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ANALYSIS OF THE FRACTAL STRUCTURES IN TURBULENT PROCESSES

On the basis of wavelet analysis and multifractal formalism it is carried out an analysis of fractal structures in the turbulent processes (parietal pressure pulsations in a turbulent flow in the pipe).
***Key words**: fractals structures, turbulent processes*

Introduction. In last years it is of a great importance the experimental and theoretical studying of the non-linear dynamical systems with aim to discover the fractal features and elements of dynamical chaos (e.g. [1-20]). One of the effective approaches to solving such a problem is the multifractal and wavelet analyses. The foundations and application information on the continuous wavelet transform-based method of multifractal analysis are presented in Ref. [12]. An extension of the concept of multifractals to irregular functions through the use of wavelet transform modulus maxima and potential and limitations of the multifractal formalism in the study of non-stationary processes and short signals are in details considered in these references. Especial attention is turned to the multifractality loss effects in the dynamics of different types of systems. A review of fundamental results on the manifestation of fractal structure in wave (turbulent) processes is presented in [1]. As it is indicated in many references (e.g. [1]) the most natral and effective illustration of the chaos effect can be observed in turbulent flows. In papers by Zaslavsky et al (e.g. [5]) the fractal properties of the sea surface have been considered on the scales which are more than the distortion correlation radius. In particular, on the basis of analyzing the aero-photo images it has been found the fractality in distribution of the zones for waves falling (d=0,5). The cited measurements were carried out in the tropical Atlantic in the opened ocean, where the tropical passates provided the stationary developed distortion during several days. A simple model is presented in Ref. [21] for the energy-cascading process in the inertial range that fits remarkably well the entire spectrum of scaling exponents for the dissipation field in fully developed turbulence. The scheme is a special case of weighted curdling and its one-dimensional version is a simple generalized two-scale Cantor set with equal scales but unequal weights (with ratio ~7/3). This set displays all the measured multifractal properties of one-dimensional sections of the dissipation field. In paper by Naugolnyh-Zosimov (see Refs. in [1] the fractal properties of the sea surface have been considered too and the laser scanning locator measurements of distribution of the mirror dots along space-temporal line, defined by the vessel running. In our paper we ipresent the results of analysing multifractal structures in the turbulent process, i.e. parietal pressure pulsations in a turbulent flow in the pipe

Method. Let us further consider Our version of the wavelet analysis and multi-fractal formalism has been in details presented in the earlier papers [11,15-18], so here we are limited only by the key aspects. The theoretical tool is in fact based on the wavelet decomposition for analyzing various signals. At present, the family of analyzing function dubbed wavelets is being increasingly used in problems of pattern recognition; in processing and synthesizing various signals; in analysis of images of any kind (X-ray picture of a kidney, an image of mineral, etc.); for study of turbulent fields, for contraction (compression) of large volumes of information, and in many other cases. Wavelets are fundamental building block functions, analogous to the sine and cosine functions. Fourier transform extracts details from the signal frequency, but all information about the location of a particular frequency within the signal is lost. At the expense of their locality the wavelets have advantages over Fourier transform when non-stationary signals are analyzed. Here, we use non-decimated wavelet transform that has temporal resolution at coarser scales.

The dilation and translation of the mother wavelet $\psi(t)$ generates the wavelet as follows: $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$. The dilation parameter j controls how large the wavelet is, and the translation parameter k controls how the wavelet is shifted along the t-axis. For a suitably chosen mother wavelet $\psi(t)$, the set $\{\psi_{j,k}\}_{j,k}$ provides an orthogonal basis, and the function f which is defined on the whole real line can be expanded as

$$f(t) = \sum_{k=-\infty}^{\infty} c_{0k}\varphi_{0,k}(t) + \sum_{j=1}^{J}\sum_{k=-\infty}^{\infty} d_{jk}\psi_{j,k}(t), \tag{1}$$

where the maximum scale J is determined by the number of data, the coefficients c_{0k} represent the lowest frequency smooth components, and the coefficients d_{jk} deliver information about the behavior of the function f concentrating on effects of scale around 2^{-j} near time $k \times 2^{-j}$. This wavelet expansion of a function is closely related to the discrete wavelet transform (DWT) of a signal observed at discrete points in time. In practice, the length of the signal, say n, is finite and, for our study, the data are available monthly, i.e. the function $f(t)$ in Eq. (1) is now a vector $f = (f(t_1),\ldots, f(t_n))$ with $t_i = i/n$ and $i = 1,\ldots,n$. With these notations, the DWT of a vector f is simply a matrix product $d = Wf$, where d is an $n \times 1$ vector of discrete wavelet coefficients indexed by 2 integers, d_{jk}, and W is an orthogonal $n \times n$ matrix associated with the wavelet basis. For computational reasons, it is simpler to perform the wavelet transform on time series of dyadic (power of 2) length. One particular problem with DWT is that, unlike the discrete Fourier transform, it is not translation invariant. This can lead to Gibbs-type phenomena and other artefacts in the reconstruction of a function. The non-decimated wavelet transform (NWT) of the data $(f(t_1), \ldots, f(t_n))$ at equally spaced points $t_i = i/n$ is defined as the set of all DWT's formed from the n possible shifts of the data by amounts i/n; $i = 1, \ldots, n$.

Thus, unlike the DWT, there are 2^j coefficients on the jth resolution level, there are n equally spaced wavelet coefficients in the NWT: $d_{jk} = n^{-1}\sum_{i=1}^{n} 2^{j/2}\psi[2^j(i/n - k/n)]y_i$, $k = 0, \ldots, n-1$, on each resolution level j. This results in $\log_2(n)$ coefficients at each location. As an immediate consequence, the NWT becomes translation invariant. Due to its structure, the NWT implies a finer sampling rate at all levels and thus provides a better exploratory tool for analyzing changes in the scale (frequency) behavior of the underlying signal in time. These advantages of the NWT over the DWT in time series analysis are demonstrated in Nason et al (e.g.[12]). As in the Fourier domain, it is important to assess the power of a signal at a given resolution. An evolutionary wavelet spectrum (EWS) quantifies the contribution to process variance at the scale j and time k. From the above paragraphs, it is easy to plot any time series into the wavelet domain. Another way of viewing the result of a NWT is to represent the temporal evolution of the data at a given scale. This type of representation is very useful to compare the temporal variation between different time series at given scale. To obtain the results, smooth signal S_0 and the detail signals D_j (j =1, …, J) are

$$S_0(t) = \sum_{k=-\infty}^{\infty} c_{0k}\varphi_{0,k}(t) \quad \text{and} \quad D_j(t) = \sum_{k=-\infty}^{\infty} d_{jk}\psi_{j,k}(t). \tag{2}$$

The fine scale features (high frequency oscillations) are captured mainly by the fine scale detail components D_J and D_{J-1}. The coarse scale components S_0, D_1, and D_2 correspond to lower frequency oscillations of the signal. Note that each band is equivalent to a band-pass filter. Further we use the Daubechies wavelet as mother wavelet [12]. This wavelet is bi-orthogonal and supports discrete wavelet transform. Using a link between wavelets and fractals, one could make calculating the multi-fractal spectrum. As usually, the homogeneous fractals are described by single fractal dimension $D(0)$. Non-homogeneous or multifractal objects are described by spectrum $D(q)$ of fractal dimensions or multifractal spectrum A problem of its calculation reduces to definition of singular spectrum $f(\alpha)$ of measure μ. It associates Haussdorff dimension and singular indicator α, that allows calculating a degree of singularity: $N_\alpha(\varepsilon)=\varepsilon^{-f(\alpha)}$. Below we use a formalism, which allows defining spectra of singularity and fractal dimension without using standard Legandre transformations. This idea at first used in ref.[8]. Wavelet transformation of some real function F can be also defined as

$$W_\Psi[F](b,a) = (1/\alpha)\int_{-\infty}^{+\infty} F(x)\Psi\left(\frac{x-b}{a}\right)dx\,, \quad (3)$$

where parameter b denotes a shift in space (a space scale). The analyzing splash Ψ has to be localized as in space as on frequency characteristics. The most correct way of estimate of the function $D(h)$, $f(\alpha)$ is in analysis of changing a dependence of the distribution function Z(q,a) on modules of maximums of the splash-transfers under scale changes

$$Z = \sum_{i=1}^{N(a)} (\omega_i(a))^q\,, \quad (4)$$

where $I=1,\ldots,N(a)$; $N(a)$ is a number of localized maximums of transformation $W_\Psi[F](b,a)$ for each scale a; function $\omega(a)$ can be defined in terms of coefficients of the splash-transformations as

$$\omega_l(a) = \max_{\substack{(x,a')\in L\\ a'<a}} |W_\Psi[F](x,a')|, \quad (5)$$

where $l_i \in L(a)$; $L(a)$ is a set of such lines, which make coupling the splash-transformation coefficient maximums (they reach or make cross-section of a level, which is corresponding to scale a). In the limit $a\to 0^+$ the distribution function $Z(q,a)$ manifests the behaviour, which is corresponding to a degree law: $Z(q,a)\sim a^{\tau(q)}$. To calculate a singularity spectrum, the standard canonical approach can be used. It is based on using such functions:

$$h(a,q) = \frac{1}{Z(a,q)}\frac{\partial Z(a,q)}{\partial q}\,, \quad (6a)$$

$$\frac{\partial Z}{\partial q} = \sum_{i=l}^{N(a)} \omega_i(a)^q \ln\omega_i(a)\,, \quad (6b)$$

$$D(a,q) = qh(a,q) - \ln Z(a,q). \quad (6c)$$

The spectra $D(q)$ and $h(q)$ are defined by standard way as follows:

$$D(q) = \lim_{a\to 0}\frac{D(a,q)}{\ln a}\,, \quad h(q) = \lim_{a\to 0}\frac{h(a,q)}{\ln a} \quad (7)$$

Other details can be found in Refs. [11,15-18].

Results and conclusions. Using the above described formalism, we have carried out a multifractal analysis of spatial spectrum of the parietal pressure pulsations during a turbulent flow in the pipe ([19]; see also [5,6,20,21]). In fig.1 it is presented the time dependence of square of the parietal pressure pulsations in a turbulent flow in the pipe, averaged on the intervals $1.6\cdot 10^{-4}$ sec (velocity of flow 10 m/sec; the size of the sensor 1 cm [19]. The process is analyzed on the time intervals which are more than the correlation scale, i.e., as one could wait for here, a intermittency has a multi-fractal nature.

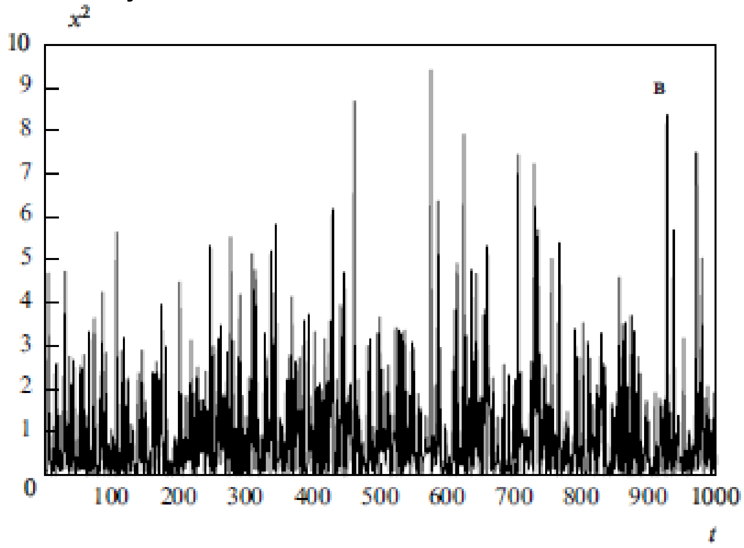

Figure 1 – The square of the parietal pressure pulsations in a turbulent flow in the pipe, averaged on the intervals $1.6 \cdot 10^{-4}$ sec [20].

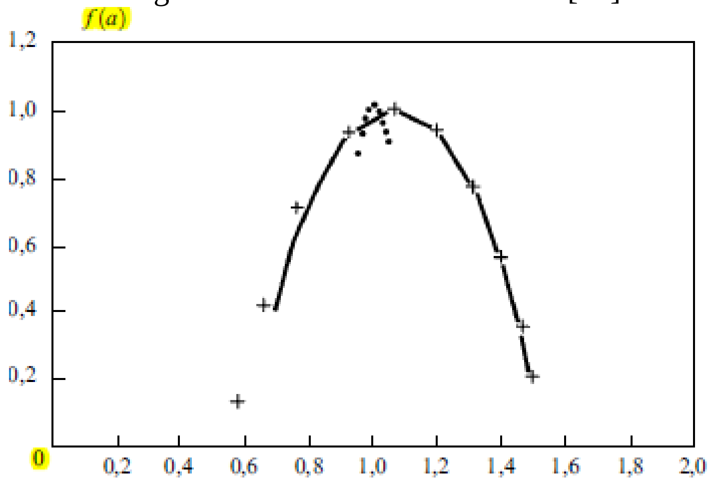

Figure 2 - The spectrum of the singularities: solid curve – theoretical data; points - experimental data.

Using the PC complex "Geomath" (c.f.[15]) we have performed the numerical calculations of the fractal spectrum for the capillary-gravitational ripple. The figure 2 shows the spectrum of the singularities: solid curve – theoretical data; points - experimental data. Our numerical estimates have shown that the singularity spectrum is situated near the point of f=0.98 and α=1.1. fractals dimensions are lying in the interval [0,65-0,88]. The experimental data are correspomdimg to a little less values (figure 2). These data are satisfactory agreed with the preliminary estimates within the simple stabdard multifractal definition modelling. Therefore, our analysis confirms the universal conclusion regarding availability of the multifractal features for the parietal pressure pulsations in a turbulent flow in the pipe.

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Аналіз фрактальних структур у турбулентних процесах

Свинаренко А.А., Хецеліус О.Ю., Мансарлійський В.Ф., Романенко С.І.

Виконано аналіз фрактальних структур у турбулентних процесах (пристінні пульсації тиску в турбулентному потоці в трубі) на підставі вейвлет - аналізу та мультіфрактального формалізму.

Ключові слова: *фрактальні структури, турбулентні процеси*

Анализ фрактальных струтур в турбулентных процессах

Свинаренко А.А., Хецеліус О.Ю., Мансарлійський В.Ф., Романенко С.И.

Выполнен анализ фрактальных структур в турбулентных процессах (пристенные пульсации давления в турбулентном потоке в трубе) на основе вейвлет - анализа и мультифрактального формализма.

Ключевые слова: *фрактальные структуры, турбулентные процессы*