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ON p-GROUPS OF NILPOTENCY CLASS 3 WHERE ALL PROPER SUBGROUPS HAVE NILPOTENCY CLASS LESS THAN 3

We obtain determination of p-groups with nilpotency class 3 where all proper subgroups have nilpotency class less or equal 2. This solves Problem Nr. 87 stated by Y. Berkovich in [1].

Keywords: p-group, subgroup, nilpotency class.

Introduction

Properties of a group are connected with its subgroup structure. Sometimes to decide whether a group has a subgroup of a given type is more difficult than to obtain the list of all groups without subgroups of this type. For example, Miller and Moreno described nonabelian groups whose all proper subgroups are abelian [2]. Due to this result one can answer the question whether a finite nonabelian group has a proper nonabelian subgroup or not.

Analogously, the determination p-groups with nilpotency class 3, where all proper subgroups have nilpotency class less than 3 gives the possibility to decide whether a finite nonabelian p-group G contains a proper subgroup H such that $cl(H) > 2$. Here $cl(G)$ denotes the nilpotency class of the group G.

In [1] Y. Berckovich formulated the following problem: "Describe all p-groups with nilpotency class 3, where all proper subgroups have nilpotency class less than 3". In this articles we solve this problem for finite p-groups.

We say that a finite p-group G is a minimal group of nilpotency class 3 if all proper subgroups of G are groups of nilpotency class less than 3. The set of such groups is denoted by $\mathcal{M}$.

Standard notation is in use.

- $\Phi(G)$ be the Frattini subgroup of G,
- $d(G)$ — the minimal number of generators of G,
- $Z_i(G)$ — the i-th member of lower central series,
- G_i — i-th member of upper central series.

We prove the following:

Theorem 1. *For a finite group $G \in \mathcal{M}$ we have*

1) $d(G) \leq 3$*;*
2) $\Phi(G) \subseteq Z_2(G)$*;*
3) $\exp(G_3) = p$.

The proof of this theorem is given in Section 2.

Using this theorem we obtain the necessary and sufficient conditions for a finite p-group to be a minimal group of nilpotency class 3. To be concrete we show that a finite p-group is a minimal group of nilpotency class 3 under the following conditions

- for $d(G) = 2$ if and only if $\Phi(G) \subseteq Z_2(G)$;
- for $d(G) = 3$ if and only if $p = 3$ and G asserts the following relations

$$
\begin{aligned}
G = \langle a_1, a_2, a_3, c_1, c_2, c_3, z \mid {} & z^3 = 1, \\
& c_1^{3^{r_1}} = z^{\mu_1}, c_2^{3^{r_2}} = z^{\mu_2}, c_3^{3^{r_3}} = z^{\mu_3}, \\
& a_1^{3^{m_1}} = c_1^{\beta_{11}} c_2^{\beta_{12}} c_3^{\beta_{13}} z^{\lambda_1}, \\
& a_2^{3^{m_2}} = c_1^{\beta_{21}} c_2^{\beta_{22}} c_3^{\beta_{23}} z^{\lambda_2}, \\
& a_3^{3^{m_3}} = c_1^{\beta_{31}} c_2^{\beta_{32}} c_3^{\beta_{33}} z^{\lambda_3}, \\
& [a_1, a_2] = c_3, [a_2, a_3] = c_1, [a_3, a_1] = c_2, \\
& [c_i, a_i] = z, [c_i, a_j] = [c_i, c_j] = 1, \\
& [c_i, z] = [a_i, z] = 1, (i, j = 1, 2, 3; i \neq j)\rangle,
\end{aligned}
$$

where parameters m_i, r_j, μ_i satisfy following conditions

a) $m_1, m_2, m_3, r_1, r_2, r_3 \geq 1$, $\lambda_i, \mu_j = 0, 1, 2$; $i, j = 1, 2, 3$.
b) $r_1 \leq \min(m_2, m_3)$, $r_2 \leq \min(m_1, m_3)$, $r_3 \leq \min(m_1, m_2)$.
c) Let $i, j, k = 1, 2, 3$; $i \neq j$, $i \neq k$, $j \neq k$ and $c_i = [a_j, a_k]$.
If $r_i < \min(m_j, m_k)$ then $\beta_{jk} \cong \beta_{kj} \cong 0 \pmod 3$, μ_i — arbitrary;
if $r_i = m_j < m_k$ then $\beta_{jk} \cong -\mu_i \pmod 3$, $\beta_{kj} \cong 0 \pmod 3$;
if $r_i = m_k < m_j$ then $\beta_{kj} \cong \mu_i \pmod 3$, $\beta_{jk} \cong 0 \pmod 3$;
if $r_i = m_j = m_k$ then $\beta_{kj} \cong \mu_i \cong -\beta_{jk} \pmod 3$.

Necessary conditions

In the proof we use the following known results

Proposition 2. *For a finite p-group G we have* $\Phi(G) = \mho(G) \cdot G_2$.

Proof of Theorem 1. 1) Let $G \in \mathcal{M}$. Since $cl(G) = 3$ there exist $g \in G_2$ such that $g \notin Z(G)$. Without loss of generality we assume that g is a commutator, i.e. there exist elements a, $b \in G$ such that $g = [a, b]$ and there is an element $c \in G$ such that $[g, c] \neq 1$. In that way the subgroup $H = \langle a, b, c\rangle$ has nilpotency class 3. So it is an improper subgroup of G, $H = G$ and $d(G) \leq 3$.

2) To prove that $\Phi(G) \subseteq Z_2(G)$ we show that $\mho(G) \subseteq Z_2(G)$. Each element g from $\mho(G)$ is a product of some set of p-th degrees of elements of G. According Propozition 2 there are elements $g_1 \in G$ and $c \in G_2$ such that ${g_1}^p = g \cdot c$. Since $c \in G_2 \subseteq Z_2(G)$ we have

$$[g, x] = [{g_1}^p \cdot c^{-1}, x] \cong [{g_1}^p, x] \cong \\ \cong [g_1, x]^p \cong [g_1, x^p] \pmod{Z(G)}.$$

Suppose that an element $g \in \mho(G)$ is not contained in $Z_2(G)$. Then there are elements $x, y \in G$ such that $[g, x] \notin Z(G)$ and $[g, x, y] \neq 1$. Since $cl(G) = 3$ we have

$$[g_1^p, x, y] = [g_1, x^p, y] = \\ = [g_1, x, y]^p = [g, x, y] \neq 1. \quad (1)$$

The subgroup $H = \langle g_1^p, x, y\rangle$ has nilpotency class 3 too, hence $H = G$. The element g_1^p is a nongenerator of the p-group G, thus G has exactly 2 generators, $G = \langle x, y\rangle$. Consequently $g_1 = x^\alpha \cdot y^\beta$ and the following equalities for commutators hold

$$[g_1, x] = [x^\alpha \cdot y^\beta, x] = [x^\alpha, x]^{y^\beta}[y^\beta, x] = [y^\beta, x].$$

From (1) we have

$$1 \neq [g_1, x^p, y] = [g_1, x, y]^p = [y^\beta, x^p, y].$$

The proper subgroup $H_1 = \langle x^p, y\rangle$ of G has nilpotency class 3. The contradiction with the condition $G \in \mathcal{M}$ gives $\mho(G) \subseteq Z_2(G)$. The condition $cl(G) = 3$ implies $G_2 \subseteq Z_2(G)$. According to Proposition 2 we have $\Phi(G) \subseteq Z_2(G)$.

3) The conditions $cl(G) = 3$ and $\mho(G) \subseteq Z_2(G)$ implies

$$1 = [a^p, b, c] = [[a, b]^p, c] = [a, b, c]^p$$

for each set of elements $\{a, b, c\} \subset G$. Hence $\exp(G_3) = p$.

Theorem 3. *Let $G \in \mathcal{M}$. If the group G is 3-generated, $G = \langle a_1, a_2, a_3\rangle$, then $p = 3$.*

Proof. 1) Let $G \in \mathcal{M}$ and $d(G) = 3$. Thus for each pair $g, h \in G$ we have

$$[g, h, h] = [h, g, g] = 1. \quad (2)$$

Let $G = \langle a_1, a_2, a_3\rangle$. Denote the commutators $[a_2, a_3]$, $[a_3, a_1]$, $[a_1, a_2]$ by c_1, c_2, c_3 correspondingly. From (2) we have

$$\begin{aligned} [c_1, a_2] = [c_1, a_3] = 1, \\ [c_2, a_1] = [c_2, a_3] = 1, \\ [c_3, a_2] = [c_3, a_1] = 1. \end{aligned} \quad (3)$$

For commutator $h_1 = [a_1 \cdot a_2, a_3]$ we have $h_1 = c_1 c_2^{-1} \pmod{Z(G)}$ and taking into consideration (2) we obtain

$$1 = [h_1, a_1 \cdot a_2] = \\ = [c_1 \cdot c_2^{-1}, a_1 \cdot a_2] = [c_1, a_1][c_2, a_2]^{-1}.$$

Thus $[c_1, a_1] = [c_2, a_2]$. Analogously $[c_1, a_1] = [c_3, a_3]$. Denote by z the element from $Z(G)$ such that

$$z = [c_1, a_1] = [c_2, a_2] = [c_3, a_3]. \quad (4)$$

On the other hand, for each group G the Witt's identity holds:

$$[x, y^{-1}, z]^y \cdot [y, z^{-1}, x]^z \cdot [z, x^{-1}, y]^x = 1$$

for all $x, y, z \in G$. Since $cl(G) = 3$ thus $[x, y, z] \in Z(G)$ and $[x, y^{-1}, z] = [x, y, z]^{-1} = [y, x, z]$. Thus for all $x, y, z \in G$ we may rewrite the Witt's identities in the form:

$$[y, x, z][z, y, x][x, z, y] = 1.$$

Therefore

$$[a_1, a_2, a_3][a_3, a_1, a_2][a_2, a_3, a_1] = 1 \quad (5)$$

So from (4) and definitions c_i, $i = 1, 2, 3$ we have $z^3 = 1$.

We obtain that the minimal groups of nilpotency class 3 with 3 generators exist for $p = 3$ only.

From the proving of the theorem we obtain

Corollary 4. *Let $G \in \mathcal{M}$, G is 3-generated, $G = \langle a_1, a_2, a_3\rangle$ and*

$$c_1 := [a_2, a_3], c_2 := [a_3, a_1], c_3 := [a_1, a_2]. \quad (6)$$

Then following conditions for the commutators hold 1)

$$\begin{aligned} [c_1, a_2] = [c_1, a_3] = 1, \\ [c_2, a_1] = [c_2, a_3] = 1, \\ [c_3, a_2] = [c_3, a_1] = 1; \end{aligned} \quad (7)$$

2) the equality

$$[c_1, a_1] = [c_2, a_2] = [c_3, a_3] = z \quad (8)$$

holds and z has the order 3;
3)

$$[c_1, c_2] = [c_2, c_3] = [c_3, c_1] = 1; \quad (9)$$

and $z \in Z(G)$

It is easy to see that for $p = 3$ a p-group $G = \langle a_1, a_2, a_3\rangle$ satisfying the conditions of Corollary 4 is a minimal group of nilpotency class 3.

The necessary and sufficient conditions for $d(G) = 2$

For $G \in \mathcal{M}$ Theorem 1 implies that $d(G) \leq 3$ and $\Phi(G) \subseteq Z_2(G)$. Now we show that last condition is sufficient for each group with $d(G) = 2$ of nilpotency class 3 to be minimal of nilpotency class 3.

Theorem 5. *Let $d(G) = 2$. Thus $G \in \mathcal{M}$ if and only if $cl(G) = 3$ and $\Phi(G) \subseteq Z_2(G)$.*

Proof. Let $d(G) = 2$, $G = \langle a, b\rangle$, $cl(G) = 3$ and $\Phi(G) \subseteq Z_2(G)$. 2-generated p-group G has $p+1$ maximal subgroups: $M_i = \langle ab^i, \Phi(G)\rangle$ where $i = 0, 1, \ldots, p-1$ and $M_p = \langle b, \Phi(G)\rangle$. Consider $M_0 = \langle a, \Phi(G)\rangle$. For each $g_1, g_2 \in M_0$ holds $g_i = a^{t_i} h_i$ where $h_i \in \Phi(G)$. Thus

$$\begin{aligned}[g_1, g_2] &= [a^{t_1}h_1, a^{t_2}h_2,] = \\ &= [a^{t_1}h_1, h_2][a^{t_1}h_1, a^{t_2}]^{h_2} = \\ &= [a^{t_1}, h_2]^{h_1}[h_1, h_2]([a^{t_1}, a^{t_2}]^{h_1}[h_1, a^{t_2}])^{h_2}.\end{aligned}$$

All factors contain $h_i \in \Phi(G) \subseteq Z_2(G)$, so all of them are the elements of $Z(G)$. From maximality M_0 we have $Z(G) \subseteq Z(M_0)$, so $[g_1, g_2] \in Z(M_0)$ for each $g_1, g_2 \in M_0$, and consequently $cl(M_0) = 2$. Analogously $cl(M_i) = 2$ for each other maximal subgroup M_i ($i = 1, \ldots, p-1, p$). Therefore each proper subgroup of G has a nilpotency class 2 and $G \in \mathcal{M}$.

The necessary and sufficient conditions for $d(G) = 3$

Consider the case $d(G) = 3$ in details.

Theorem 6. *The p-group G with $d(G) = 3$ is minimal of nilpotency class 3 ($G \in \mathcal{M}$) if and only if the next conditions hold*

1) *$p = 3$;*
2) *G asserts the representation*

$$\begin{aligned}G = \langle a_1, a_2, a_3, c_1, c_2, c_3, z \mid\ & z^3 = 1, \\ & c_1^{3^{r_1}} = z^{\mu_1}, c_2^{3^{r_2}} = z^{\mu_2}, c_3^{3^{r_3}} = z^{\mu_3}, \\ & a_1^{3^{m_1}} = c_1^{\beta_{11}} c_2^{\beta_{12}} c_3^{\beta_{13}} z^{\lambda_1}, \\ & a_2^{3^{m_2}} = c_1^{\beta_{21}} c_2^{\beta_{22}} c_3^{\beta_{23}} z^{\lambda_2}, \\ & a_3^{3^{m_3}} = c_1^{\beta_{31}} c_2^{\beta_{32}} c_3^{\beta_{33}} z^{\lambda_3}, \\ & [a_1, a_2] = c_3, [a_2, a_3] = c_1, [a_3, a_1] = c_2, \\ & [c_i, a_i] = z, [c_i, a_j] = [c_i, c_j] = 1 \\ & [c_i, z] = [a_i, z] = 1, (i, j = 1, 2, 3; i \neq j)\rangle,\end{aligned}$$

where parameters m_i, r_j, μ_j satisfy following conditions.

a) *$m_1, m_2, m_3, r_1, r_2, r_3 \geq 1$, $\lambda_i, \mu_j = 0, 1, 2$; $i, j = 1, 2, 3$.*
b) *$r_1 \leq \min(m_2, m_3)$, $r_2 \leq \min(m_1, m_3)$, $r_3 \leq \min(m_1, m_2)$.*
c) *Let $i, j, k = 1, 2, 3$; $i \neq j$, $i \neq k$, $j \neq k$ and $c_i = [a_j, a_k]$.*
   If $r_i < \min(m_j, m_k)$ then $\beta_{jk} \cong \beta_{kj} \cong 0 \pmod 3$, μ_i — arbitrary.
   If $r_i = m_j < m_k$ then $\beta_{jk} \cong -\mu_i \pmod 3$, $\beta_{kj} \cong 0 \pmod 3$.
   If $r_i = m_k < m_j$ then $\beta_{kj} \cong \mu_i \pmod 3$, $\beta_{jk} \cong 0 \pmod 3$.
   If $r_i = m_j = m_k$ then $\beta_{kj} \cong \mu_i \cong -\beta_{jk} \pmod 3$.

Proof. According the Theorem 3 and Corollary 4 the conditions $p = 3$ and commutator relations of G are the necessary conditions for G to be a minimal group of class 3. It is easy to see they are the sufficient ones too. So our aim is to establish the generating relations of such groups more precisely. To do this we will regard G as an extension of an abelian 3-generated group $A = \langle a_1\rangle_{3^{m_1}} \times \langle a_2\rangle_{3^{m_2}} \times \langle a_3\rangle_{3^{m_3}}$ by the abelian subgroup

$$D = \langle c_1, c_2, c_3, z \mid z^3 = 1, \\ c_1^{3^{r_1}} = z^{\mu_1}, c_2^{3^{r_2}} = z^{\mu_2}, c_3^{3^{r_3}} = z^{\mu_3}\rangle,$$

where parameters r_1, r_2, r_3, μ_1, μ_2, μ_3 are under the investigation.

From (7), (8) we have the group A acting on D as an operator group by the following way:

$$\begin{array}{lll} a_1\colon c_1 \mapsto c_1 z & a_2\colon c_1 \mapsto c_1 & a_3\colon c_1 \mapsto c_1 \\ \quad c_2 \mapsto c_2 & \quad c_2 \mapsto c_2 z & \quad c_2 \mapsto c_2 \\ \quad c_3 \mapsto c_3, & \quad c_3 \mapsto c_3, & \quad c_3 \mapsto c_3 z. \end{array} \tag{10}$$

To describe the extensions we use the following well known theorem.

Theorem 7 (M. Hall)**.** *Let A be a group with generators a_1, a_2, $\ldots$, a_n and relations $\varphi_1(a_1, a_2, \ldots, a_n) = 1$, $\varphi_2(a_1, a_2, \ldots, a_n) = 1$, $\ldots$, $\varphi_k(a_1, a_2, \ldots, a_n) = 1$. To determine an extension of the group A by abelian D we must determine*

1) *an acting of A on D as a group of operators;*
2) *a mapping of the relations set $\varphi_1(a_1, a_2, \ldots, a_n)$, $\varphi_2(a_1, a_2, \ldots, a_n)$, $\ldots$, $\varphi_k(a_1, a_2, \ldots, a_n)$ to D:*

$$\psi\colon \varphi_i \mapsto \alpha_i \in D, \qquad i = 1, \ldots, k, \tag{11}$$

where α_i ($i = 1, \ldots, k$) satisfy following conditions:

$$\prod_i \alpha_i^{u_i} = 1 \tag{12}$$

for each solution u_i of equation system with coefficients from integer group ring ZA

$$\sum_i s_{ji} u_i = 1, \qquad j = 1, \ldots, n, \tag{13}$$

where coefficients s_{ji} are obtained from the relation φ_i by the following way. Let ξ_j — some mutually commuting elements, then using the collections process we obtain

$$\varphi_i(a_1 \cdot \xi_1, a_2 \cdot \xi_2, \dots, a_n \cdot \xi_n) = \\ = \varphi_i(a_1, a_2, \dots, a_n) \cdot \xi_1^{s_{1i}} \cdot \xi_2^{s_{2i}} \cdot \dots \cdot \xi_n^{s_{ni}}$$

The relations set of A is

$$\varphi_1 = [a_1, a_2], \varphi_2 = [a_2, a_3], \varphi_3 = [a_3, a_1], \quad (14)$$

$$\varphi_4 = a_1^{3^{m_1}}, \varphi_5 = a_2^{3^{m_2}}, \varphi_6 = a_3^{3^{m_3}}. \quad (15)$$

Denote

$$\alpha_1 = \psi(\varphi_1), \alpha_2 = \psi(\varphi_2), \dots, \alpha_6 = \psi(\varphi_6). \quad (16)$$

Every choosing of set $\alpha_1, \alpha_2, \dots, \alpha_6 \in D$ satisfying (12) gives some group G, which is an extension A by D with action (10). Without loss of generality we may choose $\alpha_1 = c_3$, $\alpha_2 = c_1$, $\alpha_3 = c_2$. For this choosing the relations of the group G satisfy all conditions from Corollary 4.

For given group A the condition (13) may be written as

$$\begin{aligned} &a_1((a_2 - 1)u_1 + (1 - a_3)u_3) + \pi_1 u_4 = 0, \\ &a_2((1 - a_1)u_1 + (a_3 - 1)u_2) + \pi_2 u_5 = 0, \\ &a_3((1 - a_2)u_2 + (a_1 - 1)u_3) + \pi_3 u_6 = 0, \end{aligned} \quad (17)$$

where $\pi_i = 1 + a_i + a_i^2 + \dots + a_i^{p^{m_i}-1}$, $i = 1, 2, 3$.

The coefficients of this system of linear equations are elements of the group ring ZA. All of them are the divisors of zero in ZA, because

$$(a_i - 1)\pi_i = 0, \qquad i = 1, 2, 3. \quad (18)$$

Consider some solutions of the system (17)

1)

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, 0, 0, a_1 - 1, 0, 0). \quad (19)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, 0, 0, 0, a_2 - 1, 0). \quad (20)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, 0, 0, 0, 0, a_3 - 1). \quad (21)$$

2)

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (\pi_1, 0, 0, a_1(1 - a_2), 0, 0). \quad (22)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (\pi_2, 0, 0, 0, a_2(a_1 - 1), 0). \quad (23)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, \pi_2, 0, 0, a_2(1 - a_3), 0). \quad (24)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, \pi_3, 0, 0, 0, a_3(a_2 - 1)). \quad (25)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, 0, \pi_1, a_1(a_3 - 1), 0, 0). \quad (26)$$

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (0, 0, \pi_3, 0, 0, a_3(1 - a_1)). \quad (27)$$

3)

$$(u_1, u_2, u_3, u_4, u_5, u_6) = \\ = (a_3 - 1, a_1 - 1, a_2 - 1, 0, 0, 0). \quad (28)$$

According the condition (12) the solution (19) gives $\alpha_4^{a_1-1} = 1$. By the another terms an image of relation $\varphi_1 = a_1^{p^{m_1}}$ in D must commute with a_1. Thus we may suppose $\alpha_4 = c_1^{\beta_{11}} c_2^{\beta_{12}} c_3^{\beta_{13}} z^{\lambda_1}$, where $\beta_{11} \cong 0 \mod 3$. Analogously, from (20), (21) we have the images of relations $a_2^{p^{m_2}}$, $a_3^{p^{m_3}}$ must commute with a_2, a_3 respectively and suppose $\alpha_5 = c_1^{\beta_{21}} c_2^{\beta_{22}} c_3^{\beta_{23}} z^{\lambda_2}$, $\alpha_6 = c_1^{\beta_{31}} c_2^{\beta_{32}} c_3^{\beta_{33}} z^{\lambda_3}$, where $\beta_{22} \cong \beta_{33} \cong 0 \mod 3$.

The solution (22) gives $\alpha_1^{\pi_1} = \alpha_4^{a_1(a_2-1)}$. From (16) and (19) we have $c_3^{\pi_1} = [\alpha_4, a_2]$. Taking into account that $c_3^{a_1} = c_3$, $[\alpha_4, a_2] = z^{\beta_{12}}$ we obtain $c_3^{3^{m_1}} = z^{-\beta_{12}}$. Comparing with relations of D we may conclude that 1) $m_1 \geq r_3$, 2) $\beta_{12} = -\mu_3$ for $m_1 = r_3$ and $\beta_{12} = 0$ for $m_1 \leq r_3$.

The next solution gives 1) $m_2 \geq r_3$, 2) $\beta_{21} = \mu_3$ for $m_2 = r_3$ and $\beta_{21} = 0$ for $m_2 \leq r_3$.

Analogously from (24)–(27) we obtain the other conditions for β_{jk} and μ_i.

(28) gives $[a_3, \alpha_3^{-1}][a_2, \alpha_2^{-1}][a_1, \alpha_1^{-1}] = 1$, or $[a_3, c_3^{-1}][a_2, c_2^{-1}][a_1, c_1^{-1}] = 1$, which is equivalent (5).

Conclusion

We obtain the necessary and sufficient condition for finite p-group to be a minimal group of nilpotency class 3. Thus the finite p-group G is a minimal group of nilpotency class 3 if and only if G is 2-generated p-group of nilpotency class 3 with $\Phi(G) \subseteq Z_2(G)$ or G is 3-generated 3-group with relations pointed in the Theorem 5.

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p-ГРУПИ КЛАСУ НІЛЬПОТЕНТНОСТІ 3, У ЯКИХ КОЖНА ВЛАСНА ПІДГРУПА МАЄ КЛАС НІЛЬПОТЕНТНОСТІ МЕНШЕ, НІЖ 3

Визначено всі скінченні p-групи класу нільпотентності 3, у яких кожна власна підгрупа має клас нільпотентності менше, ніж 3, і таким чином вирішено проблему № 87, поставлену Я. Г. Берковичем в [1].

Ключові слова: p-група, підгрупа, клас нільпотентності.

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