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# ДО ПИТАННЯ РОЗРАХУНКУ ПРОСТОРОВИХ КОНСТРУКЦІЙ ІЗ ТОНКОСТІННИХ СТЕРЖНІВ ВІДКРИТОГО ПРОФІЛЮ

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**Анотація.** У роботі підлягала перевірці гіпотеза щодо розрахунку стержневих конструкцій, складених із тонкостінних стержнів відкритого профілю, з використанням семи вузлових невідомих. Перевірка була зведена до аналізу результатів тестових розрахунків стержневих конструкцій, поведінка яких моделювалась шляхом створення розрахункової схеми із тонких плоских скінченних елементів та її подальшого розрахунку за допомогою обчислювального комплексу SCAD. Результати виконаних досліджень показали, що припущення щодо існування «депланації вузла» часто не підтверджується навіть для тих випадків, коли розглядаються плоскі, проте просторово навантажені стержневі системи. Показано, що дійсну взаємодію стержнів у вузлі їх спряження може коректно описати лише просторова скінченно-елементна модель тонкостінної стержневої системи.

**Ключові слова:** тонкостінний стержень, депланація, бімомент, числовий експеримент, метод скінченних елементів, середньоквадратична похибка.

# К ВОПРОСУ О РАСЧЕТЕ ПРОСТРАНСТВЕННЫХ КОНСТРУКЦИЙ ИЗ ТОНКОСТЕННЫХ СТЕРЖНЕЙ ОТКРЫТОГО ПРОФИЛЯ

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**Аннотация.** В работе проверялась гипотеза о расчете конструкции, составленной из тонкостенных стержней открытого профиля, с использование семи узловых неизвестных. Проверка была сведена к анализу результатов тестовых расчетов стержневых конструкций, поведение которых моделировалось путем создания расчетной схемы из тонких плоских конечных элементов и ее дальнейшего расчета в вычислительном комплексе SCAD. Результаты выполненных исследований показали, что предположение относительно существования «депланации узла» часто не подтверждается даже для тех случаев, когда рассматриваются плоские, но пространственно нагруженные стержневые системы. Показано, что действительное взаимодействие стержней в узле их сопряжения может быть корректно описано только при помощи пространственной конечно-элементной модели тонкостенной стержневой системы.

**Ключевые слова:** тонкостенный стержень, депланация, бимомент, численный эксперимент, метод конечных элементов, среднеквадратическая ошибка.

# ON THE ISSUE OF STRUCTURAL ANALYSIS OF SPATIAL SYSTEMS FROM THIN-WALLED BARS WITH OPEN PROFILES

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**Abstract.** A working hypothesis relating to the structural analysis of the spatial structures from thin-walled bars with open profiles using seven degree of freedoms has been verified. The verification has been performed based on the results of the structural analysis of thin-walled bar systems which behavior under the external loading has been simulated using design schemes from thin shell finite elements. Structural analysis has been realized using software SCAD. The results of the performed investigation have pointed that the suggestion concerning «joint warping» existence or, in other words, the equal warping for the each end member cross-section sided to the joint under consideration often is not true even for those design cases, where plane design models with spatial application of the structural loading are considered. Only the shell finite-element model of the thin-walled bar system can describe the actual interaction of the thin-walled bars at the structural joint correctly.

**Keywords:** thin-walled bar, warping, bimoment, numerical simulation, finite element analysis, root-mean-square error.

## Problem statement

The problem of analysis of spatial structures from thin-walled bars is of interest last years. Thin-walled bar structures were the subject of investigation of different researchers, who use the finite element with seven degree of freedom at the both ends [8, 12].

Strain and stress distribution in thin-walled bars with open profiles is differ significantly from the ordinary bars, as the Euler-Bernoulli's hypothesis of the plane sections as well as principle locality of the action of Saint-Venant's balanced system of forces [3] are not true partly or completely. There is considerable warping of cross-sections in thin-walled bars with open profiles, which reflected appreciably on the structural behavior under the loading.

Structural analysis and calculation of internal forces in thin-walled structural members of open profiles accounting for bending torsion is complicated task. Modern software packages for structural analysis use finite element types which take into account up to six degrees of freedom at the structural nodes, which corresponds to the linear and angular displacements in these nodes as for the rigid bodies. Structural analysis of thin-walled bar systems can be performed using the shell finite elements. In this case, accurate selection the finite element meshing for approximation of structural members is required. Besides, number of nodes and finite elements increases comparing to the bar approximation by several digits [8].

At the same time, theory of thin-walled bars of open profile requires using additional degrees of freedom at the nodes adjoined to the thin-walled bars. These additional degrees of freedom correspond to the warping components of the total longitudinal node displacement.

A. R. Tusnin in his paper [8] considered the problem of structural analysis of spatial thin-walled bar structures with open profiles and developed thin-walled bar finite element with seven degree of freedom at the both ends (Fig. 1). He also presented stiffness matrix for such thin-walled bar taken into account the bending torsion deformation as well as coordinate transformation matrix in order to transform from the local to the global system of coordinates.

It should be noted, that the seven degree of freedom may estimate exhaustively the behavior of the thin-walled bars for the very simple cross-section types of open profiles with one web and two flanges only. For example, in C-sections of the thin-walled bar at the presence of single edge fold stiffeners the eighth degree of freedom will be required, at the presence of double edge fold stiffeners – ninth degree of freedom, etc. Besides, it begs the question about correct determination the support conditions of thin-walled finite element for each such additional degree of freedom. Finally, the problem of strain compatibility conditions at the nodes of bar design model is left in abeyance, if there are additional degrees of freedom which depend from the cross-section shapes of the bars sided to the joint under consideration [1, 11].

In last years there were lots of efforts of scientists to construct an universal algorithm for structural analysis of arbitrary thin-walled bar systems. Formulation the boundary conditions at the ends of the thin-walled bar was the main problem in this context [10]. In certain papers [6, 7] authors grounded on the principle that at the end of the thin-walled bar the warping deformation either is absent completely (rigid support relative to the warping), or free (the hinge relative to the warping).

Hypothesis concerning to uniform warping for all end cross-sections of all thin-walled bars adjoined to the considered node has been used by A. R. Tusnin in his paper [8] for certain types of joint structural decisions. V. A. Postnov and I. Ya. Kharhurim also supposed that the spatial orientation of the thin-walled bar has no an influence on warping, namely, warping deformation at the local and global system of coordinates has been assumed as equal [7].

The approach mentioned above is sufficient, for example, for plane rectangular frames without eccentricities at the nodes, when the bar axis passes through the shear center, flanges of all bars at the considered joint are parallel to the frame plane, and gusset plates welded to the flanges have got the infinite in-plane stiffness and allow the warping in out-of-plane direction [2, 4, 5] (Fig. 2).

Different types of thin-walled finite elements and calculation techniques for structural analysis of thin-walled structural systems have been considered by authors of the following papers [9, 14, 15]. In these papers have been used applicable for only particular cases approaches, which allow taking into account warping compatibility conditions and focus on using seven node unknowns (six linear and angular displacements and warping) (see Fig. 1).

However, hypothesis about availability the unified warping at the node of truly space bar structure raises serious doubts and need to be checked accurately.

## Research technique

Henceforth accurate finite-element models of the thin-walled bar systems loaded by the external

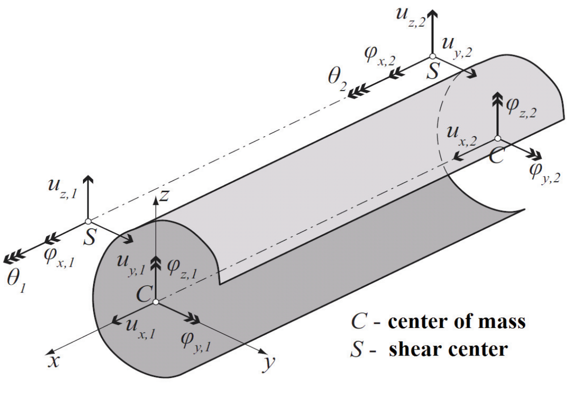



**Figure 1.** Thin-walled finite element with seven degree of freedom on each end.

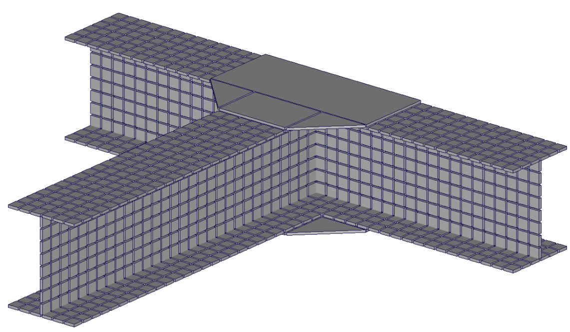


**Figure 2.** Plane rectangular frame according to [4, 5].

torque moment with different support conditions have been considered. Besides, thin-walled bars for such models have been simulated by the set of plate finite elements. Longitudinal displacements of end cross-sections points $\hat{u}_i$ of all thin-walled bars adjoined at the FE model, as well as axial stresses at these points $\hat{\sigma}_i$ have been calculated for constructed finite-element models of the thin-walled bar systems.

Comparison of the FE calculation results with the theoretical values of the longitudinal displacements $u_i$ and axial stresses $\sigma_i$ allows estimating the warping value and, in this way, calculating the bimoment value.

Let's consider base hypothesis of Vlasov's theory relating to the behavior of thin-walled bar with open profile. Longitudinal displacement for each $i^{th}$ point of the cross-section of such bar can be expressed using the following equation:

$$u_i(x,s) = \xi(x) - \eta'(x) y_i(s) - \zeta'(x) z_i(s) - \theta'(x)\omega_i(s) \quad , \ i = 1,\dots,n, \quad (1)$$

where the first three summands of the equation correspond to the hypothesis of plane sections, namely: $\xi(x)$ – longitudinal displacement of the center of mass $C$ as function of axial coordinate $x$ of the section under consideration; $\eta(x), \zeta(x)$ – lateral displacements of the pole $S$ of the section under consideration; $y_i(s), z_i(s)$ – coordinates of the $i^{th}$ section point under consideration as function of the angular position $s$. The last summand of the equation (1) corresponds to the warping component of the longitudinal displacements of section points at the direction of $x-x$ axis, where $\theta(x)$ and $\omega_i(s)$ – rotation angle of the section under consideration about the pole $S$ and sectorial coordinate for $i^{th}$ section point accordingly.

Therefore, bases on the sectorial geometrical properties of the cross-section and having the set of numerical values of longitudinal displacements $\hat{u}_i\ (i = 1,\dots,n)$ of $n$ cross-section points as a result of FE structural analysis of plate finite-element model, we can calculate the warping $\theta'(x)$ for each end cross-section of all thin-walled bars at the joint under consideration.

Violation for the results of numerical calculation using equation (1) from the results of FE structural analysis for some $i^{th}$ cross-section point can be written as, $i = 1,\dots,n$:

$$e_i^u(x,s) = \xi(x) - \eta'(x) y_i(s) - \zeta'(x) z_i(s) - \theta'(x)\omega_i(s) - \hat{u}_i . \quad (2)$$

Using the least square technique the equation (2) formulated for each $i^{th}$ cross-section point, $i = 1,\dots,n$, can be turned into the following problem of functional minimization:

$$\mathbf{E}_u = \sum_{i=1}^{n} \left(e_i^u(x,s)\right)^2 = \sum_{i=1}^{n} (\xi(x) - \eta'(x) y_i(s) - \zeta'(x) z_i(s) - \theta'(x)\omega_i(s) - \hat{u}_i)^2 \to \min . \quad (3)$$

Herewith, indispensable conditions for the minimum of the functional (3):

$$\begin{cases} \dfrac{\partial \mathbf{E}_u}{\partial \eta'(x)} = 0, \\ \dfrac{\partial \mathbf{E}_u}{\partial \zeta'(x)} = 0, \\ \dfrac{\partial \mathbf{E}_u}{\partial \theta'(x)} = 0, \\ \dfrac{\partial \mathbf{E}_u}{\partial \xi(x)} = 0, \end{cases}$$

give the system of linear equations relative to the unknown factors of the initial equation (1):

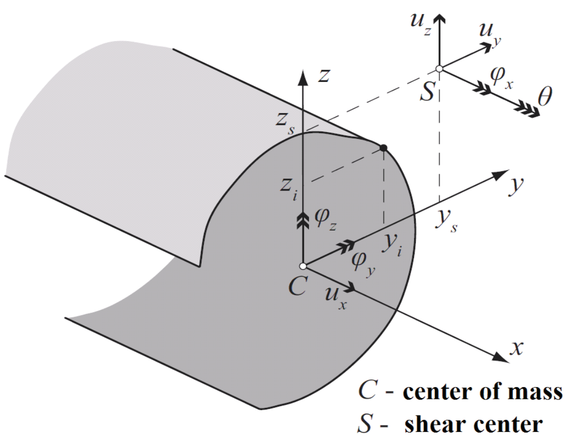



**Figure 3**. Cross-section of the thin-walled finite element with seven degree of freedom.

$$\begin{cases} -\eta'\sum_{i=1}^{n}(y_i)^2-\zeta'\sum_{i=1}^{n}y_iz_i- \\ -\theta'\sum_{i=1}^{n}y_i\omega_i+\xi\sum_{i=1}^{n}y_i-\sum_{i=1}^{n}y_i\hat{u}_i=0, \\ -\eta'\sum_{i=1}^{n}z_iy_i-\zeta'\sum_{i=1}^{n}(z_i)^2- \\ -\theta'\sum_{i=1}^{n}z_i\omega_i+\xi\sum_{i=1}^{n}z_i-\sum_{i=1}^{n}z_i\hat{u}_i=0, \\ -\eta'\sum_{i=1}^{n}\omega_iy_i-\zeta'\sum_{i=1}^{n}\omega_iz_i- \\ -\theta'\sum_{i=1}^{n}(\omega_i)^2+\xi\sum_{i=1}^{n}\omega_i-\sum_{i=1}^{n}\omega_i\hat{u}_i=0, \\ -\eta'\sum_{i=1}^{n}y_i-\zeta'\sum_{i=1}^{n}z_i- \\ -\theta'\sum_{i=1}^{n}\omega_i+n\xi-\sum_{i=1}^{n}\hat{u}_i=0; \end{cases} \quad (4)$$

here the indication on dependency from axial coordinate $x$ or angular position $s$ has been omitted and notations $\eta$, $\zeta$ and $y_i$, $z_i$ have been only used for the purpose of simplification.

Therefore, constructing and solving the system of linear algebraic equations (4) for each end cross-section of all thin-walled bars at the joint under consideration allows calculating the warping value $\theta'(x)$ for these cross-sections. In turn, comparing the warping value $\theta'(x)$ at the mentioned cross-sections gives the possibility to verify the hypothesis about its equality.

Verification of the static conditions at the node should be performed similarly comparing the values of the longitudinal (normal) stresses $\hat{\sigma}_i$ ($i=1,\dots,n$) at the considered cross-section points of the plate finite-element model with the theoretical values of the stresses $\sigma_i(x,s)$. The latter has been calculated using the known formula for thin-walled bar taken into account the value of bimoment:

$$\sigma_i(x,s)=\frac{N(x)}{A}+\frac{M_y(x)}{I_y}z_i(s)+ \\ +\frac{M_z(x)}{I_z}y_i(s)+\frac{B(x)}{I_\omega}\omega_i(s). \quad (5)$$

Violation for the results of numerical calculation $\hat{\sigma}_i$ ($i=1,\dots,n$) using equation (5) from the results of FE structural analysis for some $i^{\text{th}}$ cross-section point can be written as, $i=1,\dots,n$:

$$e_i^\sigma(x,s)=\sigma_i(x,s)-\hat{\sigma}_i=\frac{N(x)}{A}+\frac{M_y(x)}{I_y}z_i(s)+ \\ +\frac{M_z(x)}{I_z}y_i(s)+\frac{B(x)}{I_\omega}\omega_i(s)-\hat{\sigma}_i. \quad (6)$$

Comparing the theoretical values of the longitudinal (normal) stresses $\sigma_i(x,s)$ $(i=1,\dots,n)$ with the numerical values of the stresses $\hat{\sigma}_i$ derived as the results of numerical experiment when minimization of sum of squared deviations we have obtained the following:

$$\mathbf{E}^\sigma=\sum_{i=1}^{n}\left(e_i^\sigma(x,s)\right)^2\to\min,$$

$$\mathbf{E}^\sigma=\sum_{i=1}^{n}(\frac{N(x)}{A}+\frac{M_y(x)}{I_y}z_i(s)+ \quad (7)$$

$$+\frac{M_z(x)}{I_z}y_i(s)+\frac{B(x)}{I_\omega}\omega_i(s)-\hat{\sigma}_i)^2\to\min.$$

On the basis of indispensable conditions for the minimum we have obtained the system of linear algebraic equations relative to the unknowns of the longitudinal stresses equation (5) at the cross-sectional points of the thin-walled bar:

$$\begin{cases} \dfrac{\partial\mathbf{E}^\sigma}{\partial\left(\dfrac{N(x)}{A}\right)}=n\dfrac{N}{A}+\dfrac{M_z}{I_z}\sum_{i=1}^{n}y_i+ \\ +\dfrac{M_y}{I_y}\sum_{i=1}^{n}z_i+\dfrac{B}{I_\omega}\sum_{i=1}^{n}\omega_i-\sum_{i=1}^{n}\hat{\sigma}_i=0, \\ \dfrac{\partial\mathbf{E}^\sigma}{\partial\left(\dfrac{M_z(x)}{I_z}\right)}=\dfrac{N}{A}\sum_{i=1}^{n}y_i+\dfrac{M_z}{I_z}\sum_{i=1}^{n}(y_i)^2+ \\ +\dfrac{M_y}{I_y}\sum_{i=1}^{n}y_iz_i+\dfrac{B}{I_\omega}\sum_{i=1}^{n}y_i\omega_i-\sum_{i=1}^{n}y_i\hat{\sigma}_i=0, \\ \dfrac{\partial\mathbf{E}^\sigma}{\partial\left(\dfrac{M_y(x)}{I_y}\right)}=\dfrac{N}{A}\sum_{i=1}^{n}z_i+\dfrac{M_z}{I_z}\sum_{i=1}^{n}z_iy_i+ \\ +\dfrac{M_y}{I_y}\sum_{i=1}^{n}(z_i)^2+\dfrac{B}{I_\omega}\sum_{i=1}^{n}z_i\omega_i-\sum_{i=1}^{n}z_i\hat{\sigma}_i=0, \\ \dfrac{\partial\mathbf{E}^\sigma}{\partial\left(\dfrac{B(x)}{I_\omega}\right)}=\dfrac{N}{A}\sum_{i=1}^{n}\omega_i+\dfrac{M_z}{I_z}\sum_{i=1}^{n}\omega_iy_i+ \\ +\dfrac{M_y}{I_y}\sum_{i=1}^{n}\omega_iz_i+\dfrac{B}{I_\omega}\sum_{i=1}^{n}(\omega_i)^2-\sum_{i=1}^{n}\omega_i\hat{\sigma}_i=0, \end{cases} \quad (8)$$

here the indication on dependency from axial coordinate $x$ or angular position $s$ has been omitted and notations $\eta$, $\zeta$ and $y_i$, $z_i$ have been only used for the purpose of simplification.

## Hypothesis verification

The ordinary design models of the thin-walled bar systems have been examined by implementing the numerical experiment. Only structures with the rigid member-to-member joints has been undergone to the structural analysis, where flanges of the one structural member were connected to the flanges or stiffeners of another structural member in order to omit the section contour distortion. Just that very structural decision of the rigid joints ensures the clear transmitting of the bending moments and bimoments from one thin-walled structural member to another.

**Example 1.** Steel frame structure made of three thin-walled bars of I-section (Fig. 4) with flange section 600×10 mm and web section 800×10 mm have been examined. The axis (geometrical locus) of the shear centers of the I-cross-sections coincides with the axis (geometrical locus) of the centers of mass.

Numerical calculation (structural analysis) of the plate finite-element models has been performed using software package SCAD. Figure 4 presents deformed scheme of the structure, where those cross-sections of the thin-walled structural members are also indicated, for which warping values have been calculated.

Comparing the results of the numerical calculation for three end cross-sections (Table 1) adjoined to the FE-model of the joint, we can see that its warping values practically coincide with each other only for the end cross-sections of the rafters located at the one and the same horizontal plane, besides the warping values for the rafters end cross-sections differ markedly from the warping value for the column end cross-section.

**Example 2.** Γ-shaped rectangular frame with rigid supports at the ends of the column and rafter has been considered. External torque moment has been applied at the middle of the rafter span. The frame structural members had I-section with web section 300×10 mm and flange section 200×10 mm. Four structural decisions for the rigid rafter-to-column joint have been examined: (1) without stiffeners or stiffening diaphragms, (2) with one skewed stiffener, (3) with two transversal stiffeners and (4) with two transversal and one skewed stiffeners (Fig. 5).

The results of the numerical experiment shown that the structural decision of the rigid rafter-to-

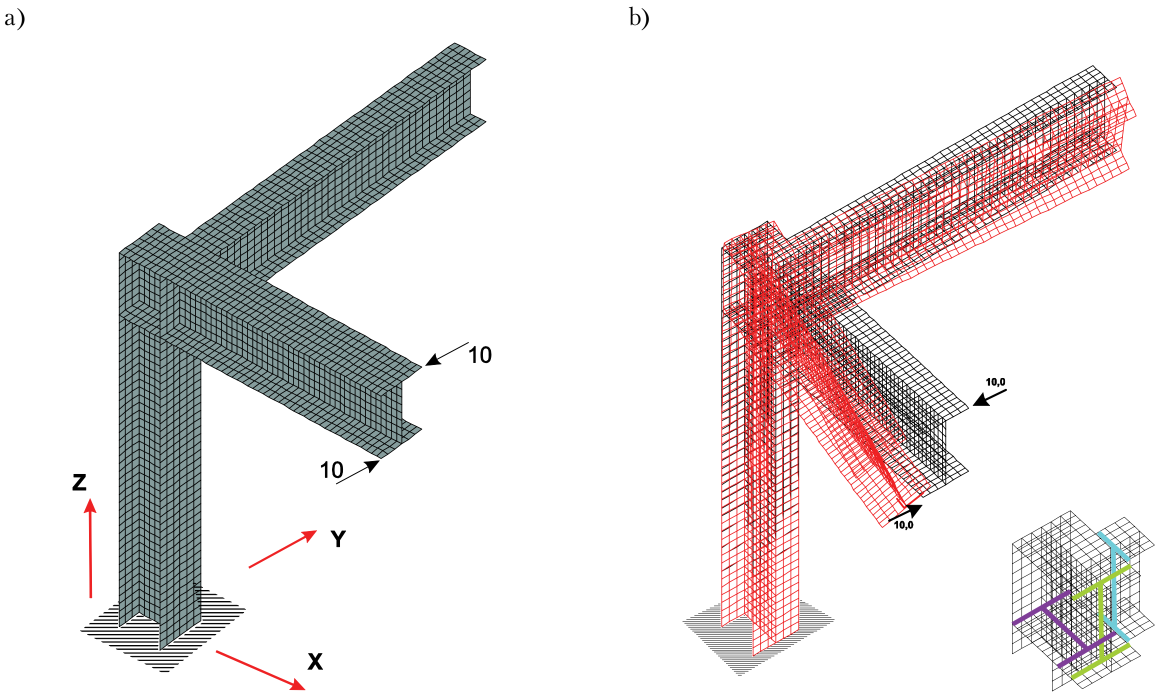



**Figure 4.** Plate finite-element model of the structure to the Example 1: a) initial; b) deformed.

column joint has significant influence on the warping and bimoment distribution in the structural system (Table 2). The values of the warping and bimoment at the end cross-sections of the rafter and column sided with rafter-to-column joint were different for all design cases.

**Example 3**. Γ-shaped frame has been examined (Fig. 6). The lower end of the frame column is rigid supported; the end of the rafter is free, where external torque moment 1 kNm was applied. Frame column had I-section with web section and flange section 300×10 mm. Frame rafter had I-section with web section 400×10 mm and flange section 300×10 mm.

Two structural decisions of the rigid rafter-to-column joint have been considered: (1) with one skewed stiffener and (2) with two transversal stiffeners. Additionally, frame design model, where the column web was oriented perpendicularly to the rafter web (Fig. 7), has been also examined.

The results of the numerical experiment have been compared in order to detect the dependence of the warping value from the load type. The warping values at the rafter and column end cross-sections sided to the joint as well as it ratio have been estimated depending on different conditions of external torque moment application: (1) at the rafter free end, (2) at the middle of the rafter span, (3) at the middle of the column height too.

Table 3 presents the results of the numerical experiment. As a result of the numerical experiment implementation has been detected that changing the design scheme of the load application on the structure caused to the significant changing not only warping values, but also the ratios of the warping

**Table 1.** Results of the numerical experiment (Example 1)

| Characteristic | Rafter along axis $y-y$ | Rafter along axis $x-x$ | Column |
|---|---|---|---|
| Warping $\theta'(x)$, $\times 10^{-5}$ мм$^{-1}$ | –11,0397 | + 11,16 | + 9,6751 |

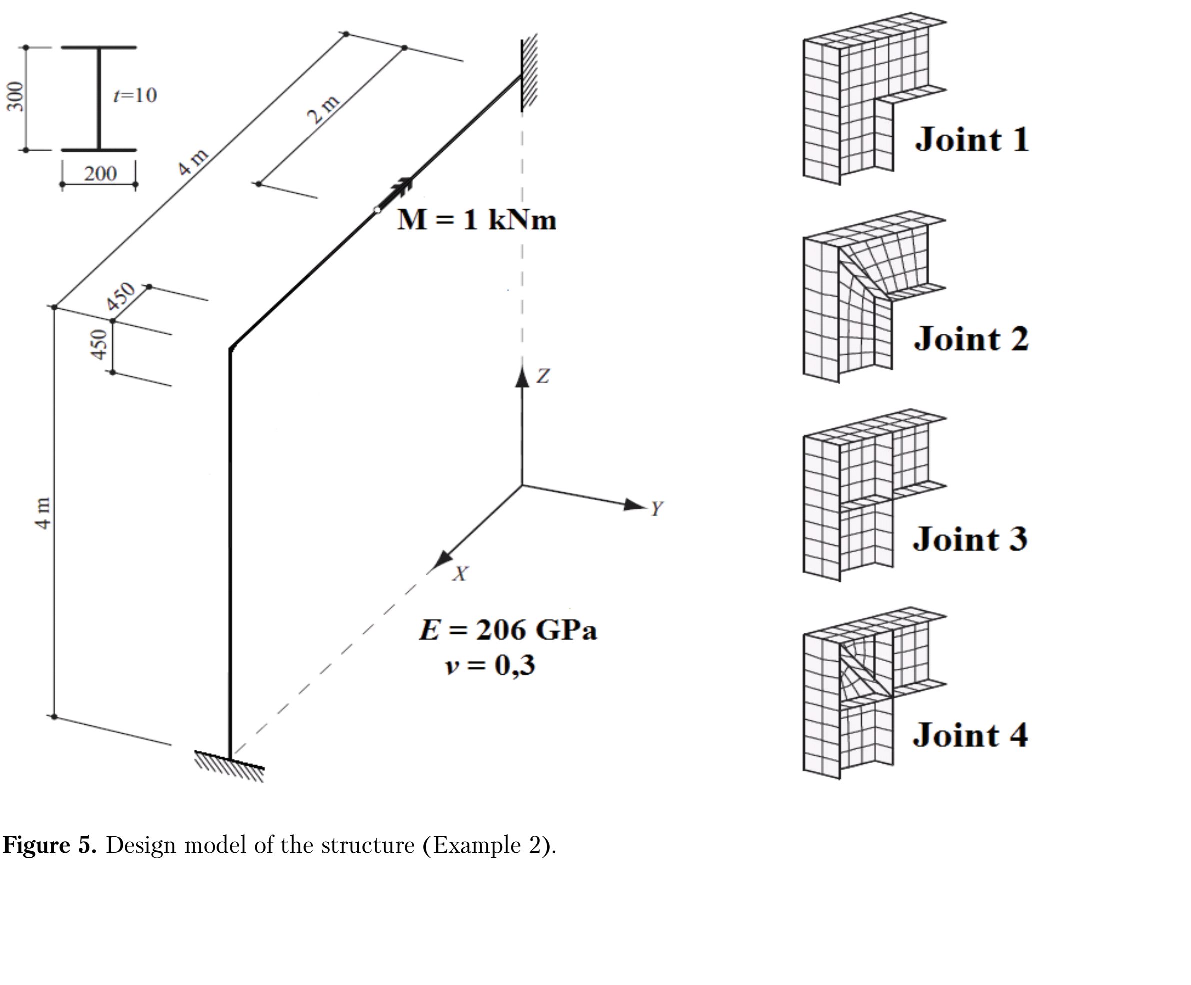
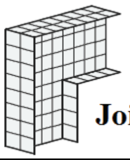
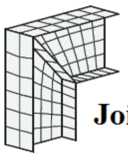
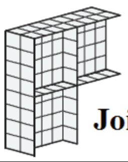
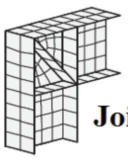



**Figure 5.** Design model of the structure (Example 2).

**Table 2.** The results of the numerical experiment (Example 2)

| Plate finite element model of the joint | Characteristic | Rafter | Column |
|---|---|---|---|
|  **Joint 1** | Warping value, × $10^{-3}$ $mm^{-1}$ | + 0,00512 | + 0,0006 |
| | Bimoment, $Nm^2$ | – 52,0886 | +7,781292 |
|  **Joint 2** | Warping value, × $10^{-3}$ $mm^{-1}$ | + 0,00541333 | + 0,00010667 |
| | Bimoment, $Nm^2$ | – 52,9171 | – 10,5611 |
|  **Joint 3** | Warping value, × $10^{-3}$ $mm^{-1}$ | + 0,00362667 | – 0,00198 |
| | Bimoment, $Nm^2$ | +118,1281 | – 60,2334 |
|  **Joint 4** | Warping value, × $10^{-3}$ $mm^{-1}$ | + 0,0020933 | – 0,00044 |
| | Bimoment, $Nm^2$ | +246,8 | –50,0292 |

values at the rafter and column end cross-section sided to the joint under consideration.

Therefore, the results of the performed investigation have pointed that the suggestion concerning «joint warping» existence or, in other words, the equal warping for the each end member cross-section sided to joint under consideration often is not true even for those design cases, where plane design models with spatial application of the structural loading are considered.

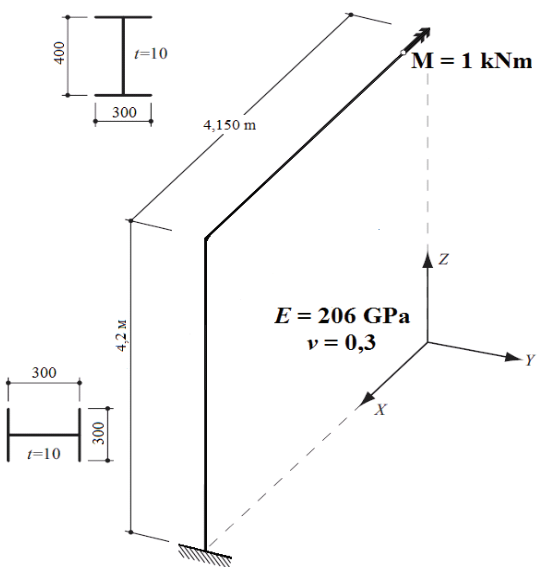



**Figure 6.** Design model of the structure (Example 3).

In general case we have no possibility to indicate so called joint center, i. e. the point, where axes passed through the shear centers of the end member cross-sections sided to the joint under consideration are intersected. This structural joint doesn't meet certain conditions of theory of plane thin-walled frames. Only the spatial finite-element model of the thin-walled bar system can describe the actual interaction of the thin-walled bars at the structural joint correctly.

In the paper [14] polish scientist S. Koczubiej has proposed an approach solved the described problem. As the full finite-element modeling of all thin-walled bars of the structural system leads to the cumbersome design models, he proposed to use shell finite elements in the joint region only and thin-walled bar finite elements in other structural regions (Fig. 8 and Fig. 9). Proposed approach reduces significantly initial data volume and, in this way, design model of the structure. At the same time, structural design model reflects truly its behavior as for the bar structure under the loading.

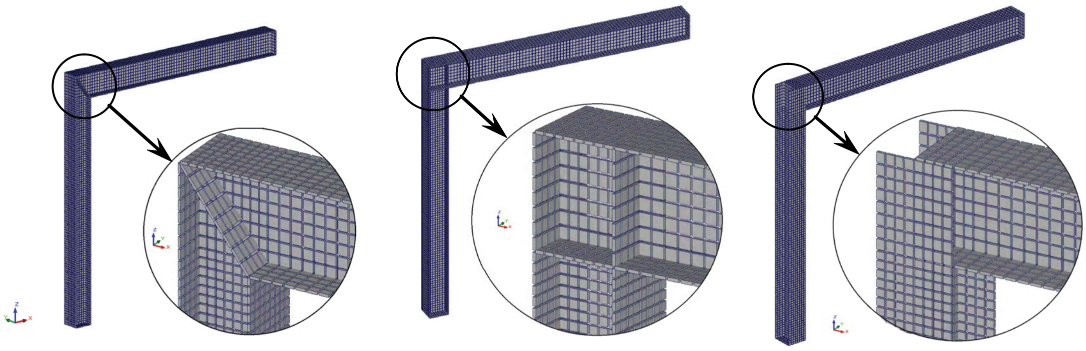


**Figure 7.** Plate finite-element models of the rafter-to-column joints (Example 3).

**Table 3.** Warping values at the end cross-sections of the rafter and column, $10^{-2}\,m^{-1}$, as well as it ratio by the different conditions of the external torque moment application

| Joint structural desicion | The location of the external torque moment application | | | | | |
|---|---|---|---|---|---|---|
| | At the end of the rafter | | At the middle of the rafter span | | At the middle of the column height | |
| | $\frac{\text{Rafter warping}}{\text{Column warping}}$ | Ratio | $\frac{\text{Rafter warping}}{\text{Column warping}}$ | Ratio | $\frac{\text{Rafter warping}}{\text{Column warping}}$ | Ratio |
| **Joint 1** | $\frac{1,6428}{1,1995}$ | 1,36957 | $\frac{0,844576}{0,60955}$ | 1,38557 | $\frac{1,78992}{2,4204}$ | 0,7395 |
| **Joint 2** | $\frac{1,61008}{-1,3974}$ | –1,1522 | $\frac{0,805494}{-0,6968}$ | –1,15599 | $\frac{-2,0744}{2,40584}$ | –0,86224 |
| **Joint 3** | $\frac{1,38199}{-1,2153}$ | 1,13716 | $\frac{0,696314}{-0,60951}$ | –1,14242 | $\frac{-1,8117}{2,10829}$ | –0,85932 |

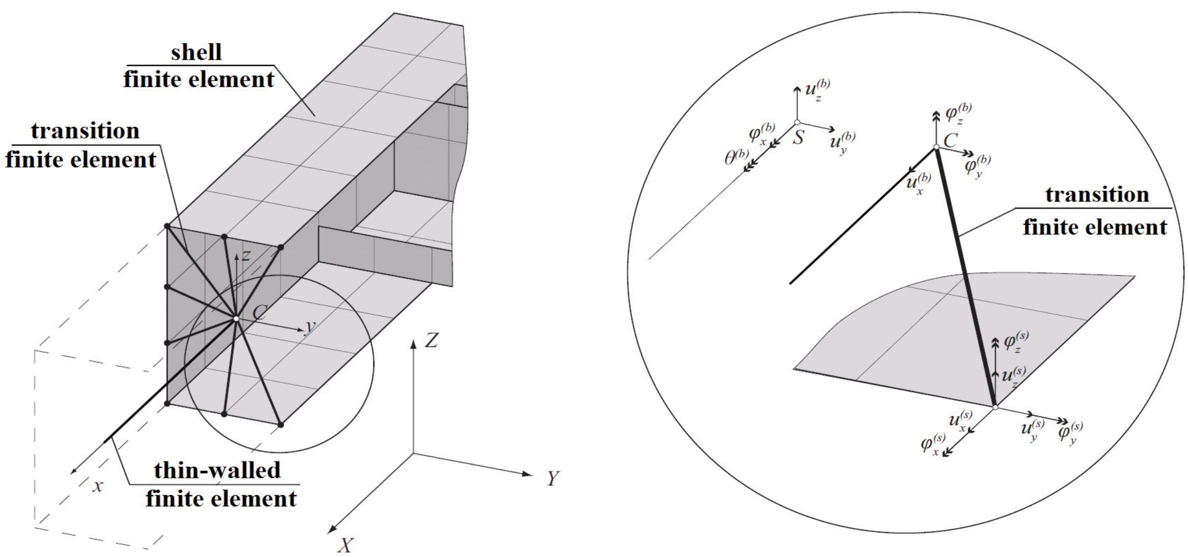



**Figure 8.** Structural modeling using shell and thin-walled finite elements [14].

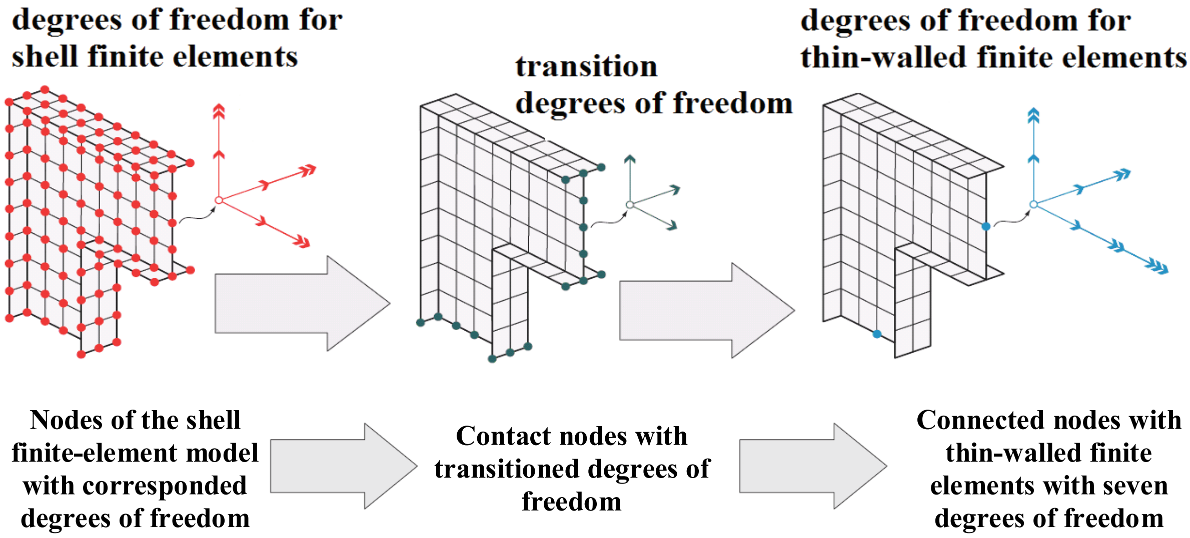



**Figure 9.** Transformationof variables of the finite-element method when structural modeling using shell and thin-walled finite elements [14].

## Conclusion

In this paper a working hypothesis relating to the structural analysis of the space structures from thin-walled open profiled bars using seven degree of freedoms has been undergone to the verification. The verification has been reduced to the analysis of the results of calculation of the certain bar systems; it behaviors under the loading have been simulated using thin plate finite element models. The results of the performed investigation have pointed that the suggestion concerning «joint warping» existence or, in other words, the equal warping for the each end member cross-section sided to joint under consideration often is not true even for those design cases, where plane design models with spatial application of the structural loading are considered. Only the spatial finite-element model of the thin-walled bar system can describe the actual interaction of the thin-walled bars at the structural joint correctly.

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