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P.P. Trokhimchuck
Lesya Ukrayinka Eastern European National University, Ukraine
13, Voly Ave., Lutsk, 43025

POLYMETRIC ANALYSIS AND CYBERNETICS

П.П. Трохимчук
Східноєвропейський національний університет імені Лесі Українки, Україна
пр. Волі, 13, м. Луцьк, 43025

ПОЛІМЕТРИЧНИЙ АНАЛІЗ ТА КІБЕРНЕТИКА

Basic concepts of Polymetric Analysis as universal system of formalization the knowledge are discussed. Correlation between Polymetrric Analysis and cybernetics is researched. It was shown that cybernetics as synthetical science is similar to Polymetric Analysis. But Polymetric Analysis may be represented as functional expansion of computer processor and therefore may be applied for the resolution the problems of artificial intelligence, pattern recognition and other chapters of modern cybernetics. This problem is analyzed with point of S. Beer centurial problem in cybernetics (problem of complexity of information) and has system nature.

Keywords: Polymetric Analysis, artificial intelligence, pattern recognition, variable hierachy, functional numbers

Розглянуто основні поняття поліметричного аналізу як універсальної системи формалізації знань. Досліджено взаємозв'язок поліметричного аналізу та кібернетики. Було показано, що кібернетика як синтетична наука схожа на поліметричний аналіз. Але поліметричний аналіз може бути представлений як функціональне розширення комп'ютерних процесорів, і тому може бути застосований для розв'язання проблем штучного інтелекту, розпізнавання образів та інших розділів сучасної кібернетики. Ця проблема проаналізована з урахуванням проблеми століття в кібернетиці С. Біра (проблема складності інформації) і має системний характер.

Ключові слова: поліметричний аналіз, штучний інтелект, розпізнавання образів, змінна ієрархія, функціональні числа

Introduction

The problem of creation the universal system of knowledge has long history [1, 2]. Firstly this problem was connected with creation of esoteric ritual system [1, 2]: Egyptian (Table of God Thot, Indian (Patanjali a. o.), Mexican (philosophy of Nagua), Jevish (decades of sephirotes). Creation the concept of modern science was beginning from Pythagor, Euclides, Plato and Aristotle. Later Pythagorean line (the synthesis) was developed by Descartes. He begin to realize the formalization of thesis "Science is science as science as it is mathematics" or Pythagorean phrase "Numbers are ruling of the world" [1]. His method was optimize by I. Newton (four rules of conclusions in physics) [1]. This method is base in the creation of modern science [1]. But Newtonian rules are rules of selection and creation of concrete science. Development of modern science shows the necessity of creation more universal sciences [1-8].

More fully realization of this idea is Polymetric Analysis (PA): theory of variable measure or theory of systems with variable hierarchy. We can say that this system was created for the formalization L. Hall phrase "All, that go from head, is intelligent".

According to [6] cybernetics is synthesis of many sciences (mathematics, physics, biology, psychology and other. Therefore methods of PA as universal theory of optimal formalized synthesis may be used for the resolutions the main cybernetical problems. Structure of PA may be represented as more deep formalization the neuronets [1].

Therefore the main problem of our paper is ascertainment of question about possible application of PA for the resolution the problems of cybernetics, including general problems (S. Beer centurial problem, problem of complexity [5]) and particular (matrix calculations, arrays sorting, pattern recognition) [1].

According to F. George "The brain is universal computer". Therefore PA as universal system formalization of knowledge may

be used for the resolution the basic problems of natural and artificial intelligence too [9, 10].

Main results and discussions

Polymetric analysis was created as alternative optimal concept to logical, formal and constructive conceptions of modern mathematics and theory of information [1]. This concept is based on the idea of triple minimum [1].

Basic elements of this theory and their bonds with other science are represented in Figure 1 [1, 2].

Basic mathematical element of polymetric analysis is functional number (generalizing elements of quadratic forms) [1]. As in Greece mathematics number is basic elements of its system. For these numbers generalizing mathematical transformations were constructed. 15 minimal types of its transformations are existed. Informative lattice is constructed on the basis if functional numbers and generalizing mathematical transformations. Theory of informative calculations is created for this lattice. Basic principle of this theory is the principle of optimal calculations.

For classification of systems of calculation hybrid theory of systems was created. This theory is based on two criterions: criterion of reciprocity – principle of creation of proper formal system, and criterion of simplicity – principle of optimality of this creation. For "inner" bond of two elements of informative lattice a parameter of connectedness was introduced. Principle of optimal informative calculation is included in criterion of simplicity. Only 10 minimal types of hybrid systems are existed [1]. But four types of these systems aren't mathematical in classical sense [1]. Hybrid theory of systems is open theory. Parameters of openness are number of generalizing mathematical transformations and parameter of connectedness. Thereby we have finite number of types of systems, but number of systems may be infinite. Hybrid theory of systems allows considering verbal and nonverbal knowledge with one point of view [1]. Roughly speaking this theory may be represented as variant of creatin variant of universal cybernertics [1-8] and resolution S. Beer centurial problem in cybernetics [1, 5].

Mathematical constructive element may be represented as generalizing knot of informative lattice. Generalizing mathematical transformations are classified as quantitative and qualitative, left and right. Calculative (quantitative) transformations are corresponded to primary measurement and qualitative transformations – to derived (secondary) measurements. It allows formalizing N.R.Campbell concept [1, 2] about primary and derived measurements. Result of this formalization was named polymetric theory of measure and measurement. Basic principles of this theory are principle of asymmetry of measurement for calculative transformations and principle of dimensional homogeneity. This theory is optimal synthesis of all famous theories of measure and measurements and dimensional analysis [1]. N.R. Campbell concept is more general as "measuring" part of quantum mechanics. Therefore L.I. Mandelstam called Quantum Mechanics as science of derivative measurements [1].

Polymetric analysis is the system of optimal formalization, synthesis and analysis of knowledge. But it is the nature of mathematics [1]. For creation of theory of foundations of mathematics we must include three aspects: synthesis, analysis and formalization. This theory must be open system. Therefore Russel – Whitehead "logic" concept, Hilbert – Bernayce "formal" concept and Brauer – Heiting "constructive" concept can't be full theories of foundations of mathematics [1]. It was cause of crisis in theory of foundations of mathematics. Therefore A.N. Whitehead made conclusion that logical concept can't be the theory of foundations of mathematics [1, 4]. But it must be "organismic" theory. Practically this concept was realized in cybernetics: theory of neuronets, systolic computers, theory of cellular automata a.o. [1]. Therefore polymetric analysis may be represented as variant of realization of Whitehead concept of "organism" mathematics [4] and formalizing unification of proper cybernetic theories.

Polymetric analysis may be represented as optimal "dynamical' formalization of Errol E. Harris polyphasic concept of modern science [3].

Thus basic concepts of awakening, creation and development of synthesis with including of historical analysis of this problem are represented in [1]. Was shown, that polymetric analysis is the necessary development of problem of formalized synthesis in modern science.

The basic axiomatic of the PA is next [1].

Definition 1. **Mathematical construction** is called set all possible elements, operations and transformations for resolution corresponding problem. The basic functional elements of this construction are called constructive elements.

Definition 2. The mathematical constructive elements $N_{x_{ij}}$ are called **the functional parameters**

$$N_{x_{ij}} = x_i \cdot \overline{x}_j, \qquad (1)$$

where x_i , $\overline{x_j}$ - the straight and opposite parameters, respectively; · - respective mathematical operation.

Definition 3. The mathematical constructive elements $N_{\varphi_{ij}}$ are called the **functional numbers**

$$N_{\varphi_{ij}} = \varphi_i \circ \overline{\varphi}_j . \qquad (2)$$

Where $\varphi_i\left(x_1,...x_n,\overline{x_1}...\overline{x_m},...N_{x_{ij}},\ldots\right)$, $\overline{\varphi_j}\left(x_1,...x_n,\overline{x_1}...\overline{x_m},...N_{x_{ij}},...\right)$ are the straight and opposite functions, respectively; ∘ – respective mathematical operation.

Remark 1. Functions φ_I , $\overline{\varphi}_j$ may be have different nature: mathematical, linguistic and other.

Definition 4. The mathematical constructive elements $N^d_{x_{ij}}$ are called the **diagonal functional parameters**

$$N^d_{x_{ij}} = \delta_{ij} N_{x_{ij}} . \qquad (3)$$

Where δ_{ij} is Cronecker symbol.

Definition 5. The mathematical constructive elements $N^d_{\varphi_{ij}}$ are called the **diagonal functional numbers**

$$N^d_{\varphi_{ij}} = \delta_{ij} N_{\varphi_{ij}} . \qquad (4)$$

Example 1. If $x_i = x^i$, $\overline{x}_j = x^{-j}$ and $\max\{i\} = \max\{j\} = m$, then $\{N^d_{\varphi_{ij}}\}$ is diagonal single matrix.

The another examples may be the orthogonal eigenfunctions of the Hermitian operator.

Remark 2. These two examples illustrate why quantities (1), (2) are called the parameters and numbers. The straight functions correspond the "straight" observation and measurement and opposite functions correspond the "opposite" observation and measurement. This procedure is included in quantum mechanics the Hilbert's spaces and Hermitian operators.

The theory of generalizing mathematical transformations is created for works on functional numbers [1].

Definition 6. **Qualitative transformations** on functional numbers $N_{\varphi_{ij}}$ (straight A_i and opposite $\overline{A}_j$) are called the next transformations. The straight qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on i units for straight parameters, and the opposite qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on j units for opposite parameters.

Definition 7. **Quantitative** (**calculative) transformations** on functional numbers $N_{\varphi_{ij}}$ (straight O_k and opposite $\overline{O}_p$) are called the next transformations. The straight calculative transformations are reduced $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on k units its measure. The opposite quantitative transformations are increased $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on l units its measure, i.e.

$$O_k \overline{O}_l N_{\varphi_{ij}} = N_{\varphi_{ij}} - k \oplus l . \qquad (5)$$

Definition 8. **Left and right transformations** are called transformations which act on left or right part of functional number respectively.

Definition 9. The maximal possible number corresponding transformations is called **the rang of this transformation**

$$rang(A_i \overline{A}_j N_{\varphi_{ij}}) = \max(i, j); \quad (6)$$

$$rang(O_k \overline{O}_p N_{\varphi_{ij}}) = \max(k, p). \quad (7)$$

Remark 3. The indexes *i,j, k,p* are called **the steps of the corresponding transformations.**

Only 15 types of generalizing mathematical transformations are existed [1, 2].

Basic elements of PA is the generalizing mathematical elements or its various presentations – informative knots [1, 2]. Generalizing mathematical element is the composition of functional numbers (generalizing quadratic forms, including complex numbers and functions) and generalizing mathematical transformations, which are acted on these functional numbers in whole or its elements [1]. Roughly speaking these elements are elements of functional matrixes.

This element ${}^{stqo}_{nmab}M_{ijkp}$ may be represented in next form

$${}^{stqo}_{nmab}M_{ijkp} = A_i \overline{A}_j O_k \overline{O}_p A^r_s \overline{A}^r_t O^r_q \overline{O}^r_o A^l_n \overline{A}^l_m O^l_a \overline{O}^l_b N_{\varphi_{ij}}. \quad (8)$$

Where $N_{\varphi_{ij}}$ – functional number; $O_k, O^r_q, O^l_a, \overline{O}_p, \overline{O}^r_o, \overline{O}^l_b$; $A_i, A^r_s, A^l_n, \overline{A}_j, \overline{A}^r_t, \overline{A}^l_m$ are quantitative and qualitative, straight and inverse (with tilde), (r) – right and (l) – left transformations.

Polyfunctional matrix, which is constructed on elements (17) is called informative lattice. For this case generalizing mathematical element was called knot of informative lattice [1]. Informative lattice is basic set of theory of informative calculations. This theory was constructed analogously to the analytical mechanics [1].

Basic elements of this theory are [1, 2]:

1. Informative computability *C* is number of possible mathematical operations, which are required for the resolution of proper problem.
2. Technical informative computability $C_t = C\sum t_i$, where t_i – realization time of proper computation.
3. Generalizing technical informative computability $C_{t0} = k_{ac}C_t$, where k_{ac} – a coefficient of algorithmic complexity [1].

Basic principle of this theory is **the principle of optimal informative calculations** [1]: any algebraic, including constructive, informative problem has optimal resolution for minimum informative computability *C*, technical informative computability C_t or generalizing technical informative computability C_{to}.

The principle of optimal informative calculations is analogous to action and entropy (second law of thermodynamics) principles in physics.

The principle of optimal informative calculation is more general than **negentropic principle the theory of the information** and **Shennon theorem** [1]. This principle is law of the open systems or systems with variable hierarchy. The negenthropic principle and Shennon theorem are the principles of systems with constant hierarchy.

Idea of this principle of optimal informative calculation may be explained on the basis de Broglie formula [1, 2]

$$S_a / \hbar = S_e / k_B \quad (9)$$

(equivalence of quantity of ordered and disorder information) [1, 2]. Where S_a – action, $\hbar$ – Planck constant, S_e – entropy and k_B – Boltzman constant. Therefore we can go from dimensional quantities (action and entropy) to undimensional quantity – number of proper quanta or after generalization to number of mathematical operations. Thus, theory of informative calculations may be represented as numerical generalization of classical theory of information.

For classification the computations on informative lattices hybrid theory of systems was created [1]. This theory allow to analyze proper system with point of view of its complexity,

The basic principles of hybrid theory of systems are next: 1) **the criterion of reciprocity;** 2) **the criterion of simplicity**.

The criterion of reciprocity is the principle of the creation the corresponding mathematical constructive system (informative lattice). The criterion of simplicity is the principle the optimization of this creation.

The basic axiomatic of hybrid theory of systems is represented below.

Definition 10. The set of functional numbers and generalizing transformations together with principles reciprocity and simplicity (informative lattice) is called **the hybrid theory of systems** (in more narrow sense the criterion of the reciprocity and principle of optimal informative calculations).

Criterion of the reciprocity for corresponding systems is signed the conservation in these systems the next categories: (1) the completeness; (2) the equilibrium; (3) the equality of the number epistemological equivalent known and unknown knotions.

Criterion of the simplicity for corresponding systems is signed the conservation in these systems the next categories: (1) the completeness; (2) the equilibrium; (3) the principle of the optimal calculative transformations.

Criterion of reciprocity is the principle of creation of proper informative lattice. Basic elements of principle reciprocity are various nuances of completeness. Criterion of the simplicity is the principle of the optimality of this creation.

For more full formalization the all famous regions of knowledge and science the **parameter of connectedness** σ_t was introduced. This parameter is meant the number of different bounds the one element of mathematical construction with other elements of this construction. For example, in classic mathematics $\sigma_t = 1$, in linguistics and semiotics $\sigma_t > 1$. The parameter of connectedness is the basic element for synthesis in one system of formalization the all famous regions of knowledge and science. It is one of the basic elements for creation the theory of functional logical automata too.

At help the criteria of reciprocity and simplicity and parameter of connectednesss the basic famous parts of knowledge and science may be represent as next 10 types of hybrid systems [1, 12, 13]:

1. The system with conservation all positions the criteria of reciprocity and simplicity for all elements of mathematical construction ($N_{\varphi_{ij}}$ and transformations) is called the **simple system**.
2. The system with conservation the criterion of simplicity only for $N_{\varphi_{ij}}$ is called the **parametric simple system**.
   Remark 4. Further in this classification reminder of criteria of reciprocity and simplicity is absented. It mean that these criteria for next types of hybrid systems are true.
3. The system with conservation the criterion of simplicity only for general mathematical transformations is called **functional simple system**.
4. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t = 1$ is called the **semisimple system**.
5. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ and with $\sigma_t = 1$ is called the **parametric semisimple system**.
6. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t = 1$ is called the **functional semisimple system**.
7. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t \neq 1$ is called **complicated system**.
8. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ is called **parametric complicated system.**
9. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t \neq 1$ is called **functional complicated system**.
10. The system with nonconservation the criteriums of reciprocity and simplicity and

with $\sigma_t \neq 1$ is called **absolute complicated system**.

With taking into account 15 basic types of generalized mathematical transformations we have 150 types of hybrid systems; practically 150 types the formalization and modeling.

In comparison with other theories of systems the hybrid theory of system has finite number of types of systems. But number of systems may be infinite. This theory is the theory of open system or theory system with variable hierarchy.

Generalizing mathematical elements (17) may be represented as the elements of variable polymetric measure, which are included the procedure of measurements [1], which are corresponded N. R. Campbell concept of measurement [1, 2]. Quantitative transformations are corresponded to primary measurement; qualitative transformations – to secondary (derivative) measurement.

Theory, which are based on generalizing mathematical elements (8), was called polymetric theory of measure and measurements [1].

The basic principles of the polymetric theory of measure and measurements are the principle of the asymmetry the measurement and the principle of dimensional homogeneously. In operational representation it has the next kind [1].

Principle of the asymmetry the measurement. When process of measurement may be represent in form (8) then

$$\begin{aligned} &|k - p| \geq 1; \\ &|q - 0| \geq 1; \\ &|a - b| \geq 1. \end{aligned} \qquad (10)$$

Strictly speaking sufficiently that one of formulas (10) was true.

Principle of the dimensional homogeneously. When process of measurement may be represent in form (8) then the next correlation for definition of dimensions the measurement must be true

$$M_{ijkp} = \delta_{ij}\left(A_i, \overline{A_j}\right)\delta_{st}\left(A_s^r, \overline{A_t^r}\right)\delta_{mn}\left(A_m^l, \overline{A_n^l}\right)\circ \\ \circ O_k \overline{O_p} O_q^r \overline{O_\theta^r} O_a^l \overline{O_b^l} N_{\varphi_{ij}} \qquad (11)$$

where $\delta_{ij}\left(A_i, \overline{A_j}\right), \delta_{st}\left(A_s^r, \overline{A_t^r}\right), \delta_{mn}\left(A_m^l, \overline{A_n^l}\right)$ are the corresponding Cronecker's symbols.

The formula (11) may be represent in more simple form

$$M_{ijkp} = \delta_{i+s+n,\, j+t+m}\left(A_i, \overline{A_j}, A_s^r, \overline{A_t^r}, A_m^l, \overline{A_n^l}\right)\circ \\ \circ O_k \overline{O_p} O_q^r \overline{O_\theta^r} O_a^l \overline{O_b^l} N_{\varphi_{ij}}. \qquad (12)$$

where $\delta_{i+s+n,\, j+t+m}\left(A_i, \overline{A_j}, A_s^r, \overline{A_t^r}, A_m^l, \overline{A_n^l}\right)$ is the generalized Cronecker's symbol.

Practically the principle of asymmetry the measurement is the principle of primary measurements while the principle of dimensional homogeneously is the principle of derivative measurements.

The basic processes of measurement in more general sense (measurement and mathematical modeling and prognostication) are included in constructive mathematical element (8).

Methods of polymetric analysis were used for the decoding of Pythagorean civilization, of VI – V B.C., which was open by German archeologists in Mediterranean in 1980–1984; and for decoding of mathematical and linguistically part of Table of God Thot (Egyptian mythology). Multiplicative (operative) mathematical system was used for these cases. Zero and infinity weren't important elements of these systems [1].

Ways of the application PA for the resolutions AI problems may be next: a search and construction the new systems with various step of complexity.

One of basic applications of PA (theory of informative calculations) in cybernetics are problems of matrix calculations and array sorting [1].

PA may be used for the problems of pattern recognition two [1].

Roughly speaking methods of PA may be used for the resolutions the various complex problems, where we must use the variable measure and variable hierarchy.

Conclusions

1. Necessity the creation of PA is discussed.
2. Basic concepts of PA are represented.
3. The bond between PA and cybernetics is observed.
4. Present and future possible applications of PA for resolutions various problems of modern cybernetics, including AI, are discussed too.

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РЕЗЮМЕ

П.П. Трохимчук

Поліметричний аналіз та кібернетика

Обговорюються основні поняття поліметричного аналізу як універсальної системи формалізації знань. Цей розділ сучасної науки був створений як відповідь на запитання про створення універсальної теоретичної концепції інформатики та варіант створення натурального підходу основ математики. Для створення цього методу була реалізована ідея потрійного мінімуму. Основними елементами ПА є функціональні числа (узагальнені квадратичні форми), узагальнені математичні перетворення (15 мінімальних типів), узагальнені конструктивні математичні елементи, інформаційна решітка, параметр зв'язності, гібридна теорія систем (існує лише 10 мінімальних типів систем формалізації знань), поліметрична теорія міри та вимірювань. Ця теорія була використана для класифікації сучасної науки з точки зору її простоти-складності. Досліджуються шляхи можливого застосування цього методу для розв'язання проблем штучного інтелекту та розпізнавання образів, включаючи розшифровку цивілізацій та міфологію. Також цю проблему проаналізовано з точки зору проблеми століття в кібернетиці С. Біра (проблеми складності).

Наведені приклади використання ПА в різних галузях кібернетики (теорія обчислень, сортування масивів та розпізнавання образів). Зроблено висновок про доцільність застосування ПА для розв'язання проблем штучного інтелекту.

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