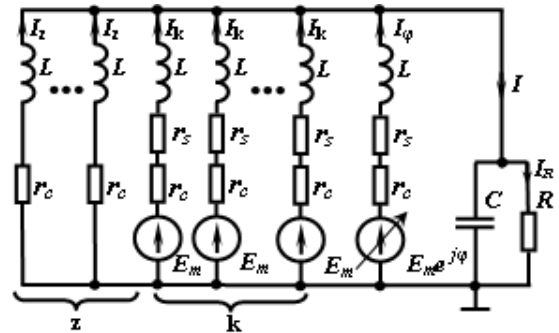


Fig. 3. Simplified circuit of *N*-section resonant inverter.

The complex amplitudes of the sources of these voltages in the ON sections and in the *N*-th continuously-controlled section are respectively equal to:

$$\underline{E}_m = \frac{2E}{\pi}, \quad \underline{E}_N = E_m e^{-j\varphi}. \quad (1)$$

There are no voltage sources in the OFF sections. Conductive losses in each section are taken into account by resistances  $r_c$ , and switching losses are taken into account by resistances  $r_s$  (Fig. 3). Switching losses in OFF sections and losses in capacitors are neglected.

The parameters of the  $N$ -section inverter are shown in Table 1. To calculate other parameters we have used operating frequency  $\omega$  and relative operating frequency

$$\Omega = \frac{\omega}{\omega_0}.$$

Table 1

Parameters of multi-stage inverter

Parallel Resonant Frequency	Parallel Characteristic Impedance	Parallel Quality Factor
$\omega_0 = \sqrt{\frac{N}{LC}}$	$Z_0 = \omega_0 L = \frac{N}{\omega_0 C}$	$Q = \frac{NR}{Z_0}$

The complex impedance of parallel connection of the capacitor  $C$  and the load resistor  $R$  is equal to:

$$\underline{Z} = \frac{1}{j\omega C + \frac{1}{R}} = \frac{QZ_0}{N(1 + j\Omega Q)}. \quad (2)$$

When  $k$  sections are switched on, the phasor of load voltage is represented by the expression:

$$\underline{U} = \frac{\sum_{i=1}^N \underline{E}_i \underline{g}_i}{\sum_{i=1}^N \underline{g}_i + \frac{1}{\underline{Z}}} = \frac{2E}{\pi N} \frac{k + \cos \varphi - j \sin \varphi}{(1 - \Omega^2) + j \frac{\Omega}{Q}}. \quad (3)$$

that makes the voltage transfer function of inverter equal to:

$$M_1 = \frac{|\underline{U}|}{E_m} = \frac{1}{N} \sqrt{\frac{(k + \cos \varphi)^2 + \sin^2 \varphi}{(1 - \Omega^2)^2 + \left(\frac{\Omega}{Q}\right)^2}}, \quad (4)$$

and its maximum value (for  $k = N-1$  and  $\varphi = 0$ ) is equal to:

$$M_{1\max} = \frac{1}{\sqrt{(1 - \Omega^2)^2 + \left(\frac{\Omega}{Q}\right)^2}}. \quad (5)$$

The quality factor which provides the maximum voltage on the load of the inverter with all sections turned ON is determined as follows:

$$Q = \frac{M\Omega}{\sqrt{1 + M^2(1 - \Omega^2)^2}}. \quad (6)$$

Let us determine phasor currents through sections of the inverter.

The phasor current of the  $k$ -th ON step-controlled section is given by

$$\underline{I}_k = \frac{E_m - \underline{U}_{ab}}{j\Omega Z_0} = \frac{2E}{\pi Z_0} \frac{1}{j(1 - \Omega^2) - \frac{\Omega}{Q}} \times \left\{ -\Omega + \frac{1}{N\Omega} (N - k - \cos \varphi) + j \left[ \frac{1}{N\Omega} \sin \varphi + \frac{1}{Q} \right] \right\}, \quad (7)$$

the phasor current of the OFF step-controlled section is express as:

$$\underline{I}_z = -\frac{2E}{\pi Z_0} \frac{1}{N\Omega} \frac{k + \cos \varphi - j \sin \varphi}{j(1 - \Omega^2) - \frac{\Omega}{Q}}, \quad (8)$$

and the phasor current of the continuously-controlled section is

$$\underline{I}_\varphi = \frac{2E}{\pi Z_0} \left\{ \frac{\left( \frac{N-1}{N\Omega} - \Omega \right) \cos \varphi + \frac{1}{Q} \sin \varphi - \frac{k}{n\Omega}}{j(1 - \Omega^2) - \frac{\Omega}{Q}} + \frac{\left( \Omega - \frac{N-1}{N\Omega} \right) \sin \varphi + \frac{1}{Q} \cos \varphi}{(1 - \Omega^2) - \frac{\Omega}{Q}} \right\}, \quad (9)$$

The phasor current through load resistance  $R$  is determined by:

$$\underline{I}_R = \frac{2E}{\pi Z_0} \frac{1}{Q} \frac{k + \cos \varphi - j \sin \varphi}{j(1 - \Omega^2) - \frac{\Omega}{Q}}, \quad (10)$$

According to the expressions (7-10), the modules of determined currents normalized to a factor  $\frac{2E}{\pi Z_0}$  as

functions of phase shift  $\varphi$  for a 5-section inverter ( $N = 5$ ) with one ON-section ( $k = 1$ ) are shown in Fig.4. The inverter parameters are:  $\Omega = 0.5$  and  $Q = 0.5$ . As it can be seen in Fig. 4, the modules of reactive currents  $I_z$  in the OFF sections are significantly smaller than the modules of currents  $I_s$  and  $I_\varphi$  in the ON sections. Therefore, the losses in the inverter are reduced by disabling the redundant sections which causes an increase in its efficiency.

The power of the inverter is

$$P = \frac{\underline{U} \underline{I}_R^*}{2} = \frac{2E^2}{\pi^2 Z_0} \frac{1}{NQ} \frac{(k + \cos \varphi)^2 + \sin^2 \varphi}{(1 - \Omega^2)^2 + \left(\frac{\Omega}{Q}\right)^2}, \quad (11)$$

The maximum inverter power (at  $\varphi=0$ ) is proportional to the number  $N$  of its sections:

$$P_{\max} = \frac{2E^2}{\pi^2 Z_0 Q} \frac{N}{(1 - \Omega^2)^2 + \left(\frac{\Omega}{Q}\right)^2}. \quad (12)$$

The maximum inverter power with  $k$  ON step-controlled sections occurs at  $\varphi = 0$ :

$$P_{k\max} = \frac{2E^2}{\pi^2 Z_0 Q} \frac{(k+1)^2}{N} \frac{1}{(1 - \Omega^2)^2 + \left(\frac{\Omega}{Q}\right)^2}. \quad (13)$$

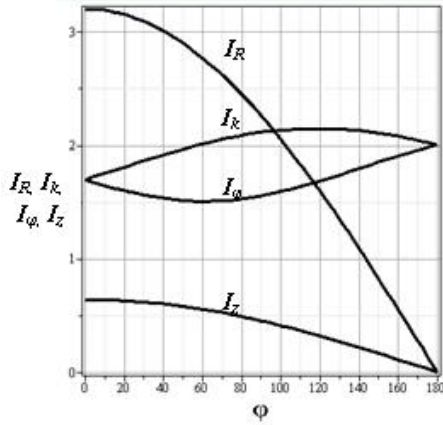


Fig. 4. Modules of currents normalized to  $\frac{2E}{\pi Z_0}$  versus phase shift of a 5-section inverter for  $k=1$ ,  $\Omega=0,5$  ma  $Q=0,5$

The control unit of the inverter must provide on/off section switching when the power reaches the switching points. Switching points are determined from the condition: *the power of the inverter does not change at the switching points of the appropriate sections*. Therefore, before switching ON the next  $k$ -th section, the power of the inverter with  $(k-1)$  sections must reach its maximum value  $P_{(k-1)\max}$  (13). Before turning OFF the excessive section, the power of the inverter with  $k$  section must reach minimum value  $P_{k\min}$ . To do this in the switching points, the phase shift of the continuously-controlled section must reach a zero value ( $\varphi=\varphi_0=0$ ) before switching ON the next section and a value  $\varphi=\varphi_{k\min}$  before turning OFF the excessive section. This condition can be written as follows:

$$P_{k\min} = P_{(k-1)\max}. \quad (14)$$

To determine the phase shift  $\varphi=\varphi_{k\min}$  at the switching points, let us express condition (14), taking into account expressions (12) and (13):

$$(k + \cos \varphi_{k\min})^2 + \sin^2 \varphi_{k\min} = k^2. \quad (15)$$

from which the phase shift can be found:

$$\varphi_{k\min} = \arccos\left(-\frac{1}{2k}\right). \quad (16)$$

The calculated values  $\varphi_{kmi}$  for different switching points of 5-section and 7-section inverters are given in Table 2 and Table 3.

The minimum inverter power with  $k$  switched ON step-controlled sections is determined using (11) at the phase shift  $\varphi_{k\min}$ :

$$P_{k\min} = \frac{2E^2}{\pi^2 Z_0} \frac{1}{NQ} \frac{(k + \cos \varphi_{k\min})^2 + \sin^2 \varphi_{k\min}}{(1 - \Omega^2)^2 + \left(\frac{\Omega}{Q}\right)^2}. \quad (17)$$

Table 2

Values of phase shift  $\varphi_{k\min}$  for 5-section inverter

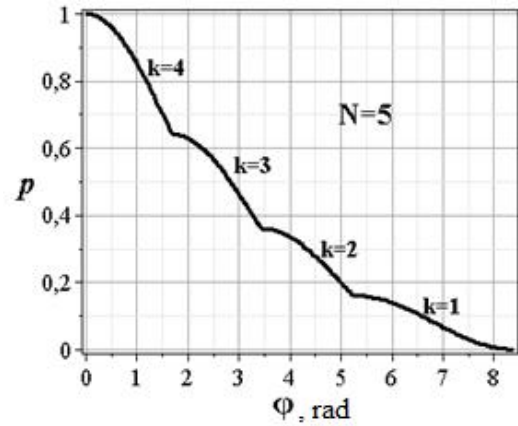
$k$	1	2	3	4	5	6
$\varphi_{k\min}$ , rad	$\pi$	1.823	1.738	1.696	1.671	1.654

Table 3

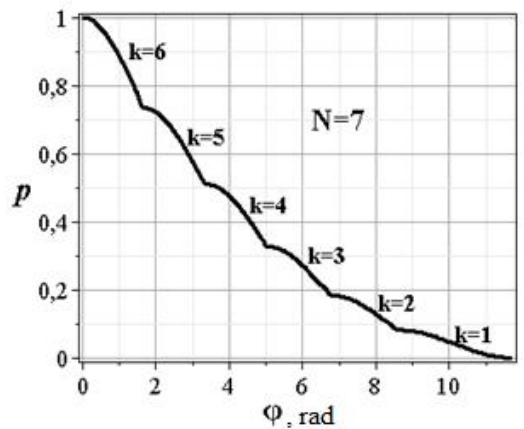
Values of phase shift  $\varphi_{k\min}$  for 7-section inverter

$k$	1	2	3	4
$\varphi_{k\min}$ , rad	$\pi$	1.823	1.738	1.696

According to expression (11) and the data in Tables 2 and 3, the control characteristics of 5- and 7-section inverters with step-continuous control as the dependences of the relative power  $p = \frac{P}{P_{\max}}$  on the total phase shift are calculated and are shown in Fig. 5. The total phase shift in Fig. 5 varies from 0 to  $\sum_{k=1}^{N-1} \varphi_{k\min}$ .



a)



b)

Fig. 5. Relative power of inverters versus phase: a – for 5-section inverter; b – for 7-section inverter

The efficiency of the inverter with step-continuous control is determined by the expression:

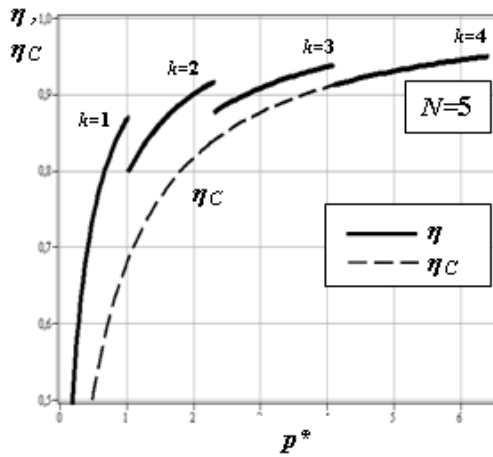
$$\eta = \frac{1}{1 + \frac{I_z^2 r_c (N - k - 1) + (I_\varphi^2 + k I_s^2)(r_s + r_c)}{I_R^2 R}} \quad (18)$$

where  $I_z$ ,  $I_\varphi$ ,  $I_s$ ,  $I_R$  are amplitudes of the corresponding currents.

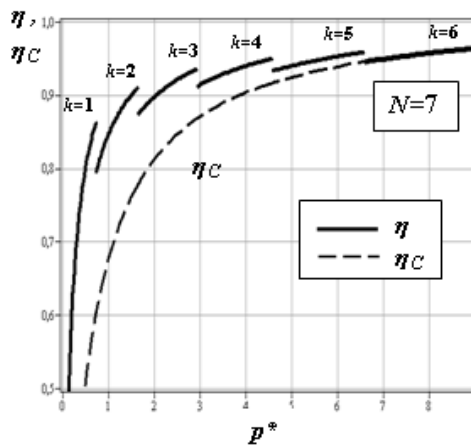
The corresponding efficiency of the N-section inverter-prototype with continuous power control is determined by the expression [4]:

$$\eta_C = \frac{1}{1 + \frac{(r_c + r_s) \sum_{i=1}^N I_i^2}{I_R^2 R}} \quad (19)$$

where  $I_i$  are the amplitudes of the currents in the N-section inverter with the continuous control.



a)



b)

Fig. 6. The plots of efficiency  $\eta$  of step-continuous control and efficiency  $\eta_C$  of continuous control

versus normalized power  $p^* = P \frac{\pi^2 Z_0}{2E^2}$ :

a – for 5-stage inverter; b – for 7-stage inverter.

A comparative analysis of expressions (18) and (19) shows that  $\eta = \eta_C$  in the case of all  $N$  sections switched on and expression (18) is actually reduced to expression (19). In the case of switching off at least one section, the component  $I_z^2 r_c (N - k - 1)$  becomes smaller than the component  $k I_s^2 (r_s + r_c)$  in expression (18). This is due to both the reduction of currents in the switched off sections compared to the currents of the switched on sections and much smaller switching losses in switched off sections.

Being based on expressions (7)–(10) and (18-19) and the data in Tables 2, 3, the efficiency dependences of 5- and 7-section inverters with step-continuous and continuous power control with the parameters  $\Omega = 0.5$  and  $Q = 0.5$  are calculated. The results of the calculations are presented in Fig. 6 as a plots of efficiencies  $\eta$ ,  $\eta_C$  versus

normalized power of the inverter  $p^* = P \frac{\pi^2 Z_0}{2E^2}$ . There,

dotted lines show such dependences for similar inverters with continuous power control.

These dependences show that step-continuous control of the multi-stage inverter provides higher inverter efficiency at lower power levels compared to its continuous control. For example, (Fig. 6), the efficiency at 50 % power level increases approximately from 87 % to 91 % for 5-stage inverter and from 90.5 % to 94.5 % for 7-stage inverter.

When operating at power levels that correspond to inverter switching points, chaotic switching may occur between adjacent power sub-bands, which will degrade the quality of the inverter and generate electromagnetic interference. This shortcoming can be eliminated by introducing hysteresis at switching points. It is possible to realize step-continuous phase control of the inverter by means of a microcontroller control unit.

#### 4. Conclusion

Step-continuous phase power control of multi-section resonant inverters combines step control by switching off the “redundant” step-controlled sections of inverter providing less power level using a smaller number of inverter sections, with continuous control at power intervals between commutations by changing the phase shift of single continuously-controlled section.

The semi-OFF state of a step-controlled half-bridge resonant inverter provides the elimination of the step changes in the inverter resonant frequency at the switching points and invariance of inverter operating frequency. It is caused by the open state of the “upper” transistor of the half-bridge which blocks the energy transmission through this section, while its “lower” transistor is in closed

state and maintains the section inductance included into the resonant system of the multi-section inverter.

The maximum power in the load takes place during in-phase parallel operation of all sections of the multi-section inverter is directly proportional to the number of included sections.

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## СТУПІНЧАТО-НЕПЕРЕРВНЕ ФАЗОВЕ РЕГУЛЮВАННЯ ПОТУЖНОСТІ БАГАТОСЕКЦІЙНОГО РЕЗОНАНСНОГО ІНВЕРТОРА

Анатолій Лупенко

Запропоновано метод ступінчато-неперервного регулювання потужності багатосекційного резонансного інвертора напруги, побудованого на базі напівмостових інверторних секцій. Ступінчата зміна його потужності полягає в комутації синфазно працюючих нерегульованих секцій, а неперервне регулювання на міжкомутаційних інтервалах здійснюється фазовим зсувом імпульсів регульовальної секції відносно імпульсів нерегульованих секцій. Таке регулювання забезпечує підвищений ККД інвертора на середніх та малих потужностях, широкий діапазон регулювання потужності та незмінну робочу частоту інвертора.



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