Generation of helical acoustic modes using anisotropic crystals

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Abstract. We have analysed a phenomenon of conical refraction for the acoustic waves on the example of trigonal α-BaB_2O_4 and β-BaB_2O_4 crystals. Different types of manifestation of this conical refraction have been shown to appear in the alternative cases when the acoustic waves propagate along the symmetry-related acoustic axes or along any other acoustic axes. It has been shown that propagation along the acoustic axes of acoustic phonons with the spins equal to $\pm\hbar$ should lead to spin-to-obit interaction, with the appearance of an orbital angular momentum and an acoustic vortex of the unit charge. We have also compared the peculiarities of dislocations of the phase front appearing for the acoustic waves propagating along the acoustic axes related to symmetry and along the rest of the acoustic axes.

Keywords: acoustic waves, conical refraction, acoustic phonons, spin-orbit interaction, crystals

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1. Introduction

A doughnut acoustic mode or an acoustic wave bearing orbital angular momentum [1–3] can be considered as an analogue of optical wave that bears a vortex, a subject that has been extensively studied in the frame of singular optics [4]. Though the acoustic and optical waves reveal a lot of common futures, propagation of the acoustic waves in material media is more complicated if compared to the optical ones. The acoustic wave characteristics, such as acoustic velocity anisotropy, polarisation etc., follow from a known Christoffel equation and are concerned with a fourth-rank elastic stiffness tensor, while the optical wave parameters can be described following from an optical indicatrix equation and a corresponding second-rank tensor of dielectric impermeability [5]. It is obvious that the difference in the ranks of the relevant tensors should give rise to a variety of different effects if one compares propagation of the acoustic waves with that of the optical ones. One of the basic differences is that the acoustic waves can have longitudinal polarisation, though the electromagnetic waves in the usual approximation reveal only a transversal polarisation. In fact, longitudinal acoustic waves can bear only some orbital angular momentum, while the corresponding spin angular momentum must be equal to zero. It is necessary to notice that one of the methods used for generation of optical vortices on the basis of anisotropic crystals consists in spin-to-orbit conversion of the angular momentum in the conditions of a conical refraction [6]. The helical optical modes can also be generated using a number of other methods, e.g., computer-synthesised holograms, *q*-plates, etc. [7–9].

Recently we have shown that the generation of helical optical modes can be carried out while utilising the crystals of trigonal or cubic classes subjected to torsion deformation [10, 11], or while applying an external, conically shaped electric field [12, 13]. These techniques allow operating the efficiency of the spin-to-orbit conversion by a torque moment or an electric field. A screw singularity of a scalar acoustic field (or an acoustic vortex) can be quite easily created even for the longitudinal waves, using piezoelectric transducers with a helical change of their thickness [14]. Despite of the fact that the longitudinal acoustic waves can propagate in liquids and gaseous media, such a simple method for generation of acoustic vortices manifests some drawbacks. For instance, the frequency of the acoustic wave is strictly determined by a transducer thickness and so it cannot be tuned, whereas the charge of the acoustic vortex thus generated is rigidly determined by a helicity of the transducer. The methods mentioned above cannot be applied for the conversion of, e. g., a plane acoustic wave to a helical one. Besides, purely longitudinal acoustic waves cannot be used for creating singularities of vector fields.

As already mentioned, one of the methods for the spin-to-orbit conversion of optical waves consists in using of a conical refraction occurring in optically biaxial crystals [6]. The charge of the optical vortex appearing due to such a conversion is equal to unity. We are also to stress that acoustic anisotropy would lead to an effect peculiar for the acoustic waves propagating along so-called acoustic axes in crystals, which should be an analogue of the conical refraction in optics. This is a conical refraction for the acoustic waves. The phenomenon can serve as a technique for creating acoustic vector field singularities and an orbital angular momentum of the acoustic waves, and so acoustic vortices. Moreover, the acoustic conical refraction is usually stronger when compared with its optical counterpart: the angles describing the conical refraction of the acoustic waves can reach tens of angular degrees, while for the optical waves we have the typical values that do not exceed several degrees.

The aim of the present work is the analysis of conditions under which the acoustic vortices can be generated in case if the acoustic conical refraction takes place in crystals. As an example, we have considered trigonal crystals of α-BaB_2O_4 (ABO) and β-BaB_2O_4 (BBO).

2. Results and discussion

2.1. Symmetry analysis

In general, the acoustic axes for the two transverse waves propagating in crystals should appear in the directions for which the velocities of these waves are equal to each other. Following from a so-called ‘first Christoffel tensor’ (a tensor associated with the phase velocities), the symmetry-related acoustic axes are only those, which are directed along N-fold symmetry axes (with $N > 2$) or perpendicular to a four-fold symmetry axis in tetragonal crystals (see, e.g., [5]). Thus, all of the three, four- and six-fold axes, along with the symmetry axes of infinite order, should simultaneously constitute the acoustic axes.

In order that the conical refraction appeared, a transverse wave propagating along these axes should be extraordinary. This condition is analogical to the statement that the wave vector of the wave should not represent the eigenvector of a so-called ‘second Christoffel tensor’ (a tensor associated with the group velocities) or, what is the same, the elastic stiffness coefficients C_{14} should not be equal to zero. This condition is satisfied for the crystals, of which symmetry groups contain three-fold axes, i.e. the crystals of the trigonal system. The semi-angle of the conical refraction in these crystals is determined by the equation

$$\psi = \arctan(C_{14} / C_{44}) . \qquad (1)$$

Notice that the point symmetry groups of crystals of the cubic system also contain three-fold axes parallel to <111> directions.

Let us rewrite the elastic stiffness tensor of the cubic crystals in the coordinate system whose Z' axis is parallel to the [111] direction. The appropriate components will be equal to

$C'_{14} = \frac{1}{6}(C_{11} - C_{12} - 2C_{44})$ and $C'_{44} = \frac{1}{3}(C_{11} - C_{12} + 2C_{44})$. Then the angle of the conical refraction for these crystals should be given by

$$\psi = \arctan\left(\frac{C'_{14}}{C'_{44}}\right) = \arctan\frac{C_{11} - C_{12} - 2C_{44}}{2(C_{11} - C_{12} + C_{44})}. \quad (2)$$

On the other hand, the equality of the phase velocities of the transverse acoustic waves can happen not only for the directions related to the symmetry, but also for some other directions determined by specific relations among the elastic stiffness components of a given crystalline material.

2.2. Acoustic wave surfaces, energy flow surfaces, and conical refraction in α-BaB_2O_4 and β-BaB_2O_4 crystals

The ABO and BBO crystals belong respectively to the point symmetry groups $\bar{3}m$ and 3m. They represent a large family of crystalline borates. We have chosen these crystals for our analysis, in particular, because they have the same chemical content. In fact, these crystals belong to different structural phases: ferroelectric in case of the BBO and ferroelastic in case of the ABO crystals [15]. Besides, the BBO is a well-known nonlinear optical material [16, 17], while the ABO represents a good acoustooptic material [18–20]. The both crystals are transparent in a wide spectral range [17, 20] and possess high enough optical damage thresholds [20]. Let us start our analysis with considering the obliquity of the acoustic energy flow with respect to the wave vector direction and constructing surfaces of acoustic phase and group velocities.

Propagation of plane acoustic waves in crystals can be described by the well-known Christoffel equation [5]:

$$C_{ijkl} m_j m_k p_l = \rho v^2 p_i , \quad (3)$$

were C_{ijkl} denote the components of the elastic stiffness tensors (numerical values of those components for the ABO and BBO crystals have been taken from our recent works [17, 19]), m_k, m_l the components of the unit wave vector, p_i, p_l the components of the unit displacement vector, ρ the density of material, and v the phase velocity. The phase acoustic velocities are nothing but the eigenvalues of Eq. (3). It follows from Eq. (1) that three acoustic waves with mutually orthogonal displacement vectors can propagate along a given direction.

The directions of the phase and group velocities are different in acoustically anisotropic media. This can manifest itself as obliquity of the acoustic energy flow direction with respect to the direction of the wave vector. The components W_j of the group velocity vector can be determined from the transformed Christoffel equation [5]:

$$W_j = \frac{1}{\rho v} C_{ijkl} p_i p_l m_k . \quad (4)$$

One can calculate the angle between the directions of the energy flow and the wave vector (i.e., the obliquity angle) using the relation [5]

$$\tan(\varphi - \psi) = \frac{1}{v(\varphi)} \frac{\partial v}{\partial \varphi}, \quad (5)$$

were φ is the angle between the crystallographic axis and the wave vector direction, ψ the angle between the same crystallographic axis and the direction of the acoustic energy flow, and $v(\varphi)$ the acoustic wave velocity. The relations describing changes in the acoustic wave velocity occurring with changing propagation direction ($\vartheta(\varphi) = \partial v / \partial \varphi$) can also be obtained from the Christoffel equation. Finally, the same Christoffel equation enables calculating the angle Δ that defines deviation of polarisation of the acoustic wave propagating in (100) plane from the purely longitudinal type:

$$\Delta = \varphi - \frac{1}{2}\arctan\left[\frac{\sin\varphi(C_{23} + C_{44})}{\cos\varphi(C_{22} - C_{44}) + \sin\varphi(C_{44} - C_{33})}\right]. \tag{6}$$

Non-orthogonality of the quasi-transverse waves can also be determined from Eq. (6), though in the latter case the angle 90 deg should be added to the r. h. s.

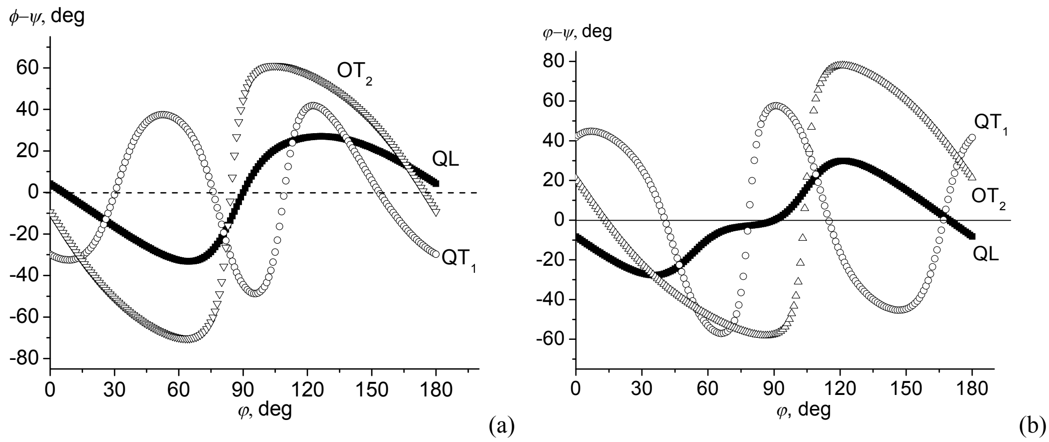

Fig. 1. Obliquity of acoustic energy flow in the ABO (a) and BBO (b) crystals taking place in (100) plane.

As one can see from Fig. 1, the obliquity of the acoustic energy flow in the ABO and BBO crystals reveals a number of peculiar features. When the angle φ equals to 90 deg (i.e., the acoustic wave propagates along [001] direction), the obliquity angles for the both transverse waves are equal by modulo and opposite in signs. Then, the obliquity angles for the both waves reach almost extreme values. For instance, this angle is equal to –44.3 deg and +44.3 deg respectively for the waves QT_1 and QT_2 propagating in the ABO crystals. In case of the BBO crystals we have –57.6 deg for the QT_2 wave and +57.6 deg for the QT_1 one. Thus, the obliquities for these transverse waves are symmetric with respect to the AA_1 axis (i.e., the Z axis). In fact, here we deal with the conical refraction occurring in the both crystals.

As seen from Fig. 2, the velocities of the transverse acoustic waves become equal at $\varphi = 90$ deg , i.e. in case when these waves propagate along the acoustic axis AA_1 coinciding with the three-fold symmetry one. Calculating the semi-angles of the internal conical refraction on the basis of Eq. (1) and our stiffness tensor data [17, 19] ($C_{14} = -5.5\times10^9$ Pa and $C_{44} = 5.6\times10^9$ Pa for the ABO and $C_{14} = 12.3\times10^9$ Pa and $C_{44} = 7.8\times10^9$ Pa for the BBO crystals), one obtains $\psi = 44.5$ and 57.6 deg respectively for the ABO and BBO.

As seen from Fig. 2, an additional acoustic axis (AA_2) exists in the both ABO and BBO crystals, which does not coincide with the directions defined by the symmetry. This axis is determined by the angles $\varphi = 64$ and 140 deg for the ABO and BBO crystals, respectively. The φ angle for the ABO crystals almost corresponds to the extreme obliquity for the QT_2 wave (the corresponding obliquity angle is equal to –70.26 deg). However, the obliquity angle for the QT_1 wave is equal to +30.37 deg in this case. The manifestations of obliquity of the energy flow in the BBO crystals are similar to those for the ABO in the case when the acoustic wave propagates along the acoustic axis AA_2. Namely, we have the obliquity –43.4 deg for the QT_1 wave and +69.3 deg for the QT_2 wave.

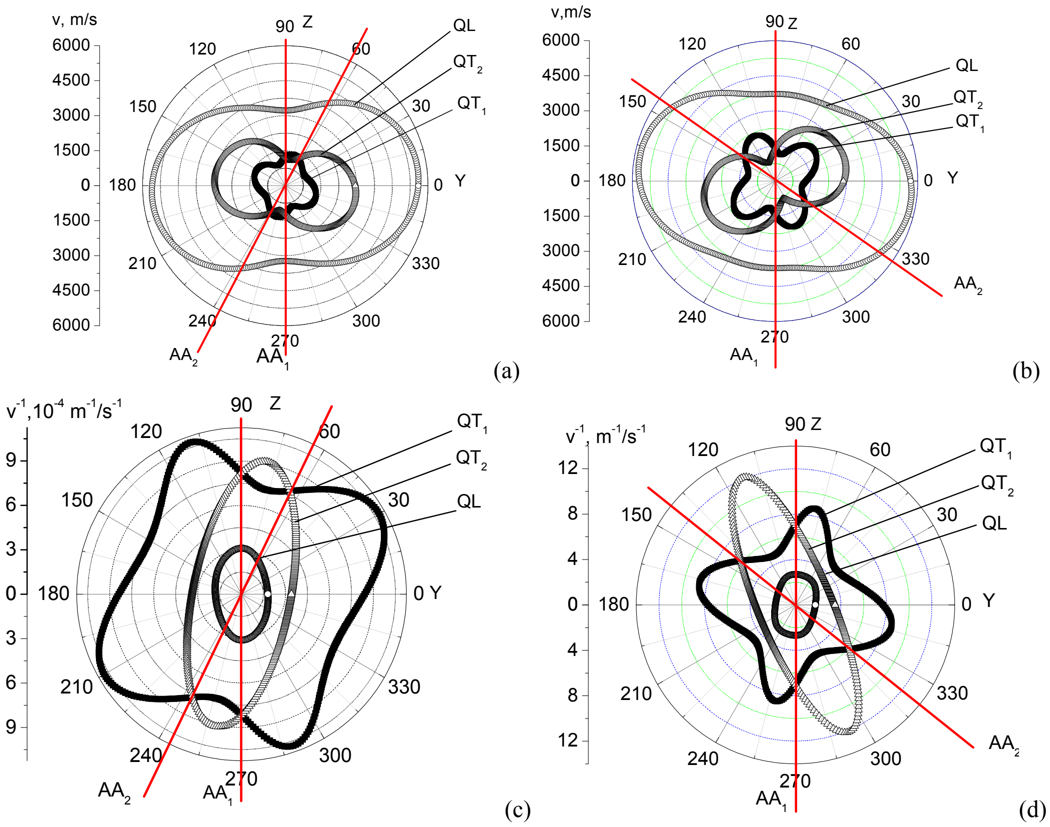

Fig. 2. Cross sections by (100) plane of the acoustic wave velocity surfaces (a, b) and the acoustic slowness surfaces (c, d) for the ABO (a, c) and BBO (b, d) crystals.

It is obvious that, according to the symmetry, the obliquity of the Poynting vectors taking place for the waves propagating along the AA_1 axis should be the same for all of the planes which contain that axis. This implies that the surface created by these Poynting vectors should have the shape of a hollow canonical cone. However, the similar surface of the acoustic beams, which is created while the acoustic waves propagate along the acoustic axis AA_2, can be more complicated: it can have the shapes of an elliptical cone or even a plane. This result conforms with that predicted for tetragonal crystals for the case when the acoustic waves propagate perpendicular to the four-fold symmetry axis [21]. In the latter case, the vortex appearing as a result of spin-orbit interaction should also be non-canonical, provided that the incoming acoustic wave is circularly polarised. At the same time, the phase front of the emergent acoustic wave should possess a mixed screw-edge dislocation, or even an edge dislocation (see, e.g., the studies [10, 22] on the similar optical vortices).

The cross sections of the group velocities obtained for the ABO and BBO crystals are shown in Fig. 3. Comparing the results of Fig. 2b, d and Fig. 3b obtained for the BBO crystals, one can see that each acoustic axis of the phase velocities has its counterpart, an acoustic axis of the group velocities. The RA_1 axis is inclined by –17 deg with respect to the AA_1 axis, while the angle between the RA_2 and AA_2 axes is equal to +17 deg. However, the two acoustic exes of the phase velocities (AA_1 and AA_2) in the ABO crystals (cf. the data displayed in Fig. 2a, c and Fig. 3a) have a common corresponding acoustic axis of the group velocities ($RA_{1,2}$), which is rotated by –17 deg with respect to the AA_1 axis. Since an effect of external conical refraction is to be observed when the incident beam propagates along the group-velocity axis, the two conical refractions (i.e., those corresponding to the AA_1 and AA_2 axes) should manifest themselves simultaneously in case of the ABO crystals.

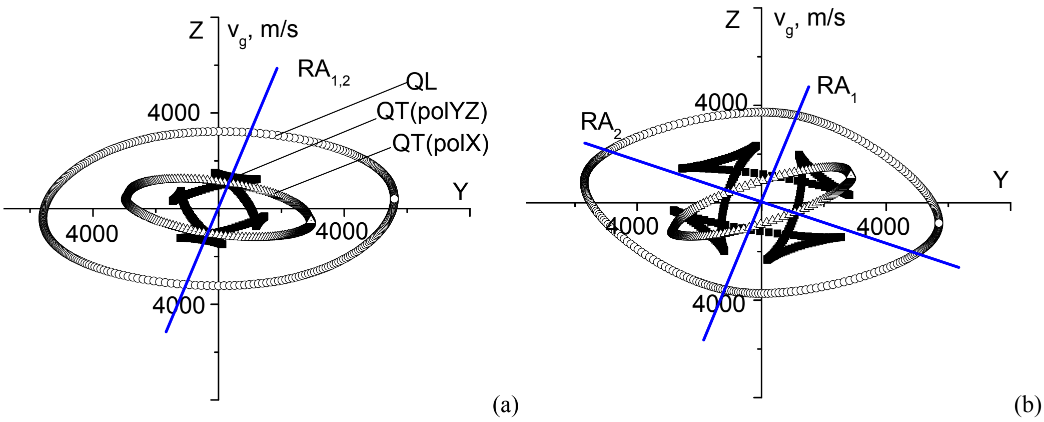

Fig. 3. Cross sections by (100) plane of the acoustic group velocity surfaces for the ABO (a) and BBO (b) crystals.

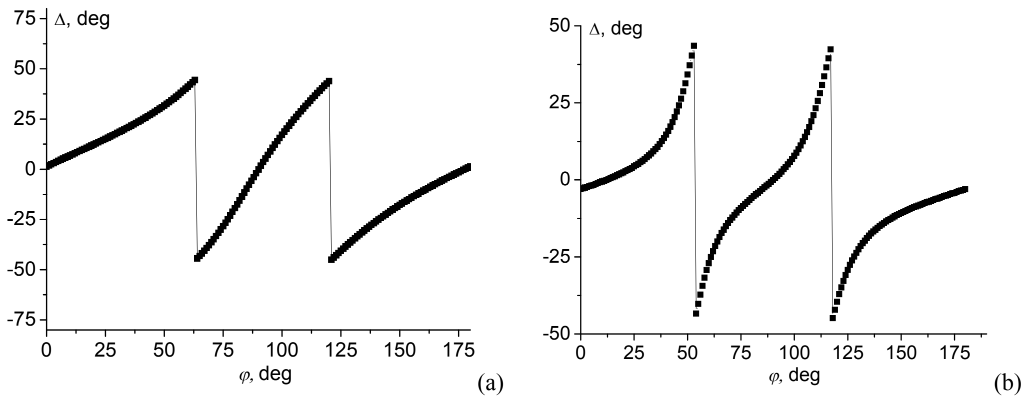

Fig. 4. Dependences of non-orthogonality angle for the QT_1 wave on the wave vector direction for the ABO (a) and BBO (b) crystals.

The deviations from orthogonality typical for the transverse wave QT_1 propagating in the both crystals are shown in Fig. 4. This wave remains purely transverse when propagating along the AA_1 axis in the ABO and BBO crystals, although it becomes quasi-transverse when the propagation along the AA_2 axis is considered. For the latter case, the angle of deviation from the purely transverse polarisation is close to zero for the ABO and equal to –15 deg for the BBO crystals. This fact means that an longitudinal wave will also propagate in the above crystals when a transverse wave is excited in the directions of acoustic axes, which are not related to the symmetry.

3. Conclusions

In the present work we have analysed the acoustic phase and group velocity surfaces for the trigonal crystals of α-BaB_2O_4 and β-BaB_2O_4. It has been found that the transverse acoustic waves of orthogonal polarisations propagating along the acoustic axes, which are parallel to the three-fold symmetry axis, reveal the obliquity angles equal in magnitudes and opposite in signs. Then the corresponding Poynting vectors form a canonical cone of the internal conical refraction. On the contrary, when the acoustic waves propagate along the acoustic axes, which are not symmetry-related, then the absolute values of the obliquity angles for the transverse waves with orthogonal polarisations are not equal to each other. It should be noticed that the surface created by the Poynting vectors in the latter case can acquire shapes of an elliptical cone or even a plane. Furthermore, these waves are not purely transverse. The latter fact should impose appearance of longitudinal acoustic waves, with their phase fronts being dislocation-free.

From our analysis it also follows, by analogy with the conical refraction of optical waves, that any incident circularly polarised acoustic wave (with the spin of the incident phonon being equal to $\pm\hbar$) propagating along the acoustic axis should give rise to spin-obit interaction of the acoustic phonons. This fact means that the emergent wave should acquire an orbital angular momentum and so should bear the acoustic vortex with the unit charge. Nonetheless, the characters of dislocations of the phase fronts differ essentially for the acoustic waves propagating along the acoustic axes related to the macroscopic symmetry and along all the other acoustic axes. The phase front contains a pure screw dislocation in the first case and a mixed screw-edge or even edge dislocation in the second one.

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Анотація. *На прикладі кристалів АВО і ВВО проаналізовано конічну рефракцію акустичних хвиль. Виявлено, що така конічна рефракція виявлятиметься по-різному у разі поширення акустичних хвиль уздовж симетрійно обумовлених акустичних осей та будь-яких інших акустичних осей. Показано, що за умови поширення акустичного фонона зі спіном* $\pm\hbar$ *уздовж акустичної осі матимемо спін-орбітальну взаємодію з появою орбітального кутового моменту акустичної хвилі та акустичного вихору з одиничним зарядом. Обговорено особливості дислокацій фазового фронту акустичної хвилі, яка поширюється вздовж акустичних осей, обумовлених симетрією, та інших акустичних осей.*