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H-functions for the analysis of models of agritechnologies

This article examines the use of N-functions and H-matrices for the analysis of models of technology obtained in the form of functional networks.

Key Words: H-function, graph, costs, functional network, fuzzy subset, index quality.

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Н-функції для аналізу моделей агротехнологій

У статті розглядається застосування Н-функцій та Н-матриць для проведення аналізу моделей агротехнологій, що отримані у вигляді функціональних мереж.

Ключові слова: Н-функція, граф, витрати, функціональна мережа, нечітка підмножина, показник якості.

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The growing of agricultural crop in the contemporary world is a technology in which the values of approximate results pawned in advance. For this purpose, in practice, we are trying to get some possible estimates for main parameters of this technology, using a system of different indicators and various models.

Models in the form of algebraic functional networks on the basis of algorithmic algebras can be used for optimization and evaluation of agricultural technological processes [1, 2]. Such models use standard algorithmic (logical) functional structures. To obtain the corresponding estimates we make the transformation of initial algorithm to some (possibly unique) operator with equivalent characteristics.

Such probability models have been considered in [3, 4]. Moreover in [4] were considered functional models in which the costs of transitions from one technological operation to another were taken as random variables, and the corresponding probabilities as fuzzy values.

For the description of agricultural technologies in the present paper also used mathematical models constructed on the basis of algebraic functional networks. In them assumed that the cost of transition from one technological operation (or its element) to the other were the fuzzy variables that correspond to the practical situation, when such values, we can to know only approximately. The specified approach provides an opportunity to assess the quality of agro-technology and make it better.

Analog of H-function method for fuzzy models of agro-technological processes

Considered the analog of the method with the using of generating H-functions [3] for modeling agricultural technology as many iterative functional structure with random cycles. Structural functions and corresponding algebraic functional networks of the technological processes consist of a fixed set of technical operations [1, 2, 5]. Therefore, the obtained functional structure is a structure without memory and a structure in which resources are replenished and not replenished. We construct a certain functional network [5, 6] to describe the technological process. The whole process is considered in N points in time.

A graph corresponds to a functional network that was constructed. The weight arcs t_{ij} are the costs transition from the node i to node j, and they are elements of some fuzzy subsets with the membership functions μ_{ij}. Then for the costs $i \to j$ in some moments $l_z, l_{z+1}, \ldots, m_z$, that are possible for this state, we have

$$\{(t_{ij}(l_z), d_{ij}(l_z)), (t_{ij}(l_{z+1}), d_{ij}(l_{z+1})), \ldots,$$

$(t_{ij}(m_z), d_{ij}(m_z))\}$.

If the expression $H_{ij}(s,\cdot) = d_{ij}(\cdot)L[\mu_{ij}(t)]$ is a H-function of the arc (i,j), where $L[\mu_{ij}(t)] = \int_0^{+\infty} e^{-pt}\mu_{ij}(t)dt$, then to replace some of the arcs on a smaller equivalent set we can to obtain following result.

Statement 1. 1. For replacement of two consecutive arcs (i,k), (k,j) on a single equivalent arc (i,j) we have

$$t_{ij} = t_{ik} + t_{kj}\,,\ \mu_{ij}(t) = \int_0^t \mu_{ik}(t-\tau)\mu_{kj}(\tau)d\tau$$

and $H_{ij}(s,\cdot) = H_{ik}(s,\cdot)\cdot H_{kj}(s,\cdot)$.

2. For replacement two parallel arcs $(i,j)'$, $(i,j)''$ on a single equivalent arc (i,j) we have

$$t_{ij} = \frac{d'_{ij}t'_{ij} + d''_{ij}t''_{ij}}{d'_{ij} + d''_{ij}},$$

$$\mu_{ij}(t) = \mu'_{ij}(t) + \mu''_{ij}(t) - \frac{d'_{ij}(\cdot)\mu'_{ij}(t) + d''_{ij}(\cdot)\mu''_{ij}(t)}{d'_{ij}(\cdot) + d''_{ij}(\cdot)}$$

and

$$H_{ij}(s,\cdot) = H'_{ij}(s,\cdot) + H''_{ij}(s,\cdot).$$

3. For replacement of a graph fragment (i,j) with arc-loop (maximal number of cycles is *M*) on single equivalent arc (i',j) we have

$$t'_{ij} = \frac{t_{ij}d_{ij} + (t_{ii}+t_{ij})d_{ii}d_{ij} + \ldots + ((M-1)t_{ii}+t_{ij})d_{ii}^{M-1}d_{ij}}{d_{ij} + d_{ii}d_{ij} + \ldots + d_{ii}^{M-1}d_{ij}}$$

and

$$H'_{ij}(s,\cdot) = H_{ij}(s,\cdot)\frac{1-(H_{ii}(s,\cdot))^M}{1-H_{ii}(s,\cdot)}.$$

The proof. In a case of replacement of two consecutive arcs (i,k), (k,j) on a single equivalent arc (i,j) taking $t_{ij} = t_{ik} + t_{kj}$ we have $d_{ij} = d_{ik}\cdot d_{kj}$ correspondently. Then, using the multiplication as a convolution

$$\mu_{ij}(t) = \int_0^t \mu_{ik}(t-\tau)\mu_{kj}(\tau)d\tau,$$

we have

$$H_{ij}(s,\cdot) = d_{ij}(\cdot)L[\mu_{ij}(t)] =$$
$$= d_{ik}(\cdot)d_{kj}(\cdot)L[\int_0^t \mu_{ik}(t-\tau)\mu_{kj}(\tau)d\tau] =$$
$$= d_{ik}(\cdot)d_{kj}(\cdot)L[\mu_{ik}(t)]L[\mu_{kj}(t)].$$

Consequently we obtain

$$H_{ij}(s,\cdot) = H_{ik}(s,\cdot)H_{kj}(s,\cdot).$$

It is easy to get a similar result for a number of consecutive arcs $r \ge 2$, that is,

$$H_{1r+1}(s,\cdot) = H_{12}(s,\cdot)H_{23}(s,\cdot)\cdot\ldots\cdot H_{rr+1}(s,\cdot).$$

In a case of replacement of two parallel arcs $(i,j)'$, $(i,j)''$ on a single equivalent arc (i,j) using $t_{ij} = \frac{d'_{ij}t'_{ij} + d''_{ij}t''_{ij}}{d'_{ij} + d''_{ij}}$ and accordingly $d_{ij} = d'_{ij} + d''_{ij}$ (for $d_{ij} \le 1$) we can get

$$\mu_{ij}(t) = \mu'_{ij}(t) + \mu''_{ij}(t) - \frac{d'_{ij}(\cdot)\mu''_{ij}(t) + d''_{ij}(\cdot)\mu'_{ij}(t)}{d'_{ij}(\cdot) + d''_{ij}(\cdot)}.$$

Then

$$H_{ij}(s,\cdot) = d_{ij}(\cdot)L[\mu_{ij}(t)] =$$
$$= (d'_{ij}(\cdot) + d''_{ij}(\cdot))L[\mu'_{ij}(t) + \mu''_{ij}(t) -$$
$$- \frac{d'_{ij}(\cdot)\mu''_{ij}(t) + d''_{ij}(\cdot)\mu'_{ij}(t)}{d'_{ij}(\cdot) + d''_{ij}(\cdot)}] =$$
$$= (d'_{ij}(\cdot) + d''_{ij}(\cdot))L[\frac{d'_{ij}(\cdot)\mu'_{ij}(t) + d''_{ij}(\cdot)\mu''_{ij}(t)}{d'_{ij}(\cdot) + d''_{ij}(\cdot)}] =$$
$$= d'_{ij}(\cdot)L[\mu'_{ij}(t)] + d''_{ij}(\cdot)L[\mu''_{ij}(t)]$$

and we obtain

$$H_{ij}(s,\cdot) = H'_{ij}(s,\cdot) + H''_{ij}(s,\cdot).$$

For the larger set of parallel arcs ($r \ge 2$) easy to get the following result

$$H_{ij}(s,\cdot) = H'_{ij}(s,\cdot) + H''_{ij}(s,\cdot) + \ldots + H_{ij}^{(r)}(s,\cdot).$$

In the case of replacement of a graph fragment with arc-loop (maximal number of cycles is *M*) on single equivalent arc (i,j) we received the values of the transition costs and the view of functions $\mu_{ij}(t)$, $H_{ij}(s,\cdot)$ in complete agreement with the deployment of the cyclic process into series-parallel.

Then

$$t'_{ij} = \frac{d_{ij}t_{ij} + d_{ii}d_{ij}(t_{ii}+t_{ij}) + \ldots + d_{ii}^{M-1}d_{ij}((M-1)t_{ii}+t_{ij})}{d_{ij} + d_{ii}d_{ij} + \ldots + d_{ii}^{M-1}d_{ij}}, \quad (1)$$

$$d'_{ij} = d_{ij} + d_{ii}d_{ij} + \ldots + d_{ii}^{M-1}d_{ij} = d_{ij}\frac{1-d_{ii}^M}{1-d_{ii}}. \quad (2)$$

Therefore

$$\mu'_{ij}(t)=\frac{1}{d_{ij}(\cdot)+d_{ii}(\cdot)d_{ij}(\cdot)+...+d_{ii}^{M-1}(\cdot)d_{ij}(\cdot)}\times$$

$$(d_{ij}(\cdot)\mu_{ij}(t)+d_{ij}(\cdot)d_{ii}(\cdot)\int_0^t\mu_{ii}(t-\tau)\mu_{ij}(\tau)d\tau+$$

$$+...+d_{ij}(\cdot)d_{ii}^{M-1}(\cdot)\int_0^t\mu_{ii}^{M-1}(t-\tau)\mu_{ij}(\tau)d\tau)=$$

$$=\frac{1-d_{ii}(\cdot)}{1-d_{ii}^{M}(\cdot)}(\mu_{ij}(t)+d_{ii}(\cdot)\int_0^t\mu_{ii}(t-\tau)\mu_{ij}(\tau)d\tau+$$

$$+...+d_{ii}^{M-1}(\cdot)\int_0^t\mu_{ii}^{M-1}(t-\tau)\mu_{ij}(\tau)d\tau). \qquad (3)$$

Taking into account (1) – (3) we obtain

$$H'_{ij}(s,\cdot)=d'_{ij}(\cdot)L[\mu'_{ij}(t)]=$$

$$=d_{ij}\frac{1-d_{ii}^{M}}{1-d_{ii}}L[\frac{1-d_{ii}(\cdot)}{1-d_{ii}^{M}(\cdot)}(\mu_{ij}(t)+$$

$$+d_{ii}(\cdot)\int_0^t\mu_{ii}(t-\tau)\mu_{ij}(\tau)d\tau+...+$$

$$+d_{ii}^{M-1}(\cdot)\int_0^t\mu_{ii}^{M-1}(t-\tau)\mu_{ij}(\tau)d\tau)]=$$

$$=d_{ij}L[\mu_{ij}(t)+d_{ii}(\cdot)\int_0^t\mu_{ii}(t-\tau)\mu_{ij}(\tau)d\tau+$$

$$+...+d_{ii}^{M-1}(\cdot)\int_0^t\mu_{ii}^{M-1}(t-\tau)\mu_{ij}(\tau)d\tau]=$$

$$=d_{ij}(\cdot)L[\mu_{ij}(t)](1+d_{ii}(\cdot)L[\mu_{ii}(t)]+$$

$$+...+d_{ii}(\cdot)^{M-1}[L[\mu_{ii}(t)]]^{M-1}).$$

Consequently

$$H'_{ij}(s,\cdot)=H_{ij}(s,\cdot)\frac{1-(H_{ii}(s,\cdot))^{M}}{1-H_{ii}(s,\cdot)}.$$

H-matrix method for models of technological processes considering index of the quality

Considered the analog of the method with the using of H-matrix [3] for modeling agro-technology as functional structure. The multidimensional graph corresponds to a functional network that was constructed. All arcs of the graph are weighted matrices, which contain fuzzy information about the quality and cost of execution for elements of the process. The weights arcs (i,j) for some moments $h_z,...,h_{z+y}$ are

$$t^{ij}(h)=\begin{bmatrix} t_{11}^{ij}\ ...\ t_{1n}^{ij} \\ ....\ ... \\ t_{n1}^{ij}\ ...\ t_{nn}^{ij}\end{bmatrix},$$

where t_{kl}^{ij} - are the cost transition from the node i to node j with the index quality at the input k and at the output l. All t_{kl}^{ij} are elements of some fuzzy subsets with the values of the membership function - d_{kl}^{ij}.

For H-matrix of the arc (i,j) as

$$H^{ij}(p,\cdot)=\begin{bmatrix} H_{11}^{ij}\ ...H_{1n}^{ij} \\ .... \\ H_{n1}^{ij}\ ...H_{nn}^{ij}\end{bmatrix},$$

$$H_{kl}^{ij}=d_{kl}^{ij}(\cdot)L[\mu_{kl}^{ij}(t)],$$

$$L[\mu_{kl}^{ij}(t)]=\int_0^{+\infty}e^{-pt}\mu_{kl}^{ij}(t)dt$$

then to replace some of the arcs on a smaller equivalent set we can to obtain following result.

Statement 2. 1. For replacement of two consecutive arcs (i,m), (m,j) on a single equivalent arc (i,j) we have

$$d_{kl}^{ij}=\sum_{p=1}^{n}d_{kp}^{im}d_{pl}^{mj}$$

and

$$H_{kl}^{ij}=\sum_{p=1}^{n}H_{kp}^{im}H_{pl}^{mj}.$$

2. For replacement two parallel arcs $(i,j)'$, $(i,j)''$ on a single equivalent arc (i,j) we have

$$d_{kl}^{ij}=d_{kl}^{ij'}+d_{kl}^{ij''}\ \text{(for } d_{kl}^{ij}\le 1\text{)}$$

and

$$H_{ij}(s,\cdot)=H'_{ij}(s,\cdot)+H''_{ij}(s,\cdot).$$

The proof. Transition through two consecutive arcs (i,m), (m,j) gives an opportunity to highlight the following constituents:

- consistent, where the cost of the transition from vertex i to vertex j are participating with a fixed index of quality k at entry and index of quality l at the output, that is,

$$t_{kl_1}^{ij}=t_{k1}^{im}+t_{1l}^{mj},\ ...,\ t_{kl_n}^{ij}=t_{kn}^{im}+t_{n}^{mj}.$$

For these we have

$$d_{kl_1}^{ij}=d_{k1}^{im}\cdot d_{1l}^{mj},\ ...,\ d_{kl_n}^{ij}=d_{kn}^{im}\cdot d_{nl}^{mj}.$$

- parallel, which is a sequence of elements after the replacement by an equivalent arc with different indices of quality in the transition, namely, elements with the costs of transition $t^{ij}_{kl_1}$, ..., $t^{ij}_{kl_n}$. Here, we have

$$d^{ij}_{kl} = d^{ij}_{kl_1} + \ldots + d^{ij}_{kl_n},$$

$$t^{ij}_{kl} = \frac{d^{ij}_{kl_1} t^{ij}_{kl_1} + \ldots + d^{ij}_{kl_n} t^{ij}_{kl_n}}{d^{ij}_{kl_1} + \ldots + d^{ij}_{kl_n}}.$$

In accordance with the Statement 1, we have

$$H^{ij}_{kl_1} = H^{im}_{k1} \cdot H^{mj}_{1l}, \ \ldots, \ H^{ij}_{kl_n} = H^{im}_{kn} \cdot H^{mj}_{nl}. \qquad (4)$$

Further, collecting parallel components

$$H^{ij}_{kl} = H^{ij}_{kl_1} + \ldots + H^{ij}_{kl_n},$$

and taking into account (4) we have

$$H^{ij}_{kl} = \sum_{p=1}^{n} H^{im}_{kp} H^{mj}_{pl}.$$

Therefore in a case of replacement of two consecutive arcs (i, m), (m, j) on equivalent arc (i, j) we received

$$d^{ij}_{kl} = \sum_{p=1}^{n} d^{im}_{kp} d^{mj}_{pl}$$

and

$$H^{ij}_{kl} = \sum_{p=1}^{n} H^{im}_{kp} H^{mj}_{pl}.$$

In a case of replacement of two parallel arcs $(i, j)'$, $(i, j)''$ on a single equivalent arc (i, j) in accordance with the Statement 1 we have $d^{ij}_{kl} = d^{ij'}_{kl} + d^{ij''}_{kl}$ (for $d^{ij}_{kl} \le 1$) and

$$H_{ij}(s,\cdot) = H'_{ij}(s,\cdot) + H''_{ij}(s,\cdot).$$

For the larger set of parallel arcs ($r \ge 2$) we got also the following result

$$H_{ij}(s,\cdot) = H'_{ij}(s,\cdot) + H''_{ij}(s,\cdot) + \ldots + H^{(r)}_{ij}(s,\cdot).$$

Conclusion

The analog of the method with the using of H-functions for modeling agricultural technology as many iterative functional structure with random cycles has been considered. The weight of arcs as the costs transition from the node i to node j are elements of some fuzzy subsets.

The analog of the method with the using of H-matrix for modeling agrotechnological processes considering index of the quality also has been considered. All arcs of the graph are weighted matrices, which contain fuzzy information about the quality and cost of execution for elements of the process.

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