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Adapted statistical experiments

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Abstract. We study statistical experiments with random change of time, which transforms a discrete stochastic basis in a continuous one. The adapted stochastic experiments are studied in continuous stochastic basis in the series scheme. The transition to limit by the series parameter generates an approximation of adapted statistical experiments by a diffusion process with evolution.

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Introduction

Statistical experiments (CE) are defined as averaged sums of random variables with a finite number of possible values. In particular, only two possible values mean that the sample values indexes the presence or absence of a certain feature.

Statistical experiments are defined in a discrete stochastic basis

$$\mathfrak{B}_{\mathbb{N}} = (\Omega,\, \mathfrak{F},\, (\mathfrak{F}_k,\, k \in \mathbb{N}),\, \mathcal{P})$$

with filtration $(\mathfrak{F}_k,\, k \in \mathbb{N} = \{0, 1, \ldots\})$ on a probability space $(\Omega,\, \mathfrak{F},\, \mathcal{P})$.

This paper deals with adapted statistical experiments defined by a random change of time [1, Ch. 1], which transforms a discrete stochastic basis $\mathfrak{B}_{\mathbb{N}}$ in a continuous one:

$$\mathfrak{B}_T = (\Omega,\, \mathfrak{G},\, (\mathfrak{G}_t,\, t \in \mathbb{R}_+),\, \mathcal{P}).$$

The adapted statistical experiments in continuous stochastic basis $\mathfrak{B}_T$ is considering in series scheme with a small series parameter $\varepsilon \to 0$, $\varepsilon > 0$. The limit passage, by $\varepsilon \to 0$, generates approximation of adapted statistical experiments by a diffusion process with evolution [2, Ch. I].

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1. Statistical experiments and evolutionary processes

Statistical experiments in a discrete stochastic basis $\mathfrak{B}_{\mathbb{N}}$ are defined as averaged sums of the sample random variables $(\delta_n(k), 1 \le n \le N)$, $k \ge 0$, identically distributed and jointly independent by different $n \in [1, N]$, for a fixed $k \ge 0$, with two possible values ± 1.

$$S_N(k) := \frac{1}{N} \sum_{n=1}^{N} \delta_n(k) \ , \quad k \ge 0. \tag{1}$$

The discrete parameter $k \in \mathbb{N}$ defines evolution by growing the discrete time k (called also stage).

Statistical experiment (1) is defined by the difference of the positive frequencies:

$$S_N(k) = S_N^+(k) - S_N^-(k) \ , \quad k \ge 0,$$
$$S_N^\pm(k) := \frac{1}{N} \sum_{n=1}^{N} \delta_n^\pm(k) \ , \quad \delta_n^\pm(k) := I\{\delta_n(k) = \pm 1\}.$$

The predictable component of binary and of frequency statistical experiments are defined by the corresponding conditional expectations:

$$C(k+1) := E[\delta_n(k+1) \,|\, S_N(k) = C(k)] \ , \quad 1 \le n \le N \ , \quad k \ge 0,$$
$$P_\pm(k+1) := E[\delta_n^\pm(k+1) \,|\, S_N^\pm(k) = P_\pm(k)] \ , \quad 1 \le n \le N \ , \quad k \ge 0$$

and does not depend on the sample size N.

The dynamics, by k of the predictable components of statistical experiment $S_N(k)$ and $S_N^\pm(k)$ are determined by *evolutionary processes*:

$$C(k+1) = E[S_N(k+1) \,|\, S_N(k) = C(k)] \ , \quad k \ge 0, \tag{2}$$

$$P_\pm(k+1) = E[S_N^\pm(k+1) \,|\, S_N^\pm(k) = P_\pm(k)] \ , \quad k \ge 0. \tag{3}$$

The following obvious identities take place:

$$C(k) = P_+(k) - P_-(k) \ , \quad P_+(k) + P_-(k) \equiv 1 \ , \quad k \ge 0.$$

Hence the relations

$$P_\pm(k) = \frac{1}{2}\big[1 \pm C(k)\big] \ , \quad k \ge 0,$$

define the relationship between evolutionary processes (2) and (3).

The predictable components evolution is given for increments as follows:

$$\Delta C(k+1) := C(k+1) - C(k) = E[\Delta S_N(k+1) \,|\, S_N(k) = C(k)],$$
$$\Delta P_{\pm}(k+1) := P_{\pm}(k+1) - P_{\pm}(k) = E[\Delta S_N^{\pm}(k+1) \,|\, S_N^{\pm}(k) = P_{\pm}(k)].$$

The form of regression functions of increments are postulated on the principle of "stimulation and deterrence":

$$V_0(p_+, p_-) := p_+ p_- (V_+ p_+ - V_- p_-) \ , \quad -1 < V_{\pm} < +1. \tag{4}$$

The basic assumption 1. The dynamics of evolutionary processes (2)–(3) is given by the difference evolution equations with regression function (4):

$$\Delta C(k+1) = -2V_0(P_+(k), P_-(k)) \ , \quad k \geq 0,$$
$$\Delta P_{\pm}(k+1) = \mp V_0(P_+(k), P_-(k)).$$

The equilibrium is determined by the balance of evolutionary processes increments.

$$\rho = \overline{V}/V \ , \quad \overline{V} := V_- - V_+ \ , \quad V := V_- + V_+,$$
$$\rho = \rho_+ - \rho_- \ , \quad \rho_{\pm} = V_{\mp}/V \ .$$

So the evolution of statistical experiments (1) is determined by the regression function of increments

$$V_0(c) = \frac{1}{4} V (1 - c^2)(c - \rho) \ , \quad |c| \leq 1, \tag{5}$$

$$V_0^{\pm}(p_{\pm}) = V p_+ p_- (p_{\pm} - \rho_{\pm})$$

given by cubic parabola with three equilibria: ± 1, ρ ($|\rho| < 1$).

Now the regression function of increments is convenient to express in terms of fluctuations, that is, through the deviations of the statistical experiment values from the equilibrium.

The basic assumption (conclusion). The evolution of statistical experiments (1) is given by the difference evolution equations

$$\Delta C(k+1) = -V_0(C(k)) \ , \quad k \geq 0, \tag{6}$$

$$\Delta P_{\pm}(k+1) = -V_0^{\pm}(P_{\pm}(k)) \ , \quad k \geq 0.$$

2. Stochastic dynamics of statistical experiments

The stochastic component is expressed by martingale-differences

$$\Delta\mu_N(k+1) := \Delta S_N(k+1) - E[\Delta S_N(k+1) \mid S_N(k)] \ , \quad k \geq 0. \tag{7}$$

Given the difference evolution equation (6), the martingale-differences (7) have the following representation

$$\Delta\mu_N(k+1) = \Delta S_N(k+1) + V_0(S_N(k)) \ , \quad k \geq 0. \tag{8}$$

Conclusion 1. The SE increments are determined by the sum two components

$$\Delta S_N(k+1) = -V_0(S_N(k)) + \Delta\mu_N(k+1) \ , \quad k \geq 0. \tag{9}$$

The predictable component $V_0(S_N(k))$, $k \geq 0$, is given by the regression function of increments (5).

The martingale-differences (7) are characterized by the first two moments

$$E\Delta\mu_N(k+1) = 0, \ E[(\Delta\mu_N(k+1))^2 \mid S_N(k)] = \sigma^2(S_N(k))/N, \ k \geq 0. \tag{10}$$

The dispersion of the stochastic components has explicit form [3]:

$$\sigma^2(c) = 1 - V^2(c) \ , \quad V(c) = c - V_0(c) \ , \quad |c| \leq 1. \tag{11}$$

The stochastic dynamics of SE $S_N(k)$, $k \geq 0$, is specified by the stochastic difference equation (9)–(11).

The properties of the stochastic component allow specifications.

Lemma 1. *The stochastic component, defined by martingale-differences* (8), *has the following representation:*

$$\Delta\mu_N(k+1) = \frac{1}{N}\sum_{n=1}^{N} \beta_n(k+1) \ , \quad k \geq 0. \tag{12}$$

The sample variables $\beta_n(k+1)$, $0 \leq n \leq N$, $k \geq 0$, *take two values:*

$$\beta_n(k+1) = \Big\{ \pm 1 - V(C(k)), \ \textit{with probability} \ \ P_\pm(k+1) \Big\}, \ k \geq 0, \tag{13}$$

where

$$P_\pm(k+1) = \frac{1}{2}\big[1 \pm C(k+1)\big] = \frac{1}{2}\big[1 \pm V(C(k))\big].$$

The latter equality provides the predictability of evolutionary process $C(k)$, $k \geq 0$.

Conclusion 2. The stochastic component (12) has Bernuolli distribution:

$$B_N(\nu;\, V(C(k))) = P\{\Delta\mu_N(k+1) = \nu - V(C(k)) \,|\, S_N(k) = C(k)\}$$
$$= \frac{N!}{N_+!N_-!} P_+^{N_+}(k+1) P_-^{N_-}(k+1), \qquad (14)$$

where

$$N_\pm/N = \frac{1}{2}[1 \pm \nu]\ , \quad \nu = \nu_+ - \nu_-\ , \quad \nu_\pm = N_\pm/N,$$

with the first two moments:

$$E[\beta_n(k+1) \,|\, S_N(k)] = 0\ , \quad \forall k \geq 0,$$
$$E[\beta_n^2(k+1) \,|\, S_N(k)] = \sigma^2(S_N(k)) = 1 - V^2(S_N(k)).$$

Now the SE dynamics has two interpretations.

- The increments $\Delta S_N(k)$ are defined by difference equation (9), in which the stochastic component has Bernoulli distribution (14).
- The probabilities (13) are defined by the Bernoulli distribution (14) of the stochastic component at a fixed k-th stage.

3. Adapted statistical experiments

The transition from discrete stochastic basis $\mathfrak{B}_\mathbb{N} = (\Omega, \mathfrak{F}, (\mathfrak{F}_k,\, k \in \mathbb{N}), \mathcal{P})$ to continuous stochastic basis $\mathfrak{B}_T = (\Omega, \mathfrak{G}, (\mathfrak{G}_t,\, t \in \mathbb{R}_+), \mathcal{P})$ is implemented by a *random change of time*

$$\nu(t),\, t \geq 0\ , \quad \nu(0) = 0.$$

The growing process $\nu(t)$, $t \geq 0$, that is everywhere right-continuous and has left limits everywhere, is determined by Markov jumping points:

$$\tau_k := \inf\{t\, :\, \nu(t) \geq k\}\ , \quad k \in \mathbb{N}.$$

The regularity of $\nu(t)$, $t \geq 0$ is provided by the following condition:

$$P\{\tau_k < +\infty\}\ , \quad \forall k > 0. \qquad (15)$$

The counting process of recovery $\nu(t)$, $t \geq 0$, is considered, for simplicity, as a stationary Poisson process with with exponentially distributed intervals of recovery $\theta_{k+1} := \tau_{k+1} - \tau_k$, $k \geq 0$:

$$P\{\theta_{k+1} \geq t\} = \exp(-qt)\ , \quad 0 < q < +\infty\ , \quad t \geq 0.$$

It is known that the compensator of Poisson process $\nu(t)$, $t \geq 0$, is given by the formula:

$$E\nu(t) = qt \ , \quad t \geq 0.$$

Definition 1. *A random change of time in discrete basis $\mathfrak{B}_{\mathbb{N}}$ is given by the filtration:*

$$\mathfrak{G}_t = \mathfrak{F}_{\nu(t)} \ , \quad t \geq 0. \tag{16}$$

According to Lemma 3.8 [2, II:3b], there has place the following properties: $\tau_0 = 0$, $\mathfrak{G}_0 = \mathfrak{F}_0$, as well as $\mathfrak{G}_{\tau_k} = \mathfrak{F}_k$ on the set $\tau_k < \infty$, where $\mathfrak{F}_{k-1} = \mathfrak{G}_{\tau_k -}$, $k > 0$.

For example, if $\nu(t) = [t]$ be an integer part of a positive number $t > 0$, then the basis $\mathfrak{B}_T$ coincides with the basis $\mathfrak{B}_{\mathbb{N}}$.

Definition 2. *The adapted statistical experiments with random change of time* (16) *is determined by the relation*

$$\alpha_N(t) := S_N(\nu(t)) \ , \quad t \geq 0,$$

or equivalently:

$$\alpha_N(t) := S_N(k) \ , \quad \tau_k \leq t < \tau_{k+1} \ , \quad t \geq 0.$$

Conclusion 3. An adapted statistical experiment is s *special semimartingale* [2, I:4c, p. 84], defined by two components:

- the predictable component defined by the regression function of increments $V(c)$, $|c| \leq 1$;
- the stochastic component defined by the Bernoulli distribution (14) of increments $\Delta\mu_N(k+1)$, $k \geq 0$.

Namely, there is a presentation (compare with (9)):

$$\alpha_N(t) = \alpha_N(0) + \mathfrak{V}_N(t) + M_N(t) \ , \quad t \geq 0,$$

$$\mathfrak{V}_N(t) := -\sum_{k=0}^{\nu(t)} V_0(\alpha_N(\tau_k)) \ , \quad M_N(t) := \sum_{k=0}^{\nu(t)} \Delta\mu_N(k+1).$$

4. Adapted statistical experiments in series scheme

The adapted statistical experimens in series scheme with small parameter $\varepsilon \to 0$, ($\varepsilon > 0$) is defined by the random change of time:

$$\nu_\varepsilon(t) := \nu(t/\varepsilon^2) \ , \quad t \geq 0, \tag{17}$$

and by normalization of the increments (9) by the series parameter ε^2.

Definition 3. *Adapted statistical experimens random change of time* (17) *in series scheme*

$$\alpha^\varepsilon(t) := S_N(\nu_\varepsilon(t)) \ , \quad \varepsilon^2 = 1/N \ , \quad t \geq 0,$$

are defined by the following predictable characteristics [2, Ch. 2]:

- *evolutionary component*

$$\mathfrak{V}_t^\varepsilon := -\varepsilon^2 \sum_{k=0}^{\nu_\varepsilon(t)} V_0(\alpha^\varepsilon(\tau_k^\varepsilon)) \ , \quad t \geq 0; \tag{18}$$

- *dispersion of stochastic component*

$$\sigma_t^\varepsilon := \varepsilon^2 \sum_{k=0}^{\nu_\varepsilon(t)} \sigma^2(\alpha^\varepsilon(\tau_k^\varepsilon)) \ , \quad \sigma^2(c) := 1 - V^2(c) \ , \quad t \geq 0, \tag{19}$$

- *compensating measure of jumps*

$$\Gamma_t^\varepsilon(g) := \varepsilon^2 \sum_{k=0}^{\nu_\varepsilon(t)} E\big[g(\alpha^\varepsilon(\tau_{k+1}^\varepsilon)) \,|\, G_{\tau_k^\varepsilon}^\varepsilon\big] \ , \quad g \in \mathcal{C}_1(R) \ , \quad t \geq 0. \tag{20}$$

Here the filtration $\mathfrak{G}_t^\varepsilon := \mathfrak{F}_{\nu_\varepsilon(t)}^\varepsilon$, $t \geq 0$.

The predictable characteristics of adapted statistical experimens (18)–(20) depend on the current value of the adapted statistical experiment $\alpha^\varepsilon(\tau_k^\varepsilon)$, $k \geq 0$ in the jumping points $\tau_k^\varepsilon = \varepsilon^2\tau_k$, $k \geq 0$ of the random change of time $\nu_\varepsilon(t)$, $t \geq 0$. So the study of convergence of adapted statistical experiments in the series scheme by $\varepsilon \to 0$ has to be implemented in two stages (see [5]).

Stage 1. One define the terms of compactness of adapted statistical experimens $\alpha^\varepsilon(t)$, $0 \leq t \leq T$, $\varepsilon \geq 0$.

Stage 2. By additional conditions at the predictable characteristics: the functions $V_0(c)$, $\sigma^2(c)$, $|c| \leq 1$ identify the limiting process, defined by the limit predictable characteristics.

At the first stage the approach proposed in [5] is used (see also [6]). That is the *compact embedding condition be firstly established.*

Lemma 2. *By the condition of the initial values boundness* $E|\alpha^\varepsilon(0)| \leq c_0$ *with a constant with a constant, independent of* ε*, there takes place the compact embedding condition [4, 4:5]:*

$$\lim_{c\to\infty} \sup_{\varepsilon>0} P\{\sup_{0\leq t\leq T} |\alpha^\varepsilon(t)| > c\} = 0. \tag{21}$$

Proof. One used semimartingale representation of the adapted statistical experimens:

$$\alpha^\varepsilon(t) = \alpha^\varepsilon(0) + \mathfrak{V}_t^\varepsilon + M_t^\varepsilon \ , \quad t \geq 0. \tag{22}$$

The evolutionary component $\mathfrak{V}_t^\varepsilon$, $t \geq 0$ is given by the sum (18), and the stochastic component is characterized by a modified component σ_t^ε, according to the formula (19).

Convergence of the random change of time (17) and regularity of the process (15) provide the boundedness of the components

$$\sup_{0\leq t\leq T} |\mathfrak{V}_t^\varepsilon|^2 \leq C_1 \ , \quad \sup_{0\leq t\leq T} |\sigma_t^\varepsilon|^2 \leq C_2.$$

Consequently, by the boundedness of the initial values, the following inequality takes place:

$$E \sup_{0\leq t\leq T} |\alpha^\varepsilon(t)|^2 \leq C$$

with a constant C, independent from ε.

Now the Kolmogorov's inequality for the adapted statistical experimens $\alpha^\varepsilon(t)$, $0 \leq t \leq T$, $\varepsilon \geq 0$, establishes the condition of compact embedding (21). □

Remark 1. Other approach of establishment the compact embedding condition (21) are presented in the monograph [4, 4:5].

Conclusion 1. Under the conditions of Lemma 2 the following estimate takes place:

$$E|\alpha^\varepsilon(t) - \alpha^\varepsilon(t')|^2 \leq C_T|t - t'| \ , \quad 0 \leq \ t, t' \ \leq T. \tag{23}$$

Under the conditions (21) and (23), the compactness of process $\alpha^\varepsilon(t)$, $0 \leq t \leq T$, $\varepsilon > 0$, takes place.

At the second stage, under the compactness condition of the adapted statistical experimens in the series scheme $\alpha^\varepsilon(t)$, $0 \leq t \leq T$, $\varepsilon > 0$, the verification of the limiting process boils down to the study of convergence (as $\varepsilon \to 0$) of the predictable characteristics (18)–(20).

First one established the convergence of the compensating measures of jumps (20):

$$\sup_{0\leq t\leq T} \Gamma_t^\varepsilon(g) \xrightarrow{D} 0 \ , \quad \varepsilon \to 0,$$

provided by Lindeberg condition:

$$\sum_{n=1}^{N} E[\beta_n^\varepsilon(k+1)]^2 \cdot I(|\beta_n(k+1)|) \geq h/\varepsilon \,|\, S_N(k) = c] \to 0 \ , \quad \varepsilon \to 0,$$

for sampling sums

$$\Delta\mu^\varepsilon(k+1) := \varepsilon \sum_{n=1}^{N} \beta_n(k+1),$$

with dispersion

$$D_N^2 := E[(\Delta\mu^\varepsilon(k+1))^2 \mid S_N(k) = c] = \sigma^2(c).$$

Next be established the convergences of the evolutionary component and of the stochastic component dispersion for semimartingale $\alpha^\varepsilon(t)$, $t \geq 0$.

Lemma 3. *In the conditions of Lemma 2 there are convergence in distribution, as $\varepsilon \to 0$:*

$$\mathfrak{V}_t^\varepsilon \xrightarrow{D} \mathfrak{V}_t^0 = -\int_0^{qt} V_0(\alpha^0(s))ds \ , \quad 0 \leq t \leq T,$$

$$\sigma_t^\varepsilon \xrightarrow{D} \sigma_t^0 = \int_0^{qt} \sigma^2(\alpha^0(s))ds \ , \quad 0 \leq t \leq T,$$

$$\sigma^2(c) = 1 - V^2(c).$$

Here the limit process $\alpha^0(t)$, $t \geq 0$, is determined by the condition of compactness (see Lemma 2):

$$\alpha^{\varepsilon_r}(t) \xrightarrow{D} \alpha^0(t) \ , \quad \varepsilon_r \to 0 \ , \quad r \to \infty.$$

Proof of Lemma 3. Since both predictable characteristics (18) and (19) have the same structure of the integral functional on the process $\alpha^\varepsilon(t)$, $t \geq 0$, there enough to explore the convergence of one of them, for example, the evolutionary component (18).

It is used martingale characterization

$$\mu_t^\varepsilon = \varphi(V_t^\varepsilon) - \int_0^t L_V^\varepsilon \varphi(V_u^\varepsilon)du \ , \quad t \geq 0.$$

The generator of integral functional (18):

$$L_V^\varepsilon \varphi(c) = \varepsilon^{-2} q[\varphi(c - \varepsilon^2 V_0(c)) - \varphi(c)] \ , \quad \varphi(c) \in C^2(\mathbb{R}),$$

admits the asymptotic representation at the class of test functions $\varphi(c) \in C^2(\mathbb{R})$:

$$L_V^\varepsilon \varphi(c) = L_V^0 \varphi(c) + R_\varepsilon \varphi(c) \ , \quad \varphi(c) \in C^2(\mathbb{R}),$$

with neglecting term:

$$R_\varepsilon \varphi(c) \to 0 \ , \quad \varepsilon \to 0 \ , \quad \varphi(c) \in C^2(\mathbb{R}).$$

The limit operator L_V^0 defines the evolution:

$$L_V^0 \varphi(c) = -qV_0(c)\varphi'(c) \ , \quad \varphi(c) \in C^2(\mathbb{R}).$$

The limit evolution is given by the following relation:

$$V_t^0 = -\int_0^{qt} V_0(\alpha^0(u))du \ , \quad t \geq 0.$$

Hence the compactness of the adapted statistical experiments (22), established by Lemma 2, provides the convergence of martingales

$$\mu_t^\varepsilon \Rightarrow \mu_t^0 = \varphi(V_t^0) - \int_0^t L_V^0 \varphi(V_u^0)du \ , \quad 0 \leq t \leq T \ , \quad \varepsilon \to 0.$$

Similarly one established the quadratic characteristic convergence (19) using the martingale characterization:

$$\mu_t^\varepsilon = \varphi(C_t^\varepsilon) - \int_0^t L_\sigma^\varepsilon \varphi(V_u^\varepsilon)du \ , \quad t \geq 0,$$

with generator

$$L_\sigma^\varepsilon \varphi(c) = \varepsilon^{-2} q[\varphi(c + \varepsilon^2 \sigma_0^2(c)) - \varphi(c)] \ , \quad \varphi(c) \in C^2(\mathbb{R}),$$

which allows asymptotic representation for the class of test functions $\varphi(c) \in C^2(\mathbb{R})$:

$$L_\sigma^\varepsilon \varphi(c) = q\sigma_0^2(c)\varphi'(c) + R_\varepsilon \varphi(c) \ , \quad \varphi(c) \in C^2(\mathbb{R}),$$

with neglecting term:

$$R_\varepsilon \varphi(c) \to 0 \ , \quad \varepsilon \to 0 \ , \quad \varphi(c) \in C^2(\mathbb{R}).$$

So the limit quadratic characteristic has the following representation:

$$\sigma_t^0 = \int_0^{qt} \sigma^2(\alpha_0(u))du \ , \quad t \geq 0 \ , \quad \sigma^2(c) = 1 - V^2(c).$$

At the final stage one use the uniqueness condition for semimartingale characterization of diffusion Markov process with evolution $\alpha^0(t)$, $t \geq 0$, given by the generator [2, Ch. IX]:

$$L_V^0 \varphi(c) = -V_0(c)\varphi'(c) + \frac{1}{2}\sigma^2(c)\varphi''(c) \ , \quad \varphi(c) \in C^2(\mathbb{R}).$$

□

Theorem 1. *Adapted statistical experimens* $\alpha^\varepsilon(t)$, $t \geq 0$ *in series scheme with small series parameter* $\varepsilon \to 0$, $(\varepsilon > 0)$, *determined by the predictable characteristics* (18)–(20) *with additional condition of convergence of initial values:*

$$\alpha^\varepsilon(0) \xrightarrow{D} \alpha_0 \ , \quad E\alpha^\varepsilon(0) \to E\alpha_0 \ , \quad \varepsilon \to 0,$$

converges, in distribution, to the diffusion process with evolution with the following time scaling

$$\alpha^\varepsilon(t) \xrightarrow{D} \alpha^0(t) \ , \quad 0 \leq t \leq T \ , \quad \varepsilon \to 0.$$

The predictable characteristics of the limiting process $\alpha^0(t)$, $t \geq 0$, *has the following representation:*

$$V_t^0 = \int_0^{qt} V_0(\alpha^0(u))du \ , \quad \sigma_t^0 = \int_0^{qt} \sigma^2(\alpha^0(u))du \ , \quad 0 \leq t \leq T,$$

and the compensating measure of jumps is absent:

$$B_t^\varepsilon(g) \to 0 \ , \quad \varepsilon \to 0 \ , \quad g(c) \in C_1(\mathbb{R}).$$

Conclusion 1. The limit diffusion process with evolution $\alpha^0(t)$, $t \geq 0$, is given by the statistical differential equation

$$d\alpha(t) = -V_0(\alpha(t))dt + \sigma(\alpha(t))dW_t \ , \quad t \geq 0,$$

with the following time scaling:

$$\alpha^0(t) = \alpha(qt) \ , \quad t \geq 0.$$

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