AVERAGE RESONANCE PARAMETERS OF TELLURIUM AND NEODYMIUM NUCLEI

M.M. PRAVDIVY, I.O. KORZH, M.T. SKLYAR

Institute for Nuclear Research, Nat. Acad. of Sci. of Ukraine
(47, Prosp. Nauky, Kyiv 03680, Ukraine; e-mail: sklyar@kinr.kiev.ua)

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Complete sets of average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ for tellurium and neodymium nuclei with natural isotope contents have been determined by analyzing the experimental differential cross-sections of neutron elastic scattering in the energy range lower than 440 keV. The data obtained, the recommended parameter values, and some literature data have been analyzed.

1. Introduction

This work is devoted to the determination of complete sets of average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ for tellurium and neodymium nuclei with a natural content of isotopes with the use of a technique developed by us earlier [1, 2]. The corresponding values were determined by analyzing the average experimental differential cross-sections obtained for the elastic scattering of neutrons with an energy lower than 440 keV by those nuclei. The technique turned out fruitful for both the measurement of new data and the verification of available literature parameters with respect to their correspondence to the average experimental cross-sections. The neutron nuclear strength functions S_0 and S_1 are mainly found by analyzing parameters known for resolved resonances. For today, a tremendous number of works were carried out for many nuclei, in which the corresponding strength functions were determined. However, the difficulties associated with the determination procedure – such as a small number of resonances, their ambiguous identification on the basis of the orbital moment l, and so on – gave rise to a considerable scatter in parameter values determined at different laboratories for the same nucleus. As a result, with the appearance of new data, the recommended parameter values often become several times larger or smaller [3–5]. The radii of the potential scattering R'_0 and R'_1 are determined using other methods and practically independently of strength functions. Actually, those parameters are determined separately and, consequently, are often not in agreement with each other. In this connection, there is the necessity to verify their correspondence to experimental cross-sections averaged over the resonances of a compound nucleus. Generally speaking, new data invoke relatively small modifications in average cross-section values, so that their satisfactory description can be considered as a sound criterion for the parameter reliability. Such a verification will promote an ultimate adoption of the dependences of resonance parameters on the mass number A, which are based now on the results of calculations in the framework of the optical model. In this work, this verification was carried out for the complete sets of resonance parameters, which were determined in work [6] by analyzing the average experimental cross-sections of elastic low-energy neutron scattering in the framework of the R-matrix theory. We also verified the values for parameters S_0, S_1, and R'_0 recommended by the Brookhaven National Laboratory (BNL) [4] and the International Atomic Energy Agency (IAEA) [5]. There are some discrepancies between those parameters, which was one of the stimuli for carrying out this research work.

The method developed by us was expounded in works [1, 2] in detail. Therefore, only its essence is briefly described below.

2. Technique for Determination of Average Resonance Parameters

The scattering of neutrons by nuclei at energies lower than about 450 keV predominantly occurs with the orbital moments $l = 0$ and 1. In this case, the differential

cross-sections of elastic scattering can be expanded in a series of Legendre polynomials, which looks like

$$\sigma_{el}(\mu) = \frac{\sigma_{el}}{4\pi}\{1 + \omega_1 P_1(\mu) + \omega_2 P_2(\mu)\}, \quad (1)$$

where $\mu = \cos\theta$, θ is the scattering angle, σ_{el} the integrated cross-section of elastic scattering, P_l the Legendre polynomials, and ω_1 and ω_2 are the expansion coefficients. Those coefficients are called the angular moments of the scattering indicatrix; they equal $\omega_l = (2l+1)\bar{P}_l$, where $\bar{P}_l$ are the Legendre polynomials averaged over the angles with the weight of the differential scattering cross-section. Provided the condition $\sigma_t \approx \sigma_{el}$ for even-even nuclei, we obtain the following expressions for the expansion coefficients:

$$\omega_1 = \frac{6\pi\lambda^2}{\sigma_{el}}(1 - \eta_{0Re} - \eta_{1Re} + \eta_{0Re}\cdot\eta_{1Re} + \eta_{0Im}\cdot\eta_{1Im}), \quad (2)$$

$$\omega_2 = \frac{2}{\sigma_{el}}(\sigma_{s1} + \pi\lambda^2 T_{1,3/2}). \quad (3)$$

Here, $\eta_l = \eta_{l\mathrm{Re}} + i\eta_{l\mathrm{Im}}$ are the diagonal elements of the average scattering matrix, σ_{s1} the potential scattering cross-sections for neutrons with $l = 1$, and $T_{1,3/2}$ the penetration factors for $l = 1$ and $j = 3/2$.

In the optical model, the cross-section σ_{el} consists of the corresponding partial cross-sections of compound and potential scatterings of neutrons, which are expressed in terms of the matrix elements η_l. In the resonance theory, the average cross-section also consists of the corresponding cross-sections of resonance and potential scatterings, which, in turn, are expressed in terms of the average resonance parameters. In the case of narrow resonances ($\Gamma \ll D$), the partial cross-sections in the optical model coincide with the corresponding cross-sections in the resonance theory [7]. This circumstance allows the matrix elements to be expressed in terms of the resonance parameters.

Hence, if the quantities σ_{el}, ω_1, and ω_2 in Eqs. (1)–(3) are written down in terms of the average resonance parameters, then, by fitting those quantities to their experimental values, it is possible to determine the average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ as fitting ones. The parameter $S_{1,1/2}$ can be determined using the relation $S_1 = (S_{1,1/2} + 2S_{1,3/2})/3$. For calculations, we applied the corresponding fitting program based on the χ^2-minimization method. All the three quantities – σ_{el}, ω_1, and ω_2 – were fitted simultaneously, whereas the χ^2-test could be used separately for each quantity.

3. Results Obtained and Their Discussion

The average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ for tellurium and neodymium nuclei with the natural content of isotopes were determined by fitting the calculated quantities σ_{el}, ω_1, and ω_2 to their experimental values published in works [8] (for tellurium) and [9] (for neodymium); the additional data averaging was done at the beginning of the energy range. All other fitting procedures described below were carried out using the data of those works as a reference. Besides determining new data, we verified the available complete sets of resonance parameters [6] and some individual recommended parameters [4, 5] for their correspondence with experimental data. For tellurium and neodymium isotopes, only the parameters S_0, S_1 [4, 5], and R'_0 [4] are recommended; we used those values to calculate the corresponding weighted average values for the natural contents of those isotopes. By fixing the values obtained, we applied the fitting procedure to calculate the other quantities from the complete set. The determined parameter sets were used to calculate the quantities σ_{el}, ω_1, and ω_2, which afterward were compared with experimental values. In all the cases, the quality of the description of experimental data was evaluated according to the χ^2-test and visually (using the corresponding plots).

3.1. Tellurium

In Fig. 1, the experimental energy dependences of σ_{el}, ω_1, and ω_2 measured in work [8] and the data obtained in works [10, 11] are depicted. One can see that the data of the cited works reveal a considerable scatter and a substantial discrepancy among themselves. Moreover, the uncertainties are observed in the energy dependences of the quantities σ_{el} and ω_1. Particularly considerable mismatches are observed for the cross-sections σ_{el}. The data on the total cross-sections do not clarify the situation, because they are scarce and also demonstrate a large scatter [12]. The figure exhibits the total cross-sections σ_t averaged in that work. The curves denote the results of calculations with various resonance parameter sets.

Curves *2* in Fig. 1 correspond to the results of calculations with the following parameters taken from work [6]: $S_0 = 0.11(3)$, $S_1 = 1.91(33)$, $R'_0 = 6.57(14)$, $R'_1 = 8.04(42)$, and $S_{1,3/2} = 1.72(28)$ (hereafter, the strength functions are expressed in terms of 10^{-4}-units and the radii in Fm units; the errors are indicated in the parentheses). The obvious scatter of experimental data and the discrepancies among the data of different

authors make it difficult to estimate the quality of the description using the given set of parameters.

Curves *3* in Fig. 1 exhibit the results of calculations with the recommended parameter values given in work [4]. For tellurium isotopes, the recommended parameters are S_0 and S_1, which we used to calculate the weighted average values $S_0 = 0.29$ and $S_1 = 1.43$. By fixing them and carrying out the fitting procedure, the following other parameters were obtained: $R'_0 = 6.29$, $R'_1 = 8.84$, and $S_{1,3/2} = 1.82$. Taking the existing scatter in experimental data into consideration, their description can be estimated as satisfactory. The values obtained turned out more optimal for the whole body of experimental data than the previous parameter set. Moreover, in work [4], besides the parameters S_0 and S_1, the parameter R'_0 was also recommended for tellurium isotopes. However, we obtained the corresponding weighted average value $R'_0 = 5.47$, which does not agree with the dependence on the mass number A. The set of parameters determined using the fitting procedure at fixed recommended values for S_0, S_1, and $R'_0 = 5.47$ is not suitable for the description of cross-sections, namely, the calculated cross-sections turn out considerably smaller even in comparison with the data of work [10].

In work [5], the recommended parameters S_0 and S_1 for tellurium isotopes were presented. We used them to calculate the weighted average values $S_0 = 0.37$ and $S_1 = 1.19$. Fixing those values and carrying out the automatic fitting procedure, the other parameters were obtained: $R'_0 = 6.28$, $R'_1 = 9.15$, and $S_{1,3/2} = 1.70$. The results of calculations are plotted in Fig. 1 as curves *4*. The parameter values in this set are close to the previous ones, and, consequently, the results of calculations are also close.

From Fig. 1, one can see that the experimental values obtained in work [8] have a considerable scatter and the uncertainty in their dependences on the energy. Under such conditions, there is no reason to determine the resonance parameters making use of automatic fitting, because the corresponding χ^2-values cannot serve as a reliable criterion for the quality of the description of experimental data. Therefore, a new set of parameters was determined by us using an individual selection of each parameter: $S_0 = 0.25$, $S_1 = 1.70$, $R'_0 = 6.22$, $R'_1 = 8.65$, and $S_{1,3/2} = 1.91$. The corresponding results of calculations are shown in Fig. 1 by curves *1*. One can see that the description of experimental data is more optimal than it was for other parameter sets, and the corresponding χ^2-values are smaller in general. The parameter values obtained by us are in agreement with the recommended ones [4] within the measurement

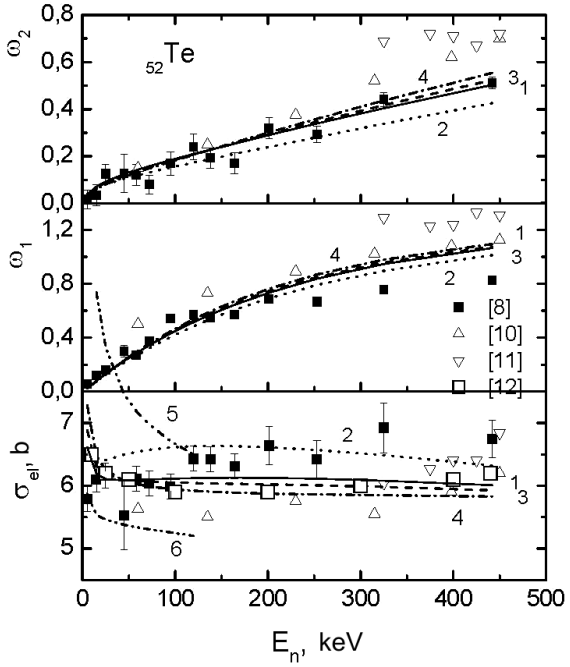

Fig. 1. Energy dependences of the quantities σ_{el}, ω_1, and ω_2 for tellurium nuclei. Symbols denote experimental data. Curves correspond to the results of calculations (see the text)

errors and, hence, confirm the reliability of the latter. However, this conclusion concerns only the weighted average quantities for tellurium with the natural isotopic content, because the recommended values of the parameters S_0 and S_1 for some isotopes differ from each other by a factor ranging from 2 to 5.

In this connection, it is necessary to mention a problem that exists for a long time, remaining still unresolved [3–5]. The largest scatter of S_0-parameter values is observed at the minimum of the S_0-dependence on A (in the interval $A \approx 90 \div 130$). As a result of this uncertainty, the improvement of calculations in the framework of the optical model becomes hampered. In general, those scatters of parameters over the isotopes have a chaotic character for the majority of elements. However, for cadmium, tin, and tellurium isotopes, the magnitudes of parameter S_0 demonstrate a regular drastic reduction with the growth of A, which contradicts the results of calculations in the framework of the optical model, according to which they must increase [3, 4].

Figure 2 demonstrates the strength functions S_0 for even isotopes of cadmium and tin obtained in our earlier researches [2], for tellurium obtained in this work (black symbols), and those recommended in works [4, 5]. One can see that the recommended values drastically decrease, as the parameter A grows. Evidently, this fact

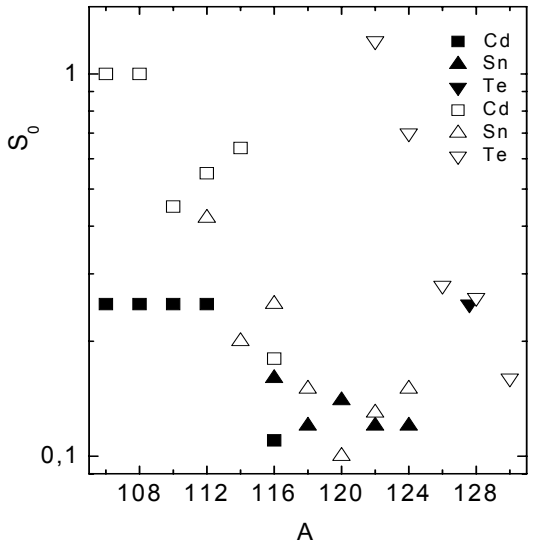

Fig. 2. Parameter S_0 for cadmium, tin, and tellurium nuclei

has to manifest itself somehow in the average experimental cross-sections as well. However, the analysis of experimental data obtained for the parameters σ_{el}, ω_1, and ω_2 of cadmium isotopes [13] showed that their dependences on the energy reveal the chaotic scatter of corresponding values, so that the appreciable regular changes of A could not be observed against such a background.

A similar conclusion was also made for the cross-sections of tin isotopes [8]. It was confirmed by the analysis of the total cross-sections measured for tin isotopes in work [14] in the energy range 20–1400 keV. The corresponding data demonstrate a smooth insignificant reduction of the cross-sections with the growth of A. In work [14], the resonance parameters were determined in two ways: by carrying out the statistical analysis of experimental data measured by the authors (the total cross-sections, the radiative capture cross-sections, and the transmittance) and calculated in the framework of the optical model. The obtained values of parameter S_0 (Fig. 2) agree with the data of work [14]. For available experimental cross-sections, they are optimal, and their enlargement to the recommended values would inevitably result in a considerable worsening of the description of cross-sections at the beginning of the energy interval. In addition, only the parameters S_0 and S_1 were recommended [4, 5], so that the other parameters have to be determined by automatically fitting the experimental data. As a result, owing to a large value $S_0 = 1.0$ for the isotope ^{108}Cd, the corresponding obtained value $R'_0 = 4.4$ does not agree with the dependence on A ($R'_0 \approx 6.5$ in the interval $A = 100 \div 110$) [4]. An analogous situation also took place for other cadmium and tin isotopes with large values of parameter S_0.

There are no experimental data in the literature concerning the parameters σ_{el}, ω_1, and ω_2 for tellurium isotopes. Therefore, we cannot estimate the reliability of S_0-parameter values recommended for them, by using our technique. However, we can demonstrate the qualitative influence of parameter magnitudes on the calculated cross-sections. In Fig. 1, we present the cross-sections σ_{el} calculated with the use of the following parameters recommended for tellurium isotopes [5]: for ^{122}Te, $S_0 = 1.22$ and $S_1 = 1.7$; for ^{130}Te, $S_0 = 0.16$ and $S_1 = 0.82$. The same value of potential scattering radius, $R'_0 = 6.19$, was adopted for both isotopes. This value is the average over all isotopes and agrees well with the recommended dependence of this parameter on A [4]. The results of calculations of cross-sections with the quoted values for the parameters S_0, S_1, and R'_0 are depicted by curves *5* and *6*, respectively. It is evident that there is a substantial difference between them, and a corresponding difference should also take place between the experimental cross-sections of those isotopes. Although such data are absent, there are no reasons to expect any isotopic effects in them, which would be different from those, which are observed for cadmium and tin isotopes.

Hence, the results of our researches testify that the mentioned anomaly in the dependences of the parameter S_0 on the mass number A for cadmium and tin isotopes does not correlate with the average cross-sections. However, the large discrepancies between the S_0-values that we calculated by fitting the average experimental cross-sections and the recommended ones [3–5] obtained by analyzing the parameters known for the resolved resonances demand to be properly explained, because it is hard to imagine that they result from experimental errors. Therefore, for this problem to be ultimately solved, the additional experimental and theoretical researches are to be carried out.

3.2. Neodymium

In Fig. 3, the experimental energy dependences for the parameters σ_{el}, ω_1, and ω_2 taken from work [9] are exhibited. Since no alternative data are available, in order to verify the reliability of the cross-sections σ_{el}, we also plotted here the averaged total cross-sections σ_t taken from work [12]. One can see that the cross-sections σ_{el} regularly exceed the cross-sections σ_t. A possible reason for this fact may consist in hydrogen

remnants in the specimen, which the authors of work [9] discussed. In addition, in that work, only the data for the cross-sections σ_{el} were reported for energies of 325 and 442 keV, whereas the corresponding data for ω_1 and ω_2 were absent at all. The figure demonstrates that, provided the available scatters, the absence of the mentioned data reduces the reliability of resonance parameter values determined by the automatic fitting, because they govern, to some extent, the corresponding dependences on the energy. Therefore, in our calculations, we used the ω_1- and ω_2-values that were obtained by a certain averaging. In the figure, they are denoted as the data of work [9]. Our calculations proved the expediency of this procedure, although the minor changes to the values do not considerably affect the determined values of parameters, but only the χ^2-values.

Curves *2* in Fig. 3 correspond to the results of calculations for the quantities σ_{el}, ω_1, and ω_2 making use of the parameters taken from work [6]: $S_0 = 2.80(20)$, $S_1 = 2.10(60)$, $R'_0 = 6.31(50)$, $R'_1 = 9.64(1.1)$, and $S_{1,3/2} = 1.51(27)$. Of experimental σ_{el}-, ω_1-, and ω_2-values, only the cross-sections σ_{el} are described satisfactorily, which was achieved owing to the enlargement of the parameter S_1, although the corresponding value does not agree with the dependence of S_1 on A [4]. For other data, the observed discrepancies are associated most probably with their scatters.

We used the resonance parameters recommended for neodymium isotopes [4] to calculate the following weighted average values for the natural isotope content: $S_0 = 3.30$, $S_1 = 0.81$, and $R'_0 = 6.27$. The other parameters were obtained by the automatic fitting: $R'_1 = 10.18$ and $S_{1,3/2} = 1.10$. The results of calculations for σ_{el}, ω_1, and ω_2 are shown in Fig. 3 by curves *3*. One can see that the calculated cross-sections are regularly smaller than the experimental σ_{el} and σ_t, as well as than those calculated with the help of the previous parameter set.

By the values for parameters S_0 and S_1 of neodymium isotopes recommended in work [5], we obtained the following weighted average values: $S_0 = 3.36$ and $S_1 = 0.81$. By fixing them and carrying out the automatic fitting, we determined the other parameters: $R'_0 = 6.74$, $R'_1 = 9.71$, and $S_{1,3/2} = 1.20$. The results of calculations are depicted in Fig. 3 by curves *4*. One can see that the total cross-sections are described satisfactorily, and the description of the coefficients ω_1 and ω_2 is approximately of the same quality, as in the case of curves *3*.

The figure testifies that all three sets of parameters almost identically describe the coefficients ω_1 and ω_2 –

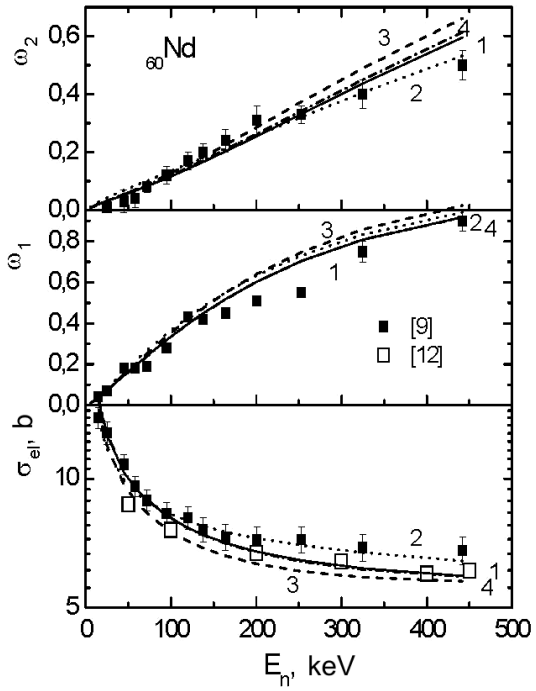

Fig. 3. The same as in Fig. 1, but for neodymium nuclei

both visually and on the basis of χ^2-test. Appreciably larger mismatches are observed for the calculated cross-sections. Owing to the uncertainty of the values for experimental cross-sections, it is difficult to judge the quality of their description. We selected such parameter values as to obtain an optimal description of experimental data. As a result, the following parameter set was obtained: $S_0 = 3.30$, $S_1 = 0.90$, $R'_0 = 6.75$, $R'_1 = 9.62$, and $S_{1,3/2} = 1.15$. The results of corresponding calculations are shown in Fig. 3 by curves *1*. One can see that the calculated cross-sections are considerably smaller than the data of work [9], the agreement with total cross-sections from work [12] is better, and the descriptions of quantities ω_1 and ω_2 are not worse than for the previous parameter sets – both visually and on the basis of χ^2-test. The obtained values of parameters S_0, S_1, and R'_0 are in agreement with the recommended ones, within the error limits, which confirms their reliability.

Hence, the results of our calculations testify that those parameters, which describe well the average cross-sections, agree, in general, with their dependences on the mass number A calculated in the framework of the optical model. At the same time, if any of the parameter sets, which were analyzed in this work, contains parameters with appreciable deviations from those de-

Average resonance parameters for tellurium and neodymium nuclei

Nucleus	$S_0 \times 10^4$	$S_1 \times 10^4$	R'_0, Fm	R'_1, Fm	$S_{1,1/2} \times 10^4$	$S_{1,3/2} \times 10^4$
Te	0.25(10)	1.70(30)	6.22(25)	8.65(45)	1.28(1.20)	1.91(40)
Nd	3.30(20)	0.90(20)	6.75(22)	9.62(65)	0.40(85)	1.15(30)

pendences, the corresponding description of experimental cross-sections turns out inevitably worse.

The resonance parameters obtained in this work are quoted in the Table.

4. Conclusions

In this work, new complete sets of average resonance parameters S_0, S_1, R'_0, R'_1 and $S_{1,3/2}$ were determined for tellurium and neodymium nuclei with the natural isotope contents. In general, the parameters obtained for both nuclei satisfactorily describe the available experimental data and agree with the dependences on the mass number A calculated in the framework of the optical model. As a whole, the values of parameters S_0 and S_1 recommended for both nuclei were confirmed. We also confirmed the value of parameter R'_0 recommended for neodymium nuclei [4]. For tellurium, the recommended value of parameter R'_0 (5.47) was not confirmed, because it does not coordinate with the dependence on A, and the parameter set obtained by fitting, making use of this value, unsatisfactorily describes experimental cross-sections.

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СЕРЕДНІ РЕЗОНАНСНІ ПАРАМЕТРИ ЯДЕР ТЕЛУРУ І НЕОДИМУ

М.М. Правдивий, І.О. Корж, М.Т. Скляр

Р е з ю м е

Із аналізу експериментальних диференціальних перерізів пружного розсіяння нейтронів у області енергії до 440 кеВ визначено повні набори середніх резонансних параметрів S_0, S_1, R'_0, R'_1, $S_{1,3/2}$ ядер телуру і неодиму з природним складом ізотопів. Проведено аналіз отриманих результатів, а також рекомендованих параметрів та деяких літературних даних.