УДК 669.15.26.74-196

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Stress state of multilayered hollow cylinder under internal and external pressure

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Анализ напряжённого состояния многослойного полого цилиндра под действием внутреннего и внешнего давления

This paper proposes a new method by using the program of MathCAD-14 in order to determine the elastic constants of anisotropic material, which consists of a set of reinforced layers. There were reviewed ten variants of multilayer anisotropic hollow cylinders with different structure reinforcement. For each variant of reinforcement there was determined the stress state of the hollow cylinder under the action of internal and external pressure. It was revealed that tangential stress, normal stress and shear stress decrease with increasing thickness of the hollow cylinder.

***Keywords**: composite material, elastic coefficients, stress state, multi-layered cylinder.*

Предложен новый метод с использованием программы MathCAD-14 для определения упругих констант анизотропного материала, который состоит из множества армированных слоев. Рассмотрены десять вариантов многослойных анизотропных полых цилиндров с различной структурой армирования. Для каждого варианта определено упрочнение напряженным состоянием полого цилиндра под действием внутреннего и внешнего давления. Определены касательное, нормальное напряжение и напряжение сдвига, уменьшение напряжения при увеличении толщины полого цилиндра.

***Ключевые слова**: композиционный материал, упругие коэффициенты, напряженное состояние, многослойный цилиндр.*

Пропонується новий метод використання програми MathCAD-14 для визначення пружних констант анізотропного матеріалу, який складається з безлічі армованих шарів. Розглянуті десять варіантів багатошарових анізотропних порожнистих циліндрів з різною структурою армування. Для кожного варіанту визначено зміцнення напруженим станом порожнистого циліндра під дією внутрішнього і зовнішнього тисків. Визначено дотична, нормальна напруга і напруга зсуву, зменшення напруги при збільшенні товщини порожнистого циліндра.

***Ключові слова**: композиційний матеріал, пружні коефіцієнти, напружений стан, багатошаровий циліндр.*

Introduction

There is a great need in the high pressure gas cylinder for compressor, power and aerospace industry in order to find materials that minimize the mass of machines, spacecraft, rocket, missiles and launch vehicle structures. For the last 25 years, the development of fiber-reinforced composite systems has led to the manufacture of such structures, but their development has presented the new design challenges over traditional metallic structures. One design challenge is the determination of the stress-rupture life of the composite. This lifetime is the amount of time a composite can withstand a constant applied tensile load without breaking. While stress-rupture failures occur in structures that are exposed to constant tensile stress for long periods of time, structures for use on long-term space missions are particularly susceptible to stress-rupture failures.

Designing constructions of composite materials, one meets a great number of possible versions and schemes for reinforcement. Therefore, a theoretical problem to determine optimal deformation and strength properties of such materials in a combination with a minimum cost of experiment seems to be urgent.

In a composite material with a regular structure, as a rule, repeated elements are present in the form of single-directed layers. Neglecting a structural non-uniformity at the micro-level of each layer, one could find efficient characteristics of individual layers at the macro-level. In this case, a material deformation model would have a quasi-uniform structure composed of single-directed layers with various angles of arrangement [1-3].

Analysis of different approaches [4-6] to the calculation of elastic characteristics of a composite material demonstrated that a correct evaluation of an effect of reinforcement arrangement schemes on physical and mechanical characteristics of a material could be derived by solving boundary-value problems of an elasticity theory for a multiply-connected region. However, such calculation cannot exclude errors conditioned by deviation of a real material structure from its idealized model and is associated with a laborious numerical analysis.

A principle of summation of the repeated elemental layers serves as a basis for an approximated calculation of elastic characteristics of composite materials. The elastic characteristics of an elemental layer, as a rule, can be determined in two stages. First, one should find the characteristics of a reduced matrix by averaging elastic properties of fibers of an orthogonal-reinforced material layer. It is assumed that material components (fiber and matrix) are isotropic, linearly elastic, and work jointly at all deformation stages. In addition, the following assumptions were accepted: the stresses, which were perpendicular to fibers, when a normal load was acting along the fibers, were not taken into account; lateral strains occurring under tension-compression of every component were proportional to its volume content in a matrix; consideration of stress concentration at a fiber-matrix interface should be excluded. At the second stage, calculation of the layer characteristics was performed on the basis of elastic properties of fibers and the modified matrix.

Theoretical bases of calculation of elastic characteristics of multilayer materials

For orthotropic material calculated according to the elastic properties of high-modulus fiber-reinforced layer.

Matrix stiffness and pliability K-rank orthotropic layer in the direction of symmetry axes 1′, 2′, 3′ respectively fig 1.

If the thin-walled element consists of an unidirectional reinforced layers, the axis of local coordinate systems that do not coincide with the axes of the global coordinate system, that is, for example, in cross-reinforced shells, it is possible to vary the material properties due to the angle of reinforcement

The first index of Poisson's coefficient indicates a direction of load application; the second one demonstrates a direction of the lateral deformation, which was induced by this force. On the basis of the proposed algorithm, using an applied packet of PC MATHCAD-14 programs, we obtained numerical values of elastic characteristics of the reinforced material. A carbon-plastic [8, 9] composed of 31 layers with a code $[0^\circ_2/90^\circ/0^\circ_2/\pm 45^\circ/(0^\circ_2/90)_2/\pm 45^\circ/\bar{0}^\circ]_S$, and a glass-plastic with a longitudinal-transversal scheme of arrangement of 19 single layers $[(0^\circ/90^\circ)_S/\bar{0}^\circ]_S$ were used as an example for a calculation of elastic characteristics of a cross-reinforced material.

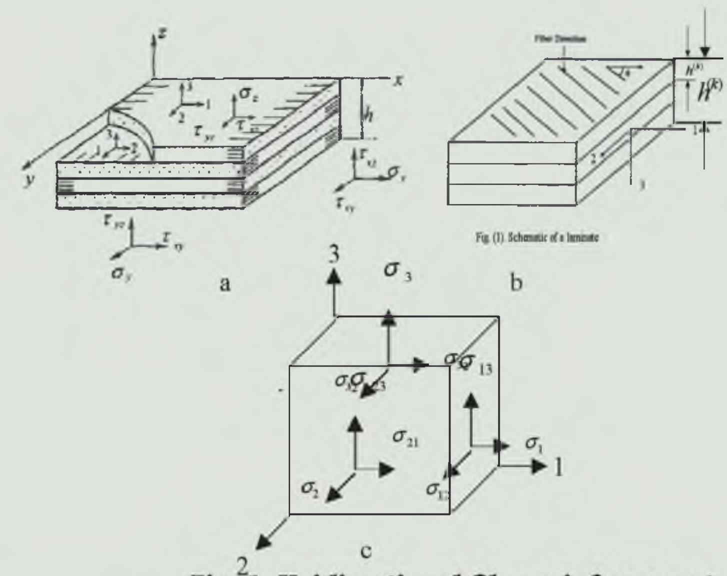

Fig. 1. Unidirectional fiber reinforcement scheme

Properties of component compositions:

Carbon-Plastic. According to the certificate data, an elastic modulus $E_в$, a shear $G_в$, and Poisson's coefficient $\nu_в$ for the carbon fiber LU-03 were equal to 235000 MPa, 90400 MPa and 0,3 respectively. The mechanical characteristics of the carbon-plastic binder (with three phenol epoxy resin and aniline formaldehyde resins) were the following: $E_м$=350 MPa, $G_м$=1320 MPa, $\nu_м$=0,32. In every mono-layer with thickness 0,171 mm, a volume occupied by fibers was 55% of the total volume.

Glass-Plastic. An epoxy polymer 5-211B having the following parameters $E_м$=4200 MPa, $G_м$=1500 MPa, $\nu_м$=0,4 was used as a matrix of the glass plastic. A tissue of a satin structure T-10-80 was used as a reinforcing element. The tissue thickness was 0,25mm. Its base density was 36 filaments/cm and 20 filaments/cm for the bundle. The tissue was fabricated from weaved aluminum-boron-silicate fibers БС6-26×1×1(E glass). The fiber diameter was 6 x 10-3mm. The fiber mechanical characteristics were $E_в$=74800 MPa, $G_в$=31000 MPa, $\nu_м$=0,2. An amount of fibers in one stream was 800. Results of calculations performed by the authors [8, 9] demonstrated that the elastic modulus of the filament was equal to 74506 MPa, the shear modulus and Poisson's coefficient of the filament were assumed such as they were for the fibers.

Technical constants of elasticity of the considered multilayer composites obtained on the basis of the relationships (1)-(16) are summarized in Table 1.

In this case, the glass plastic(fiberglass) had to represent a traversal isotropic material, which

Table 1. ***Elastic characteristics of carbon plastic and fiberglass***

Material	E_{ii}, MPa	G_{ij}, MPa	ν_{ij}, MPa	ν_{ji}, MPa
CFRP	E_{11}=84457 E_{22}=42026 E_{33}=14703	G_{12}=12410 G_{13}=4287 G_{23}=3677	ν_{12}=0,21 ν_{13}=0,28 ν_{23}=0,3	ν_{21}=0,11 ν_{31}=0,049 ν_{32}=0,1
GRP	E_{11}=24260 E_{22}=24260 E_{33}=9989	G_{12}=4254 G_{13}=2947 G_{23}=2947	ν_{12}=0,15 ν_{13}=0,42 ν_{23}=0,42	ν_{21}=0,15 ν_{31}=0,17 ν_{32}=0,17

was composed of 19 single-directed bases corresponding to experimentally determined value of the elastic modulus $E_{11}^{э}$. A relative volume content of the layer reinforcement towards the axis 3 direction was assumed to be $\psi_3^{(k)} = 0.05\psi_1^{(k)}$. A comparison of results presented in Table 1 with those obtained by the authors of [8, 9] confirmed that the technique, used to determine averaged technical parameters of the multi-layered composite, was correct. Physical and mechanical characteristics for the transversal shear and reduction G_{13}, G_{23}, E_{33}, ν_{13}, ν_{23} were ruled out.

Stressed State of Multi-Layered Cylinders under Action of Internal and external Pressure

If the hollow cylinder considered in the preceding section is rigidly fixed at its ends and is deformed by pressure P and q distributed uniform over the inner, middle and outer surfaces, $r_i = a$, $r_m = a + \dfrac{b-a}{2}$, and $r_o = b$ it is in a state of generalized plane strain (Fig. 2).

The solution to the problem for a type of material with cylindrical anisotropy of a special kind was obtained by [10].

On the basis of the presented calculation models and techniques, which were developed for calculation of such class of problems, the stressed state of the carbon plastic cylinder with the internal surface radius r_1=0,1m was studied. The shell was fabricated by reeling up a single-directed glass strip. As a whole, such cylindrical shell was composed of 31 single-directed layers. The reeling angle of every layer was deter-

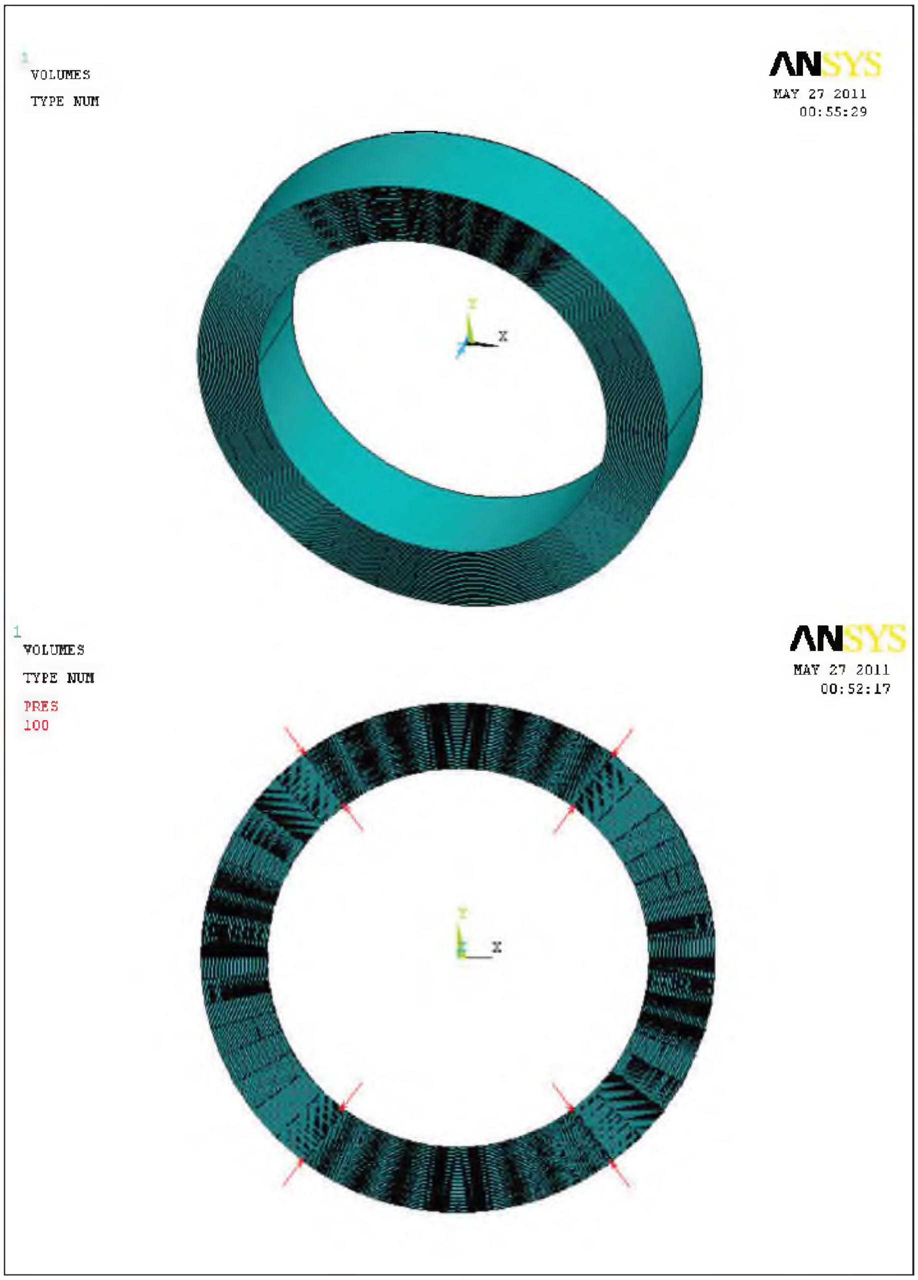

Fig. 2. Design scheme of the shell in a cylindrical coordinate system in ANSYS

Table 2. ***The structure of the laminate cylinder***

№	Code
1	$[0^\circ_2 / 90^\circ / 0^\circ_2 / \pm 30^\circ / 0^\circ_2 / 45^\circ / 30^\circ_2 / 0^\circ_2 / -30^\circ / \overline{0}^\circ]_S$
2	$[0^\circ_2 / 45^\circ / 0^\circ_2 / 45^\circ / 0^\circ_3 / 90^\circ / 0_2 / 90^\circ / 0^\circ_2 / \overline{0}^\circ]_S$
3	$[0^\circ_2 / -30^\circ / 0^\circ_2 / \pm 90^\circ / 0^\circ_2 / 30^\circ / 0^\circ_2 / \pm 30^\circ / 0^\circ / -9\overline{0}^\circ]_S$
4	$[0^\circ_2 / 30^\circ / 90^\circ / 45^\circ / 0^\circ_5 / 90^\circ / 0^\circ_2 / 90^\circ / 0\ ^\circ / 9\overline{0}^\circ]_S$
5	$[0^\circ_2 / 45^\circ / 0^\circ_2 / \pm 45^\circ / 0^\circ_2 / 90^\circ / 0^\circ_2 / 90^\circ / \pm 30^\circ / 9\overline{0}^\circ]_S$
6	$[0^\circ_2 / 45^\circ / 0^\circ_2 / \pm 45^\circ / 0^\circ_2 / 90^\circ / 0^\circ_2 / 90^\circ / \pm 30^\circ / 9\overline{0}^\circ]_S$
7	$[0^\circ_2 / 45^\circ / 0^\circ_2 / \pm 45^\circ / 0^\circ_2 / 90^\circ / 0^\circ_2 / 90^\circ / \pm 30^\circ / 9\overline{0}^\circ]_S$
8	$[0^\circ_2 / 45^\circ / 0^\circ_2 / \pm 45^\circ / 0^\circ_2 / 90^\circ / 0^\circ_2 / 90^\circ / \pm 30^\circ / 9\overline{0}^\circ]_S$
9	$[0^\circ_2 / 45^\circ / 0^\circ_2 / \pm 45^\circ / 0^\circ_2 / 90^\circ / 0^\circ_2 / 90^\circ / \pm 30^\circ / 9\overline{0}^\circ]_S$
10	$[0^\circ_2 / 45^\circ / 0^\circ_2 / \pm 45^\circ / 0^\circ_2 / 90^\circ / 0^\circ_2 / 90^\circ / \pm 30^\circ / 9\overline{0}^\circ]_S$

Table 3. ***The elastic constants of carbon fiber***

№ Code	E_z, MPa	E_θ, MPa	Er, MPa	$G_{\theta z}$, MPa	G_{zr} MPa	$G_{\theta r}$, MPa	$\nu_{z\theta}$	ν_{zr}	$\nu_{\theta r}$	$\nu_{\theta z}$	ν_{rz}	$\nu_{r\theta}$
1	102900	24300	16740	14480	4344	3282	0.393	0.201	0.292	0.093	0.033	0.201
2	110100	31890	16870	8277	4317	3309	0.09	0.299	0.314	0.026	0.046	0.166
3	104600	35350	17000	11090	4267	3364	0.196	0.265	0.301	0.066	0.043	0.145
4	103700	39550	17020	7946	4237	3388	0.066	0.306	0.315	0.025	0.05	0.135
5	91650	39420	17100	14250	4160	3470	0.247	0.247	0.289	0.106	0.046	0.125
6	90360	45510	17140	11820	4105	3521	0.153	0.277	0.298	0.077	0.053	0.112
7	88330	47230	17170	12580	4080	3550	0.176	0.270	0.293	0.094	0.052	0.107
8	88230	51450	17190	10970	4054	3576	0.128	0.285	0.3	0.074	0.056	0.1
9	75870	58440	17220	12290	3919	3707	0.128	0.284	0.292	0.098	0.065	0.086
10	54100	71840	17240	16580	3697	3934	0.167	0.270	0.254	0.222	0.086	0.061

mined by using a code of material structure. Totally, 10 versions of the shell reinforcement were considered (Table 2).

Thickness is δ=0,17mm. The other characteristics of the monolayer components are given earlier. For any given structure of carbon fiber there were found physical and mechanical characteristics as a composite material with one plane of elastic symmetry (Table 3). The Cartesian coordinate system (Fig. 1) was replaced by a cylindrical (Fig. 2).

Values of normal and tangential stresses at points of the internal, middle and external cylinder surface under internal pressure intensity p=20 MPa and external pressure p=5 MPa are presented in Table 4. Analysis of results demonstrated that a changed code did not practically influence the values of normal stresses in a circle direction σ_θ. In this case, an essential change of stress values $\sigma_{\theta z}$ of a transversal shear $\sigma_{\theta z}$ and normal axial stresses σ_z took place. Investigations performed for a stressed state of multi-layered shell with $[0^\circ_2/45^\circ/0^\circ_2/\pm 45^\circ/0^\circ_2/90^\circ/0^\circ_2/90^\circ_2/-90^\circ/\bar{0}^\circ]_S$ code, when the layer thickness was successively increased to a given value (Fig. 3-6), are of interest. As a whole, the shell thickness was determined by an expression $h=r_2-r_1$. The value r_2 shown in Fig. 3-6, a value of the shell internal surface, did not change and was equal to r_1=0,1 m. Elastic constants for the presented set of layers did not depend on the shell thickness.

We would like to note that the increased shell thickness did not practically change a difference of tangential stresses in the circle direction at points of the internal, middle and external surfaces. So, for example, if the shell thickness was $r_2-r_1=h$=5,4 mm, this difference would be 15.4 MPa, if it was h=13,7mm, the difference would be 17 MPa. Analyzing dependences of $\sigma_{\theta z}$ and σ_z stresses on the shell thickness, one could notice that in anisotropic shells, when r/h>20, essentially high stress values $\sigma_{\theta z}$ and σ_z arose. Such stresses could be a reason for destruction of the binder in a considered reinforced material. In this case, conditions for an ideal contact between the layers considered in a continuous-structure theory of anisotropic plates and shells turned out to be essentially violated.

Conclusions

The paper proposed a method for determination of the elastic constants of anisotropic material, which consists of a set of reinforced layers. There were reviewed ten variants of multilayer anisotropic hollow cylinders with different structure reinforcement. For each variant of reinforcement there was determined the stress state of the cylinder under the action of internal pressure. It was revealed that transverse shear stresses and normal stresses decrease in the longitudinal direction with increasing thickness of the hollow cylinder.

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Table 4. ***Stress state of the thickness cylinder (h=5.4 mm)***

№ Code	$\rho=r_1$				$\rho = r_1 + \frac{r2 - r1}{2}$				$\rho=r_2$			
	σ_r, MPa	σ_θ, MPa	σ_z, MPa	$\sigma_{\theta z}$, MPa	σ_r, MPa	σ_θ, MPa	σ_z, MPa	$\sigma_{\theta z}$, MPa	$\sigma_{\theta r}$, MPa	σ_θ, MPa	σ_z, MPa	$\sigma_{\theta z}$, MPa
1	-20	285.9	59.7	2.79	-12.2	277.9	58.29	3.59	-5	270.5	57	4.34
2	-20	286.1	50.4	-0.94	-12.2	277.8	49.3	0.35	-5	270.4	48.32	1.55
3	-20	286.1	40.58	0	-12.2	277.8	39.71	0	-5	270.3	38.94	0
4	-20	286.2	39.74	3.98	-12.2	277.8	38.96	4.62	-5	270.3	39.29	5.22
5	-20	286.2	35.68	0.21	-12.2	277.8	34.97	0.74	-5	270.3	34.35	1.23
6	-20	286.4	33.13	1.09	-12.2	277.8	32.55	1.94	-5	270.2	32.04	2.72
7	-20	286.4	30.29	-3.43	-12.2	277.8	29.76	-2.59	-5	270.2	29.31	-1.82
8	-20	286.5	28.18	0.92	-12.2	277.8	27.73	1.37	-5	270.2	27.36	1.79
9	-20	286.7	25.76	-0.95	-12.2	277.8	25.45	0.15	-5	270	25.21	1.17
10	-20	286.9	15.75	0	-12.2	277.8	15.87	0	-5	269.9	16	0

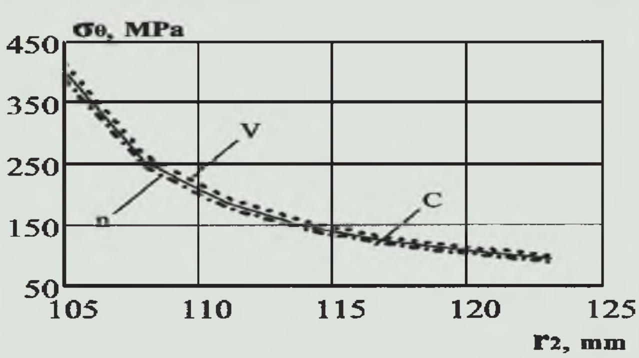

Fig. 3. The relation between radial stresses σr and the shell thickness (V – inner surface, C – middle surface, n – outer surface)

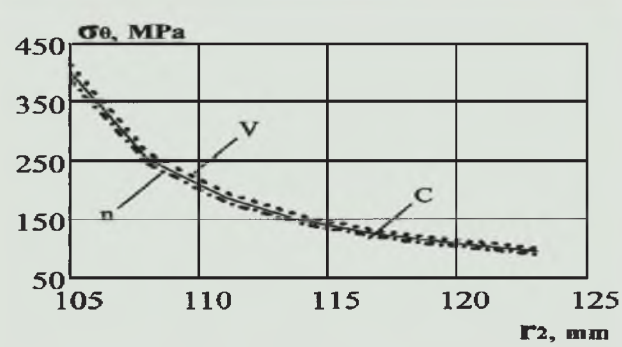

Fig. 4. The relation between tangential stress σ_θ and the shell thickness (V – inner surface, C – middle surface, n – outer surface)

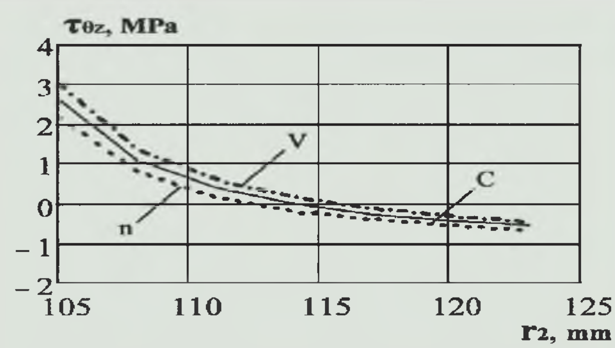

Fig. 5. The relation between radial stress $\tau_{\theta z}$ and the shell thickness (V – inner surface, C – middle surface, n – outer surface)

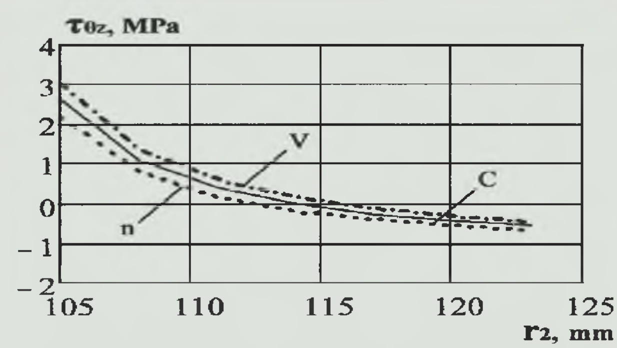

Fig. 6. The relation between radial stress $\tau_{\theta z}$ and the shell thickness (V – inner surface, C – middle surface, n – outer surface)

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