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Type B uncertainty of two-channel measurements

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Abstract

The paper presents the problems of evaluating the standard uncertainty of measuring a quantity using the type B method, the result of which is the average value of the results obtained from two channels with the same parameters, for example, as the indications of two measuring instruments of the same type. It is shown that for given values of maximum permissible errors (MPE) of measuring instruments and their readings x_1 and x_2, the uncertainty of the result determined *a posteriori* is not equal to the uncertainty determined by the conventional method (GUM). It is shown that when the measurement result is determined as arithmetic mean $y=(x_1+x_2)/2$, additional information as the half distance of readings $v=|x_1-x_2|/2$ an be used to correctly determine the standard uncertainty of such measurement. Depending on the half distance of readings, the standard uncertainty can theoretically vary from its maximum value (the readings of both meters are equal) to zero (with maximum difference in readings). The analysis of the uncertainty was carried out for uniform distributions of possible deviations of the readings of measuring instruments within their MPE. The results of simulations by the modified Monte-Carlo method, which show good convergence with theoretical results, are given.

Keywords: Type B; uncertainty; measurement; measurand; Monte-Carlo simulation.

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1. Introduction

1.1. Type B uncertainty

It is well known that the standard uncertainty $u(X)$ is evaluated based on scientific judgment about all available information on possible variability of X. The pool of information may include previous measurement data; experience with or general knowledge of the behaviour and properties of relevant materials and instruments; manufacturer's specifications; data provided in calibration and other certificates; uncertainties assigned to the reference data taken from handbooks [GUM, 1]. According to the definition of uncertainty [GUM, 1], [VIM, 2]: *uncertainty is a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand*. To correctly evaluate the standard measurement uncertainty from a formal view, the procedure of a non-statistical type B method should be based on the probability density function (PDF) $f_X(X|x)=f_X(X–x)$ of possible X-values of the measurand around the observed measurement result x (and, of course, the parameters of the used instrument, as well as the conditions of measurement, etc.):

$$u_B\left(X|x\right)=\sqrt{\int_{-\infty}^{\infty}\left(X-x\right)^2 f_X\left(X|x\right)dX}. \qquad (1)$$

In this relation, x (result value) is known and non-random, while the value of the measurand X is random [3]. I.e., to evaluate Type B uncertainty, we must use a method, which is similar to that used to evaluate Type A uncertainty. However, in the case of type B uncertainty, the derivation of the distribution $f_X(X–x)$ of a measurand is different from that of type A uncertainty, which is based on the distributions obtained from estimation theory [3].

Since manufacturers usually provide maximal permissible errors (MPE) $\pm\cdot\Delta_{MPE}$ of their instruments, in practice, another probability distribution $f_\Delta(\Delta|X)=f_x(x-X)$ of the measuring instrument errors ($\Delta=x-X$) is used, which describes the distribution of possible x-values of the measurement results at a known value of the measured quantity X. In [GUM, 1] this distribution is called "*a priori* distribution". This distribution (its estimations or at least its parameters) can be determined experimentally based on the tests carried out, for example, as in the case of calibration. It should be noted that for distribution $f_x\ (x–X)$, the x-value of a result is random, while the value of the measurand X is known, i.e. not random.

For known values of $\pm\Delta_{MPE}$, the PDFs $p_X(X–-x_i|\Delta_{MPE})$ and $p_x(x–X|\Delta_{MPE})$ are described by the dependences:

$$p_X\left(X-x_i|\Delta_{MPE}\right)=\begin{cases} f_X\left(X-x_i\right), & \left|X-x_i\right|\le\Delta_{MPE}, \\ 0 & \text{otherwise.}\end{cases} \qquad (2)$$

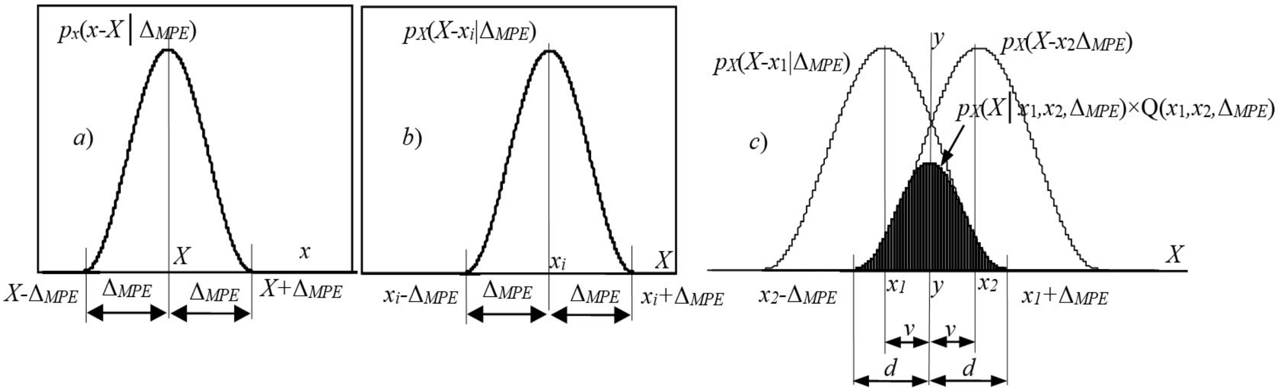

Fig. 1. *A priori* PDF $p_x(x–X|\Delta_{MPE})$ of possible x-values of the instrument readings at a known measurand $X(a)$, and PDF $p_X(X–x_i|\Delta_{MPE})$ of possible X-values of a measurand at known instrument readings $x_i(b)$, *a posteriori* distributions $p_X(X–x_1|\Delta_{MPE})$, $p_X(X–x_2|\Delta_{MPE})$, and joint distribution $p_X(X|x_1,x_2,\Delta_{MPE})$ without a normalizing factor $Q(x_1,x_2,\Delta_{MPE})$ − shaded

$$p_x\left(x-X\middle|\Delta_{MPE}\right)=\begin{cases} f_x\left(x-X\right), \left|x-X\right|\le\Delta_{MPE}, \\ 0 \qquad\qquad \text{otherwise.}\end{cases} \quad (3)$$

Despite a certain similarity, the PDFs $p_X(X–x_i|\Delta_{MPE})$ and $p_x(x–X|\Delta_{MPE})$ are not the same. The differences of these PDFs are shown in Fig. 1*a*, *b*. For these PDFs, the range $x(X,\Delta_{MPE})$ for indicating, when the values of the measurand X and MPE are known, is:

$$X-\Delta_{MPE}\le x\left(X,\Delta_{MPE}\right)=X+\Delta_{MPE}, \quad (4)$$

the range $X(x_i,\Delta_{MPE})$ for the value of the measurand X, when the indication x_i and MPE are known, is:

$$x_i-\Delta_{MPE}\le X\left(x_i,\Delta_{MPE}\right)=x_i+\Delta_{MPE}. \quad (5)$$

Although here, in the simplest case of measuring the width of both ranges is the same ($2\cdot\Delta_{MPE}$), the standard uncertainty of the observed result x is equal to the standard deviation of the indication error:

$$\begin{aligned} u_B\left(X|x_i\right)&=\sqrt{\int_{x_i-\Delta_{MPE}}^{x_i+\Delta_{MPE}}\left(X-x_i\right)^2 p_X\left(X-x_i|\Delta_{MPE}\right)dX}= \\ &=s\left(\Delta|X\right)=\sqrt{\int_{X-\Delta_{MPE}}^{X+\Delta_{MPE}}\left(x-X\right)^2 p_x\left(x-X|\Delta_{MPE}\right)dx}, \end{aligned} \quad (6)$$

however, as it will be shown below, in case of more complex measurements, this approach can lead to incorrect values.

1.2. Simultaneous measurements of quantities using two measuring instruments of the same type

Such measurements are used to increase the level of reliability on the results obtained in measurements for important objects. The measuring channel may be the same (with the same$\cdot\Delta_{MPE}$) or different, i.e. with different $\Delta_{MPE,1}$ and $\Delta_{MPE,2}$. Let the same quantity X be measured simultaneously using two measuring instruments of the same type, with the same value of MPE: $\Delta_{MPE,1}=\Delta_{MPE,2}=\Delta_{MPE}$ and the same PDF $p_\Delta(x_i–X|\Delta_{MPE})$ of possible readings x_1, x_2 around X. Then for independent readings x_1, x_2 of both instruments, their average value (the result of a given measurement) is equal:

$$y=\frac{x_1+x_2}{2}=X+\frac{\Delta x_1+\Delta x_2}{2}=X+\Delta_y, \quad (7)$$

where $\Delta_y=(\Delta x_1+\Delta x_2)/2$ is an error of the result.

This error value is described by a PDF in the form of a convolution of the PDFs of the error possible values of the readings of these two measuring instruments:

$$\begin{aligned} p_y\left(\left(y-X\right)\middle|\Delta_{MPE}\right)=2\int_{-\Delta_{MPE}}^{\Delta_{MPE}} p_x\left(\left(x_1-X\right)\middle|\Delta_{MPE}\right)\times \\ \times p_x\left(\left(2y-X-x_1\right)\middle|\Delta_{MPE}\right)dx_1. \end{aligned} \quad (8)$$

Distribution (8) characterizes the dispersion of possible y-values of result (6) around the measurand value X. Since, using both instruments of the same type, the distributions of two readings are the same, the standard deviation of possible y-values (7) is $\sqrt{2}$ times smaller than the standard deviation $\sigma(x_1)=\sigma(x_1)=\sigma(x)$ of the readings of each instrument:

$$\sigma(y)=\sqrt{\frac{\sigma^2\left(x_1\right)+\sigma^2\left(x_2\right)}{4}}=\frac{\sigma\left(x_1\right)}{\sqrt{2}}=\frac{\sigma\left(x_2\right)}{\sqrt{2}}=\frac{\sigma\left(x\right)}{\sqrt{2}}. \quad (9)$$

In the conventional approach [GUM, 1], this value is taken as the standard uncertainty of such measurement:

$$u_c(y)=\sigma(y)=\frac{\sigma\left(x\right)}{\sqrt{2}}. \quad (10)$$

This is where the basic problem arises, related to the fact that such approach, based on the principles of classical probability theory, quite fails to correspond to the definition of the uncertainty, as a measure of the dispersion of possible measurand values (X) around the obtained result (y), and not vice versa.

The aim of the study. The purpose of the following study is to accordingly derive relations for correct evaluation of the standard uncertainty of a two-channel measurement and to perform a Monte-

Carlo test for the correctness of the obtained relations.

2. *A posteriori* probability distribution and standard uncertainty of measurand

2.1. *A priori* distribution for two-channel measurement

If two meters of the same type are used to measure the same (unknown) value of the measurand X and two observations (instruments readings) x_1 and x_2 are obtained, then for these two readings there are two distributions (2) $p_X(X–x_1|\Delta_{MPE})$, $p_X(X–x_2|\Delta_{MPE})$ of possible values of the measurand X around these readings (Fig. 1*c*). As it was mentioned above, to correctly evaluate the standard measurement uncertainty using two meters, one should first derive the PDF $f_X(X-y)=p_X(X|x_1, x_2, \Delta_{MPE})$ of possible X-values of a measurand around the observed measurement result y. In other words, one needs to answer the question: what X-values could the measurand have if the measurement result was determined as the average y-value from the readings x_1, x_2 of the corresponding instruments?

Since the readings of two meters are consistent with the values of the maximum permissible error Δ_{MPE} and with the observed readings x_1 and x_2, the interval, in which the value of the measurand X can be measured by ordering the meter readings $x_1 \leqslant x_2$, exists in the intervals (Fig. 1*c*):

$$x_2 - \Delta_{MPE} \le X \le x_1 + \Delta_{MPE}. \quad (11)$$

These two distributions form a combined distribution $p_X(X|x_1, x_2, \Delta_{MPE})$ of possible values of the measurand X (Fig. 1*c*), which in interval (11) is described by the relation:

$$p_X\left(X|x_1,x_2,\Delta_{MPE}\right) = \frac{p_X\left(X-x_1|\Delta_{MPE}\right)\cdot p_X\left(X-x_2|\Delta_{MPE}\right)}{Q\left(x_1,x_2,\Delta_{MPE}\right)},$$
$$x_2 - \Delta_{MPE} \le X \le x_1 + \Delta_{MPE}, \quad (12)$$

where $Q(x_1, x_2, \Delta_{MPE})$ is a constant that ensures the normalization of the distribution:

$$Q\left(x_1,x_2,\Delta_{MPE}\right) = \int_{x_2-\Delta_{MPE}}^{x_1+\Delta_{MPE}} p_X\left(X|x_1,x_2,\Delta_{MPE}\right)dX. \quad (13)$$

2.2. Example for a uniform *a priori* distribution

A specific example of measuring the same voltage with two voltmeters of the same type with $\Delta_{MPE} = \pm 0.05$ V will be studied first. Let us assume that the indications of voltmeters are: $u_1=2.265$ V and $u_2=2.345$ V. Therefore, the measurement result is (7):

$$y = U = \frac{u_1+u_2}{2} = 2.305\,\text{V}. \quad (14)$$

The interpretations of the uncertainties of the voltage measurements by both voltmeters and the uncertainty of a two-channel measurement are given in Fig. 2. For $u_1=2.265$ V of the 1st voltmeter and $\Delta_{MPE} = \pm 0.05$ V, the range of possible values of the measured voltage (Fig. 1*c* and (5) accordingly) is between: 2.265 V − 0.005 = 2.215 V and 2.265 V + + 0.005 = 2.315 V. In Fig. 2 this interval (left side) is filled by grey colour. For $u_2 = 2.345$ V of the 2nd voltmeter and $\Delta_{MPE} = \pm 0.05$ V, the range of possible values of the measured voltage is between: 2.345 V − 0.005 = 2.295 V and 2.345 V + 0.005 = 2.395 V. In Fig. 2 this interval (right side) is filled by grey colour too.

Because for both voltmeters the measured voltage is the same, having determined result (14), the range of the uncertainty is the result of the operation & both ranges in Fig. 2, i.e., between 2.345 V − 0.005 = 2.295 V and 2.265 V + 0.005 = 2.315 V. The width of this range is $2{\cdot}d = 2.315 - 2.295 = 0.02$ V and its half width is $d = 1$ V. Consequently, the standard uncertainty of such a two-channel measurement with the result $U = 2.305$ V is:

$$u_c\left(U\right) = \frac{d}{\sqrt{3}} = \frac{0.01\,\text{V}}{\sqrt{3}} \approx 0.00577\,\text{V}. \quad (15)$$

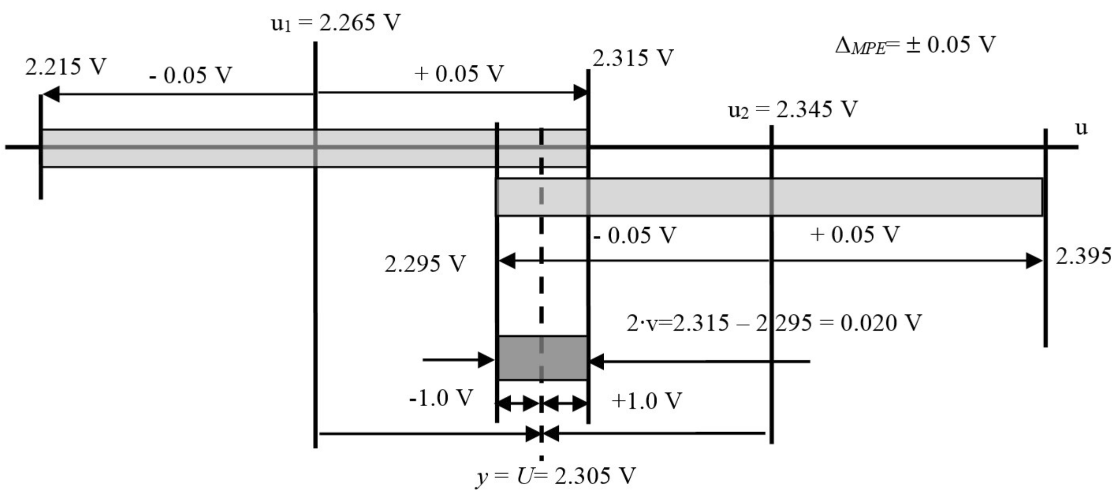

Fig. 2. Interpretation of uncertainties of the voltage measurement by both voltmeters (u_1=2.265 V and u_2 = 2.345 V), Δ_{MPE} = ± 0.05 V and of two-channel measurement

In the conventional approach [GUM, 1] (10), accounting for the independency of the indications of both voltmeters, the standard uncertainty of the result (10) is:

$$u_{c,us}(U) = \frac{\Delta_{MPE}}{\sqrt{2\cdot 3}} = \frac{0.05\ \text{V}}{\sqrt{6}} \approx 2.041\ \text{V}. \quad (16)$$

One can see that when evaluating the uncertainty by different approaches, we obtain different values, about 3.4 times.

2.3. Standard uncertainty for *a priori* uniform distribution in general case. Using indications x_1, x_2 of both instruments and the MPE value Δ_{MPE} as in the example presented above, the width of the interval within the located measurand is:

$$2d = 2\Delta_{MPE} - |x_1 - x_2| = 2\Delta_{MPE}\left(1 - \frac{|x_1 - x_2|}{2\Delta_{MPE}}\right) = 2\Delta_{MPE}\left(1 - \frac{v}{\Delta_{MPE}}\right), \quad (17)$$

where $v = |x_1 - x_2|/2$ is a half of the distance between the indications.

Based on (17), the *a posteriori* uniform distribution (12), (13) for the measurand is given by the expression:

$$p_X\left(X|y, v; \Delta_{MPE}\right) = \frac{1}{2\Delta_{MPE}\cdot\left(1 - \frac{v}{\Delta_{MPE}}\right)} \times \\ \times \begin{cases} 1, & 0 \le |X - y| \le \Delta_{MPE} - v,\ 0 \le v \le \Delta_{MPE} \\ & 0,\ \text{otherwise};\ v > \Delta_{MPE} \end{cases} \quad (18)$$

Therefore, in general case (18), the standard uncertainty of such a two-channel measurement is:

$$u\left(X|v; \Delta_{MPE}\right) = \frac{\Delta_{MPE}}{\sqrt{3}}\cdot\left(1 - \frac{v}{\Delta_{MPE}}\right), \\ 0 \le v \le \Delta_{MPE}. \quad (19)$$

The dependency (19) is shown in Fig. 3*a*. We can see, that for a two-channel measurement with instruments of the same type (same Δ_{MPE}), the standard measurement uncertainty decreases when the interval between the readings of instruments increases. The standard uncertainty ($\Delta_{MPE}/\sqrt{6}$) determined by the conventional method [GUM, 1] is shown by the dash line.

3. Monte-Carlo simulation results

A modified Monte-Carlo method (MCM) was used to verify the obtained relation (19) of the combined standard uncertainty of the measurement result. In general, the conventional MCM is applied to study simple problems, not inverse ones. This is a consequence of the fact that in the conventional MCM [4], the parameters of the distributions of input quantities and the function, according to which the output quantities are determined, are given, and then by multiple draws we determine the parameters of the distributions of output quantities. On the other hand, for inverse problems, the parameters of the distributions of input quantities must be determined.

Regarding the verification of the dependence of the standard uncertainty of the mean value of the readings of two instruments of the same type with a given probability distribution of possible deviations of the readings of instruments within the limits of the maximum permissible error, the modification consists in repeated application of the conventional MCM.

The ***first step*** is to assign (i) the value of the maximum permissible error Δ_{MPE}, (ii) the value of the measurement result y, (iii) the variation range of the measurand:

$$y - \Delta_{MPE} = X_{01} \le X \le X_K = y + \Delta_{MPE}. \quad (20)$$

With this, the successive X_k-values of the input quantity are determined as:

$$X_k = X_{k-1} + hx = y - \Delta_{MPE} + k\cdot hx,\ \ k = 0,1,2,...,\ K, \quad (21)$$

where $hx = 2\Delta_{MPE}/K$ is the value of the step of a changing measurand, K is the number of the measurand values

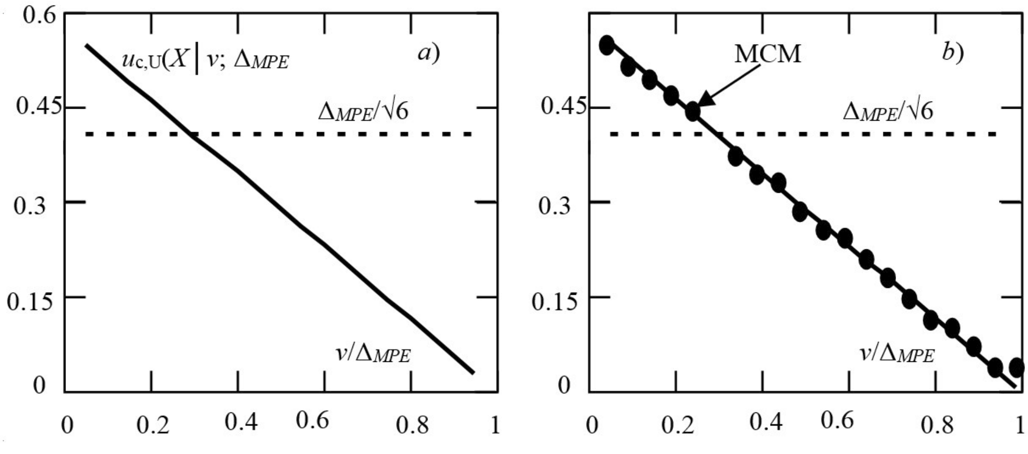

Fig. 3. Theoretical values of standard uncertainty vs ratio v/Δ_{MPE} (for *a priori* uniform distribution) (*a*), results of Monte-Carlo simulation (*b*)

around the y-result, and (iv) N values $v_i=\Delta v\cdot i-hv/2$ (i = 1,...,N) with step $hv=\Delta_{MPE}/N$ of a half distance between the previously determined results.

The ***second step*** involves K times (with changed parameters) of the MCM simulation with M trials. For this, during each k (k = 0,1,...,K) of the MCM simulations (j = 1,2,..,M) the following procedure is realized:

1) for a given value of X_k (k = 0,1,...,K) of the input quantity, M pairs of random values of $x_{1;k,j}$, $x_{2;k,j}$ of the readings of both measuring instruments within $X_k \pm \Delta_{MPE}$ are generated;

2) based on the values of $x_{1;k,j}$, $x_{2;k,j}$, M values of the output quantity are calculated – measurement results:

$$y_{k,j} = \left(x_{1;k,j} + x_{2;k,j}\right)/2, \tag{22}$$

and the modulus of the half-width between the indications:

$$v_{k,j} = \left(x_{2;k,j} - x_{1;k,j}\right)/2; \tag{23}$$

3) the number of $L_{k,i}$ events, for which two conditions:

$$\begin{gathered}\left(y - hx/2 \le y_{k,j} \le y + hx/2\right) \;\; \& \\ \left(v_i - hv/2 \le v_{j;k} \le v_i + hv/2\right)\end{gathered} \tag{24}$$

(when the observed measurement results $y_{k,j}$ fall in the interval $y \pm hx/2$ around the set y-value and the estimated half-width $v_{k,j}$ falls in the interval $v_i \pm hv/2$) are determined.

The ***third step*** concerns the determination (from the results obtained by simulations) of the standard uncertainty:

1) for different input X_k-values and half width v_i, the matrix **P** of probability values is estimated:

$$P_{k,i} \approx \frac{L_{k,i}}{\sum_{k=1}^{K} L_{k,i}}; \tag{25}$$

2) using X_k-values and corresponding estimated probabilities $P_{k,i}$ at the given values of the half-width v_i, the standard uncertainty as the standard deviation of the measurand is evaluated as:

$$\begin{gathered} u\left(X|v_i\right) \approx s\left(X|v_i\right) = \sqrt{\sum_{k=1}^{K}\left(X_k - \overline{X_k}\right)^2 \cdot P_{k,i}}, \\ \overline{X_k} = \sum_{k=0}^{K} X_k \cdot P_{k,i}. \end{gathered} \tag{26}$$

The obtained results of a Monte-Carlo simulation (M=3×10[4]) when K=20, N=20 are shown in Fig. 3*b*. We can see that the standard uncertainties evaluated by simulations are very close to the theoretical values.

4. Notes on other applications of a similar method

It should be noted, that in [5], after referring to [6], the so-called minimax method of evaluating the result with multiple observations (of number n), which have limited maximum errors, is presented and analysed. In this method, the result of a measurement is determined as the centre of a segment that includes all intervals around each result. That is, although this method is applied to multiple measurements, but the principle of determining the result is similar to the two-channel measurement presented. Fig. 2 also can be applied to this method, only for two results ($n=2$). However, there is an important difference between the one discussed and the one presented in these papers. These differences are a consequence of the fact that the discussed method provides measuring the same quantity with two meters at the same time, while the presented method provides the processing of successive measurement results over time. Besides, the paper analyses in detail the standard uncertainty, which is dependent on the half distance between the indications of two measuring instruments.

Subsequently, this method was discussed in [7] and developed in [8]. However, no detailed analysis of the uncertainty of such measurements was carried out in these works. In addition, it should be noted that this method leads to a well-known method based on positional statistics [6], often used to develop observations with uniform distributions [9].

5. Conclusions

Based on the analysis, we can conclude that in the case of a two-channel measurement with identical measuring instruments, the uncertainty, which results in the average value of the readings of these instruments, the standard uncertainty cannot be determined in the conventional way, i.e. as the standard uncertainty of the readings divided by the root of 2.

To correctly determine the standard uncertainty, first it is necessary to determine the *a posteriori* distribution of the measurand, the shape of which depends on the shape of the *a priori* distribution, and its parameters depend on the distance between the readings of the used instruments. Therefore, the standard measurement uncertainty is a function of this distance.

With equal readings of both meters, the standard uncertainty and the standard uncertainty determined from the *a priori* distribution of an individual instrument are equal. On the other hand, when the interval between the readings increases, the standard uncertainty decreases. Theoretically, the uncertainty tends to zero if the distance between the readings of both instruments take extreme values with an interval of $2\Delta_{MPE}$.

The simulations by the Monte-Carlo method confirmed the correctness of the derived relations for calculating the standard uncertainty of a two-channel measurement.

The results are obtained for the uniform *a priori* distribution of the instrument readings, but the same approach can be used for other *a priori* distributions. The general character of the dependence of the standard measurement uncertainty, determined from the *a posteriori* distribution, will be similar. For small distances of the instrument readings, the standard uncertainty will be larger, and with increasing distances, the uncertainty will decrease.

Невизначеність типу В двоканального вимірювання

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Анотація

У статті розглянуто проблеми оцінювання стандартної невизначеності вимірювання величини методом типу В, результатом якого є середнє значення результатів, отриманих із двох каналів з однаковими параметрами, наприклад, як покази двох однотипних засобів вимірювання. Такі вимірювання виконують, наприклад, з метою покращення надійності отримуваних результатів. Загалом для правильного оцінювання невизначеності типу В, подібно як і у випадку оцінювання невизначеності типу А, потрібно мати розподіл *a posteriori* можливих значень вимірюваної величини навколо отриманого результату. Показано, що коли результат вимірювання визначається як середнє арифметичне показів x_1 та x_2 засобів, $y=(x_1+x_2)/2$, то для отримання потрібного розподілу *a posteriori* може бути використано додаткову інформацію у вигляді половини відстані $v=|x_1-x_2|/2$ між показами засобів. У такому разі стандартна невизначеність типу В є функцією від цієї відстані й теоретично вона може змінюватись від максимального значення (при однакових показах обох засобів) до нуля (при максимальній різниці показів). Теоретичний аналіз стандартної невизначеності проводився для рівномірного *a priori* розподілу (розподілу можливих відхилень показів засобів у межах $\Delta_{МДП}$), для якого розподіл *a posteriori* є теж рівномірним, але в інших межах: $y-(\Delta_{MPE}-v)\leq X\leq y+(\Delta_{MPE}-v)$. У цьому випадку маємо лінійне зменшення стандартної невизначеності від збільшення половини відстані v між показами засобів. З метою перевірки правильності отриманих результатів виконано моделювання модифікованим методом Монте-Карло. Модифікація полягає у тому, що звичайний метод Монте-Карло виконується багаторазово для різних можливих значень вимірюваної величини навколо отриманого результату в межах $\pm\Delta_{МДП}$ та різних можливих значень $2v$ відстані між показами засобів. У результаті такого моделювання отримується оцінка розподілу *a posteriori* вимірюваних величин, на основі якого визначають оцінки стандартної невизначеності для різних значень половини відстані v. Представлено результати такого моделювання, які показали добру збіжність із теоретичними результатами.

Ключові слова: тип В; невизначеність; вимірювання; вимірювана величина; Монте-Карло моделювання.

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