K.A. BUGAEV,[1] D.R. OLIINYCHENKO,[2] A.S. SORIN[2]
[1] Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14b, Metrolohichna Str., Kyiv 03680, Ukraine; e-mail: Bugaev@th.physik.uni-frankfurt.de)
[2] Bogoliubov Laboratory of Theoretical Physics
(JINR, 6, Joliot-Curie Str., Dubna, Russia)

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A SCIENTIFIC ANALYSIS OF THE PREPRINT arXiv:1301.1828v1 [nucl-th]

Below, we analyze the "critic" statements made in Preprint arXiv:1301.1828v1 [nucl-th]. The doubtful scientific argumentation of the authors of Preprint arXiv:1301.1828v1 [nucl-th] is also discussed.

Keywords: hadron resonance gas, hadronic multiplicities.

1. Introduction

Recently, there appeared the Comment arXiv: 1301.1828v1 [nucl-th] by A. Tawfik, E. Gamal and H. Magdy [1] to our recent work [2]. Since this Comment is based on an obsolete and highly unrealistic version of the hadron resonance gas model (HRGM), the critical remarks presented in [1] look like an attempt to "prove" that the results of more elaborate and more realistic versions of the HRGM [2–4] are wrong. This very fact forced us to analyze the main statements of opus [1] in order to clearly demonstrate its original pitfalls.

The main "critique" statements made in Comment [1] are as follows: **No. 1.** The authors of [2] "entirely disregarded the experimental results in baryo-chemical potentials μ_b and their corresponding temperatures T".

No. 2. The chemical freeze out criterion of constant entropy per hadron $\frac{s}{\rho_p} \simeq 7.18$ which was found to be robust in [2] is simply wrong.

No. 3. A few popular chemical freeze-out criteria (see later) agree well with the condition $s/T^3 = 7$ suggested in [5, 6].

No. 4. In addition, the authors of Comment [1] claim that a criterion of constant entropy per hadron is an *ad hoc* one and it has no explanation.

All other statements made in Comment [1] are hard to discuss, since the above statements Nos. 1–4 clearly demonstrate us that the authors of Comment [1] do not know about the recent development of the HRGM made in [2–4]. Hence, we concentrate only on the statements Nos. 1–4 listed above.

2. Scientific vs. Nonscientific Statements in [1]

First of all, it is necessary to recall that, in contrast to the statement **No. 1** of the authors of Comment [1], there are NO any "experimental results in baryo-chemical potentials μ_b and their corresponding temperatures T" at the chemical freeze out or at any other stage of a heavy-ion reaction. This is because such quantities (and all other thermodynamic quantities) cannot be directly measured in the experiments. All of them require some model, which, with some success, may allow us to extract the particle or charge densities, or μ_b and T, by fitting the experimental data on hadron multiplicities by a model. If a model has a realistic physical input, then an extracted information is a reliable one, otherwise any result can be obtained. Therefore, the statement **No. 1** is a nonscientific one.

Furthermore, due to the absence of the first-principle theoretical arguments in the phenomenological analysis of the experimental data, any statement like **No. 2** that some phenomenological result is wrong indicates that the authors of Comment [1] (HRGM1 hereafter) have no any solid scientific arguments against the results of work [2] (HRGM2 hereafter). A detailed analysis of their model outlined in [5] completely supports such a conclusion. The worst, however, is that the authors of Comment [1] claim wrong not only the results of [2], but many years of research to formulate the most successful version of the HRGM [3, 4] (HRGM3 hereafter), on which our formulation HRGM2 [2] is mainly based. Although the particle table and the treatment of the resonance width in the HRGM2 [2] are slightly different, as compared with the HRGM3 [3, 4], the main results of these models are very close to each other.

Usually, the HRGM is used to extract the thermodynamic quantities from the hadron yields measured under certain conditions (at midrapidity or in 4π solid angle). At present, there are many different formulations of the HRGM, but the most successful one, the HRGM3, was developed by A. Andronic, P. Braun-Munzinger and J. Stachel in [3, 4]. A great success of the HRGM3 [3, 4] is naturally explained by its realistic features. The most important of them are as follows:

I. The presence of the hard core repulsion between hadrons. This feature is of a principal importance [7, 8], since, in the absence of the hard core repulsion between hadrons, the hadronic pressure becomes so huge that there is no transition to the quark gluon matter, if all hadrons with masses up to 2 GeV are accounted. Evidently, such a model simply contradicts QCD, and, hence, it cannot be used at temperatures exceeding the pion mass. The last statement is based on the fact that the hard core repulsion essentially reduces the particle densities compared to the ideal gas. See, for instance, Fig. 3 in [9], where it was shown that such a reduction can be up 90 % (!), and, hence, the ignorance of the hadron hard core repulsion may lead to unrealistic values of such thermodynamic parameters as the chemical freeze out volume or the ratios between the yields of the most abundant hadrons (pions) and the less abundant ones (multistrange baryons).

II. All hadronic resonances with masses up to 2.5 GeV should be accounted. This is necessary to successfully describe the hadronic multiplicities for the center-of-mass energies per nucleon $\sqrt{s_{NN}} > 6$ GeV [3, 4]. It is also evident that Properties I and II are closely related, because, if more resonances are taken into account, then the stronger deviation from the mixture of ideal gases should be expected.

III. It is also important that wide hadronic resonances are accounted in a proper way. In other words, the wide resonances should not be treated as stable particles, but their spectral functions up to a threshold of the leading channel of decay should be implemented into a model. Usually, it is believed that the width of wide resonances is important at low temperatures [3] only. However, it was shown recently [10, 11] that the heavy and wide resonances should be taken into account up to temperatures of about 170 MeV.

IV. The full hadronic multiplicities at the chemical freeze out should take both the thermal hadronic yields and the yields coming from the decays of heavier resonances into account. Otherwise, it is impossible to describe the measured hadronic multiplicities. For instance, it is well known that, without inclusion of $\sigma(600)$ meson into the HRGM, it is hard to correctly describe the pion yield at energies $\sqrt{s} < 6$ GeV [3], because just this meson alone provides up to 5 % of total pion yield at these energies.

V. The conservation laws. Usually only the strangeness conservation is taken into account explicitly by finding out the chemical potential of the strange charge from the condition of vanishing strangeness.

As one can judge from [2], one of the main purposes of this paper was to demonstrate that the form of conservation laws (5) and (6) suggested in [3] and used afterwards leads to unrealistically small volumes at the chemical freeze out (see Fig. 3 in [2]). The critique is strong, but convincing. Moreover, as one can see from [12], the critique put forward in [2] is accepted, and the corresponding conservation laws are modified.

3. Doubtful Scientific Argumentation in [1]

The HRGM1 used by the authors of Comment [1] is highly unrealistic, since it does not possess Properties I-IV. Hence, any physical conclusion drawn out of it is simply unrealistic. Moreover, the main critique of the authors of Comment [1] is based on the parametrization [8]

$$T(\mu_b) = a - b\,\mu_b^2 - c\,\mu_b^4, \tag{1}$$

with $a = 0.166 \pm 0.002$ GeV, $b = 0.139 \pm 0.016$ GeV^{-1}, and $c = 0.053 \pm 0.021$ GeV^{-3}. Parametrization (1) is based on a compilation of results of a few models, and not all of them are supplemented by Properties I–IV. As it is clearly seen from Fig. 1, the chemical freeze out temperature dependence of models [2, 3] differs from (1), and, hence, any critique of Comment [1] based on Eq. (1) is not eligible.

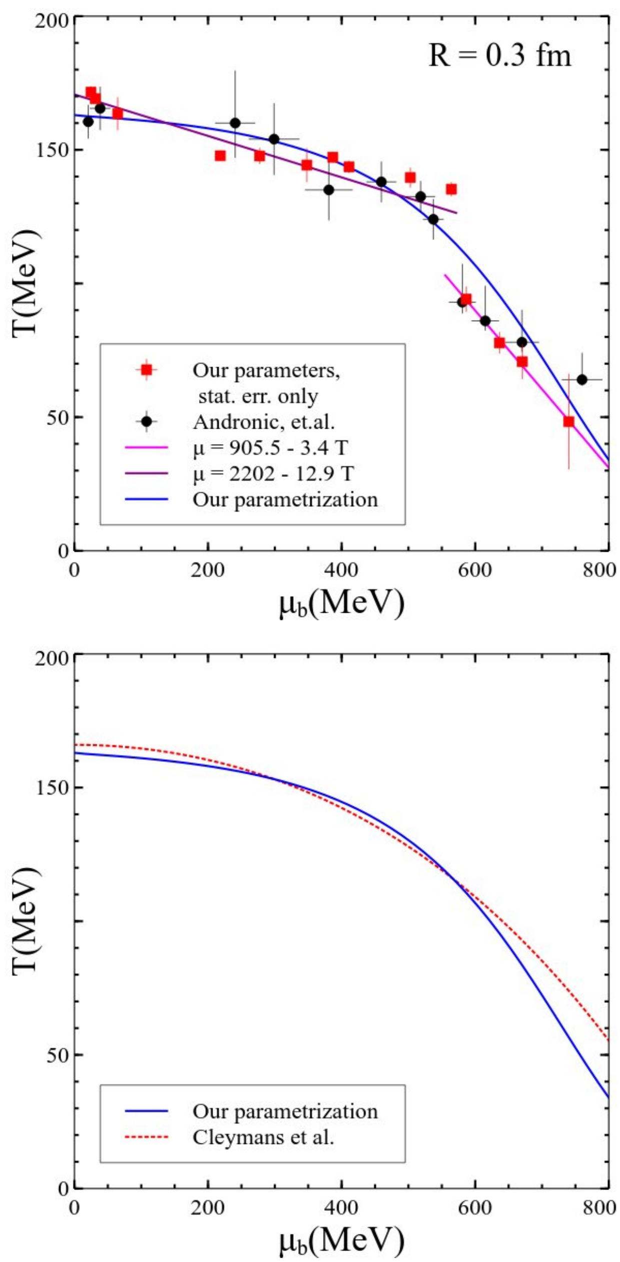

Fig. 1. Chemical freeze out temperature dependence on the baryonic chemical potential μ_b. The symbols in the upper panel correspond to the fit of hadron yield ratios obtained in [2] (squares) and in [3] (circles) for the same value of the hard core radius of all hadrons $R = 0.3$ fm. The solid curve in the upper panel is a fit to the results of [2] and [3] by Eq. (2). The dashed and solid curves in the lower panel correspond to Eqs. (1) and (2), respectively. In fact, the straight lines with the parameters specified in the upper panel describe well the μ_b dependence of the chemical freeze out temperature

A functional dependence relating the values of T and μ_b at the chemical freeze out, which approximately describes the results found by the HRGM2 [2] and the HRGM3 [3], is as follows:

$$T(\mu_b) = \frac{T_0}{1 + \frac{2.44}{\left[\ln\left(\frac{1}{a}\left[\frac{\mu_0}{\mu_b} - 1\right]\right)\right]^4}} \quad \text{for} \quad \mu_b \leq 750 \text{ MeV}. \tag{2}$$

Here, $T_0 \simeq 163$ MeV, $a \simeq 0.31$, and $\mu_0 \simeq 1407$ MeV. At first glance, it seems that the curves defined by Eqs. (1) and (2) and shown in Fig. 1 do not differ essentially. Indeed, for $\mu_b < 750$ MeV, the difference of two freeze out temperatures is below 25 MeV. However, due to the absence of Properties I-IV in the HRGM1 employed by the authors of [1], the corresponding particle densities found within the HRGM1 [1] and the HRGM2 [2] may differ essentially. A more accurate parametrization $\mu_b(T)$ at the chemical freeze out found in [2] is given in the upper panel of Fig. 1.

In "criticizing" the chemical freeze out condition $\frac{s}{\rho_p} \simeq 7.18$ [2], the authors of [1] use the doubtful scientific argumentation. First of all, they simply ignore the results of the lower panel of Fig. 6 in [2], which clearly demonstrates that such a criterion is valid even at low center-of-mass energies of collision $\sqrt{s_{NN}} \geq 2.3$ GeV, i.e. at large values of baryonic chemical potential $\mu_b > 500$ MeV. Instead, the authors of Comment [1] claim that "It is obvious that s/n never reaches 7.18 at $\mu_b > 500$ MeV", forgetting to mention that this conclusion is obtained not within the realistic HRGM2 [2], but within the unrealistic HRGM1 [1, 5, 6].

The second example of the doubtful scientific argumentation used by the authors of Comment [1] is as follows. In order to prove the validity of the statement **No. 2**, the authors of Comment [1] substitute the particle number density ρ_p by the baryonic charge density n. In fact, Sect. III of Comment [1] is called as "PHYSICS OF CONSTANT ENTROPY PER NUMBER DENSITY", but as one can judge from Eq.(3) in [1], which is written as

$$\frac{s}{n} = \frac{1}{T}\left(\frac{p}{n} + \frac{\epsilon}{n} - \mu_b\right), \tag{3}$$

either n is a baryonic charge density and, hence, the authors of the Comment are criticizing not the condition of constant entropy per particle, or n is, indeed, the particle number density, but then Eq. (3)

in Comment [1] has nothing to do with the standard thermodynamics. Since the authors of [1] failed to specify their notations used in (3), here we also assumed that they consider p as the system pressure and ϵ as its energy density.

The third example of the doubtful scientific argumentation in Comment [1] requires a special attention. In order to "prove" the validity of their statement **No. 2**, the authors of Comment [1] simply extrapolate (with the help of parametrization (1)!) the HRGM1 results of [1, 5, 6] to the chemical freeze out temperatures somewhat well below 50 MeV and demonstrate that the entropy per baryonic charge is essentially larger than 7.18. From such a procedure, the authors of Comment [1] conclude that the chemical freeze out criterion $\frac{s}{\rho_p} \simeq 7.18$ [2] cannot be used at the AGS and SIS energies. However, we have to stress here that, to our best knowledge, none of the realistic thermal models, including the HRGM3, which are able to describe the particle ratios at the SIS energies $\sqrt{s_{NN}} = 2.24$ GeV and $\sqrt{s_{NN}} = 2.32$ GeV ever showed the chemical freeze out temperatures below 49 MeV (see, e.g., [8]). Hence, there is no need to worry about the behavior of the ratio $\frac{s}{\rho_p}$ at $T < 50$ MeV!

4. Special Role of the Chemical Freeze Out Criterion $s/T^3 = 7$

The authors of Comment [1] are considering a few traditional chemical freeze out criteria, namely, of constant energy per particle, but they write it as $\epsilon/n \simeq 1$ GeV, and of constant number of baryons and antibaryons $n_b + n_{\bar{b}} \simeq 0.12$ fm^{-3}, but the main attention is paid to the criterion $s/T^3 = 7$ suggested in [5, 6]. It is necessary to recall that the criterion $s/T^3 = 7$ was heavily criticized already in [8], where it was demonstrated that the inclusion of the hard core repulsion into the HRGM1 essentially modifies relation (1) between the chemical freeze out parameters for the criterion $s/T^3 = 7$. This is clearly seen in Fig. 2.

Probably, the authors of Comment [1] think it is a great advantage of their model that all the chemical freeze out criteria shown in the left panel of Fig. 2 calculated in [1] at the curve $s/T^3 = 7$ demonstrate a constant behavior for all values of the baryonic chemical potential from $\mu_b \simeq 5$ MeV to $\mu_b \simeq 10000$ MeV. We, however, would like to recall that,

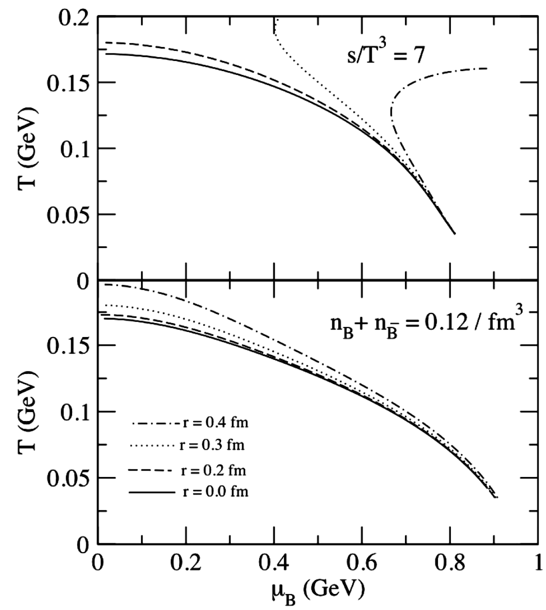

Fig. 2. The effect of excluded volume corrections on the constant $n_B + n_{\bar{B}}$ (bottom) and constant s/T^3 (top) freeze-out criterion. This figure is taken from the preprint J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, arXive:hep-ph/0511094v2 of Ref. [8] in order to demonstrate the unrealistic behavior of the chemical freeze-out criterion $s/T^3 = 7$, if the hard core repulsion is taken into account. For the hard core radius $R = 0.3$ fm, which was used in [2–4] to fit the data, the chemical freeze-out criterion $s/T^3 = 7$ does not work

at so huge values of the baryonic chemical potential ($\mu_b \gg 1000$ MeV), there is no reason to discuss both the chemical freeze out and the HRGM, since, according to the contemporary QCD, there should exist other state of matter at this region, and, hence, the hadron resonance gas is simply inapplicable.

Also it is necessary to stress that the chemical freeze out criterion $s/T^3 = 7$ is not observed in the HRGM2 [2] and in the HRGM3 [3], and a similar conclusion is also confirmed by the recent analysis of [9]. In Ref. [9], parametrization (1) is used for the HRGM which is similar to the HRGM1 [1]. As one can see from the right panel of Fig. 2 in [9], the ideal gas model gives $s/T^3 \simeq 7$ in this case for the lab energies of collision above 4 GeV per nucleon and just for a strangeness suppression factor equal to 1 (no suppression); while for smaller lab energies, the ratio s/T^3 is essentially larger than 7! If, however, one introduces the strangeness suppression factor dependence as suggested in [13], then $s/T^3 = 6$ for all lab energies above 8 GeV per nucleon. Finally, if one employs parametrization (1) for the HRGM with the hard core repulsion, then, as one can see from the right panel of Fig. 4 in [9], s/T^3 varies from 3.6 to 6, depending on the set of hard core radii.

Therefore, in order to prove the claim **No. 3**, the authors of Comment [1] forget about parametrization (1), which they used to "criticize" the HRGM2 and HRGM3 results presented in [2]. Thus, the authors of Comment [1] use the double standards.

Finally, before claiming that a criterion of constant entropy per hadron is an *ad hoc* one (claim **No. 4**), it would be nice, if the authors of Comment [1] could follow their own advice in the first place and could not ignore the existing literature on this subject. Probably, the authors of Comment [1] should have looked into a recent work [11] to study the suggested explanation for a criterion of constant entropy per hadron.

5. Conclusions

The above analysis clearly shows us that Comment [1] lacks any new result, and its authors are trying to prove an impossible thing, namely that their obsolete formulation of the HRGM1 has some advantages over more elaborate ones. In contrast to their own calls to lift up the scientific standards, the authors of Comment [1] use the doubtful scientific argumentation to "prove" the validity of the unrealistic model of Refs. [1, 5, 6] and to claim wrong the results of the advanced HRGM formulations worked out in [2–4].

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1. A. Tawfik, E. Gamal, and H. Magdy, arXiv:1301.1828v1 [nucl-th].
2. D.R. Oliinychenko, K.A. Bugaev, and A.S. Sorin, Ukr. J. Phys. **58**, 211 (2013) [arXiv:1204.0103 [hep-ph]].
3. A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl. Phys. A **772**, 167 (2006) and references therein.
4. A. Andronic, P. Braun-Munzinger, and J. Stachel, Phys. Lett. B **673**, 142 (2009) and references therein.
5. A. Tawfik, Europhys. Lett. **75**, 420 (2006).
6. A. Tawfik, Nucl. Phys. A **764**, 387 (2006).
7. J.I. Kapusta, Phys. Rev. D **10**, 2444 (1981).
8. J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C **73**, 034905 (2006).
9. V.V. Begun, M. Gazdzicki, and M.I. Gorenstein, arXiv:1208.4107 [nucl-th].
10. K.A. Bugaev, D.R. Oliinychenko, A.S. Sorin, and G.M. Zinovjev, Eur. Phys. J. A **49**, 30 (2013) [arXiv:1208.5968 [hep-ph]].
11. K.A. Bugaev, D.R. Oliinychenko, E.G. Nikonov, A.S. Sorin, and G.M. Zinovjev, PoS Baldin ISHEPP XXI (2012) 017, 1-14 [arXiv:1212.0132 [hep-ph]]
12. A. Andronic, P. Braun-Munzinger, J. Stachel, and M. Winn, arXiv:1201.0693 [nucl-th].
13. J. Manninen, F. Becattini, and M. Gazdzicki, Phys. Rev. C **73**, 044905 (2006).

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К.О. Бугаєв, Д.Р. Олійниченко, О.С. Сорін

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Р е з ю м е

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К.А. Бугаев, Д.Р. Олийниченко, А.С. Сорин

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Р е з ю м е

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