B. BOUTABIA-CHÉRAITIA,[1] T. BOUDJEDAA[2]

[1] Laboratoire de Probabilités et Statistiques (LaPS), Université Badji-Mokhtar, Faculté de Médecine d'Annaba
(Annaba, Algérie; e-mail: bboutheina@hotmail.com)

[2] Laboratoire de Physique Théorique, Faculté des Sciences, Université de Jijel
(Algérie; e-mail: boudjedaa@gmail.com)

PACS 03.65.Ca, 03.65.Pm, 03.65.Nk

SCATTERING MATRIX IN THE DKP THEORY. BARRIER POTENTIAL CASE

We construct a pseudo-unitary scattering matrix for the Duffin–Kemmer–Pétiau (DKP) particles when interacting with a barrier potential.

Keywords: DKP equation, barrier potential, coefficient of transmission, coefficient of reflection, scattering matrix.

1. Introduction

The computation of the S-matrix is the main aim of the scattering theory. It finds its physical interpretation when we connect it with the coefficients of reflection and transmission. The realistic cases in the study of the problems of scattering are generally three-dimensional, the one-dimensional case being an approximation representing this reality, given that the one and three-dimensional scattering problems have interesting similarities, like the case of the scattering from a monosymmetric potential in one dimension which is similar to that of the triplet nucleon-nucleon scattering (in three dimensions).

The S-matrix for the Feshbach–Villars (FV) equation for spin 0 and $1/2$ particles in the presence of a Woods-Saxon potential was constructed in [1] using an unified approach, and the transmission and reflection coefficients were deduced.

The physical matrix elements of an S-matrix in the Duffin–Kemmer–Pétiau and Klein–Gordon–Fock theories coincide in cases of spin 0 particles interacting with external and quantized Maxwell and Yang-Mills fields and in case of an external gravitational field (without and with torsions) [2].

In this paper, we will construct the scattering matrix for the one-dimensional DKP equation in case of the interaction of spin 0 and spin 1 particles with a barrier potential. The related Green function was given in [3] by using the Sakata and Taketani (ST) decomposition, which brings out the "particle components" into one distinct Hamiltonian constructed on the basis of the FV analogy. Note that the particle components contain all what is necessary to describe the particle and exhibit the particle-antiparticle nature of the DKP equation which is revealed by the charge density.

Let us recall that the DKP equation [4–7], which is similar in structure to the Dirac equation, describes scalar (spin 0) and vectorial (spin 1) particles coupled to an electromagnetic field and will be written as ($\hbar = c = 1$):

$$\left[i\beta^{\mu}\left(\partial_{\mu}+ieA_{\mu}\right)-m\right]\psi\left(\mathbf{r},t\right)=0, \tag{1}$$

and the matrices β^{μ} satisfy the DKP algebra

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda}+\beta^{\lambda}\beta^{\nu}\beta^{\mu}=g^{\mu\nu}\beta^{\lambda}+g^{\nu\lambda}\beta^{\mu}. \tag{2}$$

The convention for the metric tensor is $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. Algebra (2) has three irreductible representations, whose degrees are 1, 5, and 10. The first one is trivial, having no physical content, the second and third ones correspond, respectively, to the scalar and vectorial representations. For the spin 0, the β^μ are given by

$$\beta^0 = \begin{pmatrix} \theta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}; \quad \beta^i = \begin{pmatrix} \mathbf{0} & \rho^i \\ -\rho_T^i & \mathbf{0} \end{pmatrix}; \quad i = 1, 2, 3, \tag{3}$$

with

$$\rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4}$$

The ρ_T denotes the transposed matrix of ρ, and $\mathbf{0}$ denotes the zero matrix. For the case of spin 1, β^μ are given by

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \bar{0}^T & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix};$$
$$\beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix}; \quad i = 1, 2, 3, \tag{5}$$

with

$$e_1 = (1, 0, 0); \quad e_2 = (0, 1, 0);$$
$$e_3 = (0, 0, 1); \quad \bar{0} = (0, 0, 0). \tag{6}$$

The s_i are the standard nonrelativistic (3×3) spin 1 matrices, and $\mathbf{0}$ and $\mathbf{1}$ denotes, respectively, the zero matrix and the unity matrix. When the interaction is scalar and independent of time, namely the square barrier potential

$$V(z) = V_0 \theta (a - |z|), \tag{7}$$

we choose the form $e^{-iEt}\kappa(z)$ for $\psi(z,t)$, and we obtain the following eigenvalue equation [8]:

$$\left[\beta^0 (E - eV) + i\beta^3 \frac{d}{dz} - m\right] \kappa(z) = 0. \tag{8}$$

Here, $\kappa(z)^T = (\varphi, \mathbf{A}, \mathbf{B}, \mathbf{C})$, A, B, and C are, respectively, vectors with components A_i, B_i, and C_i; $i = 1, 2, 3$. The square barrier potential is one of the simple models of potentials realizing the specific physical situation of the pair-creation phenomena [9–11], which is intimately related to the tunnel phenomena.

According to the equations they satisfy, one gathers the components of $\kappa(z)$ in this way:

$$\Psi^T = (A_1, A_2, B_3), \quad \Phi^T = (B_1, B_2, A_3),$$
$$\Theta^T = (C_2, -C_1, \varphi); \quad \text{and} \quad C_3 = 0, \tag{9}$$

with

$$\mathbf{O}_{\text{KG}} \Psi = 0; \quad \begin{pmatrix} \Phi \\ \Theta \end{pmatrix} = \begin{pmatrix} \frac{E - eV}{m} \\ \frac{i}{m} \frac{d}{dz} \end{pmatrix} \otimes \Psi. \tag{10}$$

Then one will designate, by $\phi(z)^T = (\Psi, \Phi, \Theta)$, the solution of (8), $O_{\text{KG}} = \frac{d^2}{dz^2} + \left[(E - eV)^2 - m^2\right]$ being the Klein–Gordon (KG) operator.

2. Scattering Matrix

When the wave representing a particle is incident on a potential, it is partially transmitted and partially reflected. The asymptotic forms of diverging waves can be determined at the time very long after the interaction and, therefore, determined by the S-matrix [12]. We consider a vectorial DKP particle to be subjected to the barrier potential (7). As $|z| \to \infty, V(z) \to 0$ sufficiently fast so that $\phi(z)$, the solution of Eq. (8) becomes that for a free particle. In what follows, we will try to examine the transmission-reflection problem, in which the particle is incident, say, from the left $(-\infty)$ or from the right $(+\infty)$. The asymptotic form for the wave function $\phi(z)$ is given by [8]

$$\phi_\pm(z) = \begin{cases} e^{\pm ik(z+a)} M \otimes \mathbf{V} + \\ + e^{\mp ik(z+a)} N' \otimes \mathbf{V} \begin{cases} z \to -\infty \text{ for } (+), \\ z \to +\infty \text{ for } (-), \end{cases} \\ e^{\pm ik(z-a)} M' \otimes \mathbf{V} + \\ + e^{\mp ik(z-a)} N \otimes \mathbf{V} \begin{cases} z \to +\infty \text{ for } (+), \\ z \to -\infty \text{ for } (-). \end{cases} \end{cases} \tag{11}$$

So, the incoming and outgoing parts of the wave function will be written as

$$\begin{cases} \phi_\pm^{\text{in}}(z) = \theta(-z) e^{\pm ik(z+a)} M \otimes \mathbf{V} + \\ + \theta(z) e^{\mp ik(z-a)} N \otimes \mathbf{V}, \\ \phi_\pm^{\text{out}}(z) = \theta(z) e^{\pm ik(z-a)} M' \otimes \mathbf{V} + \\ + \theta(-z) e^{\mp ik(z+a)} N' \otimes \mathbf{V}, \end{cases} \tag{12}$$

where $k = \sqrt{E^2 - m^2}$. The suffix $\pm$ refers to the situation where the wave is incident from $\mp\infty$, and $\mathbf{V}$

is a constant vector of dimension 3×1, whose components are related to the three directions of the spin 1. The vectors $M; M'; N; N'$ are defined by

$$M = \begin{pmatrix} B \\ B\frac{E}{m} \\ -B\frac{k}{m} \end{pmatrix}; \quad M' = \begin{pmatrix} C \\ C\frac{E}{m} \\ -C\frac{k}{m} \end{pmatrix}; \\ N = \begin{pmatrix} D \\ D\frac{E}{m} \\ D\frac{k}{m} \end{pmatrix}; \quad N' = \begin{pmatrix} A \\ A\frac{E}{m} \\ A\frac{k}{m} \end{pmatrix}. \tag{13}$$

The S-matrix relating $\phi_{\pm}^{\text{in}}(z)$ to $\phi_{\pm}^{\text{out}}(z)$ will be written as

$$\begin{pmatrix} M' \\ N' \end{pmatrix} \otimes \mathbf{V} = \begin{pmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{pmatrix} \left[\begin{pmatrix} M \\ N \end{pmatrix} \otimes \mathbf{V} \right], \tag{14}$$

where S_{aa}, S_{ba}, S_{ab}, and S_{bb} are block matrices of dimension 9×9. Applied to the two special cases $(M, N) = (\mathbf{0}, \mathbf{1})$ and $(M, N) = (\mathbf{1}, \mathbf{0})$, where $\mathbf{1}^T = (1, 1, 1)$ and $\mathbf{0}^T = (0, 0, 0)$, the relation above leads to

$$S = \begin{pmatrix} T_+ & R_- \\ R_+ & T_- \end{pmatrix} \otimes I_9. \tag{15}$$

We define the pseudo-unitarity of S by

$$S\overline{S} = \overline{S}S = \overline{I_{18}},$$

where

$$\overline{S} = \left(I_2 \otimes \widetilde{\beta^0} \right) S^+ \left(I_2 \otimes \widetilde{\beta^0} \right), \tag{16}$$

and

$$\widetilde{\beta^0} = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}; \quad \overline{I_{18}} = \left(I_2 \otimes \widetilde{\beta^0} \right)^2, \tag{17}$$

where $\mathbf{1}$ and $\mathbf{0}$ are, respectively, the (3×3) unit and null matrices. This leads to

$$|T_+| = |T_-|; \quad |R_+| = |R_-|. \tag{18}$$

The parity of $V(z)$ involves $SS^* = S^*S = 1$. This implies that

$$R_+ = R_-; \quad T_+ = T_-. \tag{19}$$

Finally, S takes the form

$$S = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \otimes I_9. \tag{20}$$

While putting $D = 0$ in Eq. (11), we obtain $\mathbf{R}$ and $\mathbf{T}$:

$$\mathbf{R} = |R|^2 = \left| \frac{A}{B} \right|^2; \quad \mathbf{T} = |T|^2 = \left| \frac{C}{B} \right|^2. \tag{21}$$

Note that $\mathbf{R}$ and $\mathbf{T}$ are given in [8] by

$$\mathbf{R} = \frac{\left(\frac{k^2 - p^2}{2pk} \right)^2 \sin^2 2pa}{1 + \left(\frac{k^2 - p^2}{2pk} \right)^2 \sin^2 2pa}, \tag{22}$$

$$\mathbf{T} = \frac{1}{1 + \left(\frac{k^2 - p^2}{2pk} \right)^2 \sin^2 2pa}, \tag{23}$$

with $p = \sqrt{(E - eV_0)^2 - m^2}$.

The scattering matrix for the spin 0 case would be deduced from that for the spin 1. Instead of being 18×18, it would only be 6×6, since we remove the tensorial product $(\otimes \mathbf{V})$ everywhere it appears in the relations above, since the particle of spin 0 is obviously scalar, not vectorial. Therefore,

$$S = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \otimes I_2. \tag{24}$$

The pseudo-unitarity of S is defined by

$$S\overline{S} = \overline{S}S = \overline{I_6} \tag{25}$$

with

$$\overline{S} = \left(I_2 \otimes \widetilde{\beta^0} \right) S^+ \left(I_2 \otimes \widetilde{\beta^0} \right) \tag{26}$$

and

$$\widetilde{\beta^0} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \overline{I_6} = \left(I_2 \otimes \widetilde{\beta^0} \right)^2. \tag{27}$$

Remark that if we consider only the two first components of M, M', N and N', S becomes of dimension 4×4 identically to the case of FV [13].

3. Conclusion

We have constructed the DKP scattering matrix for the time-like Lorentz vector interaction. For a scalar particle, the S-matrix has been yet constructed in the literature using the FV and the KG theories. In this paper, it is deduced from that of a vectorial particle using the DKP theory in case of the interaction with a barrier potential, and its physical elements coincide with those of the FV theory.

1. K. Khounfais, T. Boudjedaa, and L. Chetouani, Czech. J. Phys. **54**, 697 (2004).
2. B.M. Pimental and V.Ya. Fainberg, Theor. Math. Phys. **124**, No. 3, 1234 (2000).
3. B. Boutabia-Chéraitia and T. Boudjedaa, J. of Geometry and Phys. **62**, 2038 (2012).
4. R.Y. Duffin, Phys. Rev. **54**, 1114 (1938).
5. N. Kemmer, Proc. Roy. Soc. A **173**, 91 (1939).
6. J. Géhéniau, Acad. R. Belg. Cl. Sci. Mém. Collect. **18**, No. 1, 15 (1938).
7. G. Pétiau, Acad. Roy. de Belg. Mem. Collect. **16**, No. 2, 1 (1936).
8. B. Boutabia-Chéraitia and T. Boudjedaa, Phys. Lett. A **338**, 97 (2005).
9. A. Calogeracos and N. Dombey, Int. J. Mod. Phys. A **14**, 631 (1999).
10. N. Dombey and A. Calogeracos, Phys. Rep. **315**, 41 (1999).
11. R.K. Su, G.G. Siu, and X. Chou, J. Phys. A **26**, 1001 (1993).
12. Y. Nogami and C.K. Ross, Am. J. Phys **64**, 923 (1996).
13. H. Feshbach and F. Villars, Rev. Mod. Phys. **30**, 24 (1958).

Received 30.09.13

Б. Бутабіа-Черетіа, Т. Будьеда

МАТРИЦЯ РОЗСІЯННЯ В ТЕОРІЇ ДКП.
ВИПАДОК БАР'ЄРНОГО ПОТЕНЦІАЛУ

Р е з ю м е

Побудована псевдоунітарна матриця розсіювання для Дуффін–Кеммер–Петі (ДКП) частинок, що взаємодіють за допомогою бар'єрного потенціалу.