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On stabilization of neutral type nonlinear indirect scalar processes

By employing the direct Lyapunov method, using Lyapunov-Krasovskii functional approach, stabilization problem of nonlinear scalar Lurie-type indirect control systems of neutral type is considered. Conditions are constructed in terms of matrix algebraic inequalities.

Key Words: Lyapunov's functional, absolute stability, neutral type time-delay, stabilization.

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Стабілізація скалярних процесів непрямого регулювання нейтрального типу

На основі прямого методу Ляпунова, з використанням функціоналу Ляпунова-Красовського, розглянуто задачу стабілізації до стану абсолютної стійкості нелінійних скалярних процесів непрямого регулювання нейтрального типу. Результати сформульовано у вигляді матричних алгебраїчних нерівностей..

Ключові слова: функціонал Ляпунова, абсолютна стійкість, відхилення аргументу нейтрального типу, стабілізація

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1 Introduction

One of the problems of stability motion is the problem of absolute stability. It arise in solving practical tasks. In some technical control systems, the control function is the nonlinear function of one variable located between two lines in the first and third quarters of the coordinate plane. Originally, the control systems of ordinary differential equation were considered [1-5].

Time-delay naturally appears in many real control processes, and it is frequently a source of instability. But the systems with deviating arguments better describe the real systems. In recent years, the absolute stability problem of nonlinear control systems with aftereffect is a topic of great practical importance which has attracted a lot of interests [3-7]. Some neutral type nonlinear systems with indirect regulation are considered in [6-10]. The sufficient conditions of absolute stability are derived in the papers [8-10] by direct Lypunov method with using Lyapunov-Krasovskii functionals in the type of the sum of the quadratic form from current coordinates plus the integral from delay coordinates and the integral of nonlinear components of the considered system. And the coefficients of the exponential decay of solutions are calculated. All results [7-10] has unified form of matrix algebraic inequalities.

But reasonable question has arise. What we will do in case if there is no any positive answer on absolute stability of investigated systems, using all previous results. There are two traditional next steps:

we need to change method of investigation (radical decision) – for example we will use Popov-Yakubovich frequency method [11,12];

we need to change Lyapunov function or functional.

But there is other new interesting way: we can try to introduce new linear control and to stabilize close-loop system at previous chosen Lyapunov function or functional. There are some interesting papers devoted to investigation of stability and stabilization tasks in this rezone [12-17].

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Present article is direct extension of [14-16]. Using the techniques of paper [17], the regulatory processes that can be described by scalar equation of neutral type was considered. It is believed that the solution is not absolutely stable, or could not find the correct parameters of Lyapunov-Krasovskii functional to establish this fact. We gives the solution to the stabilization problem to absolute stability state in the selected class of Lyapunov-Krasovskii functionals, by introducing feedback for phase coordinates, taken in the present and previous points in time.

2 Problems formulation and main results

Main goal of this article is consider a $n-$dimensional regulation process x described by the system of $n+1$ equations,

$$\frac{d}{dt}[x(t)-Dx(t-\tau)]=Ax(t)+Bx(t-\tau)+bf(\sigma(t))$$
$$\frac{d}{dt}\sigma(t)=c^T x(t)-\rho\sigma(t),$$

where $t\ge t_0\ge 0$, $x=(x_1,x_2,...,x_n)^T$ is the $n-$dimensional column vector of the state, σ is the scalar function of the control defined on $[t_0,\infty)$, A, B and D are $n\times n$-constant matrices, $|D|<1$, $b=(b_1,b_2,...,b_n)^T$ is an $n-$dimensional constant column vector, $c=(c_1,c_2,...,c_n)$ is an $n-$dimensional constant row vector, $\tau>0$ and $\rho>0$ are constants, $f(\sigma)$ is continuous function on $\Re$ satisfying so-called linear sector condition.

But for the results visibility, all next investigation will present for one-dimensional case. And its generalization is not so difficult to do by himself.

Therefore, let us consider an indirect control process described by a system of two scalar equations with delay argument of neutral type in the form

$$\frac{d}{dt}[x(t)-dx(t-\tau)]=a_1x(t)+a_2x(t-\tau)+bf(\sigma(t))$$
$$\frac{d}{dt}\sigma(t)=cx(t)-\rho f\sigma(t), \qquad (1)$$

where $t\ge t_0\ge 0$, x is the state function, σ is the control defined on $[t_0,\infty)$, a_1, a_2, b, c, d, $\rho>0$, $\tau>0$ are constants, $|d|<1$, $f(\sigma)$ is continuous nonlinear function on $\Re$ satisfying sector condition

$$k_1\sigma^2\le f(\sigma)\sigma\le k_2\sigma^2, \qquad (2)$$

where $k_2,k_1-const$: $k_2>k_1>0$.

Definition 1 The continuous vector function $(x,\sigma):[t_0-\tau,\infty)\to\Re^2$ is said to be a solution of (1) on $[t_0,\infty)$ if (x,σ) is continuously differentiable on $[t_0,\infty)$ and satisfies the system (1) on $[t_0,\infty)$.

Definition 2 The system (1) is called absolutely stable if the trivial solution $(x,\sigma)=(0,0)$ of the system (1) is globally asymptotically stable for an arbitrary function $f(\sigma)$ satisfying (2).

For this case Lyapunov-Krasovskii functional, which was used for investigation in papers [8-10], will present in next form

$$V[x(t),\sigma(t),t]=hx^2(t)+\beta\int_0^{\sigma(t)}f(\sigma)d\sigma+$$
$$+\int_{t-\tau}^{t}e^{-\varsigma(t-s)}\{g_1x^2(s)+g_2\dot{x}^2(s)\}ds \qquad (3)$$

where $h>0$, $g_1>0$, $g_2>0$, $\beta>0$, $\varsigma>0$ are constants, (x,σ) is a solution of (1), and $t\ge t_0\ge 0$. Define, using the coefficients of the functional (3), auxiliary numbers

$$s_{11}^1=-2a_1h-g_1-a_1^2g_2,\quad s_{12}^1=-ha_2-a_1a_2g_2,$$
$$s_{13}^1=-hd-a_1dg_2,\quad s_{14}^1=-hb-a_1g_2b-\frac{1}{2}\beta c,$$
$$s_{22}^1=e^{-\varsigma\tau}g_1-a_2^2g_2,\quad s_{23}^1=-a_1a_2g_2,$$
$$s_{24}^1=-a_2g_1b,$$
$$s_{33}^1=e^{-\varsigma\tau}g_2-d^2g_2,\quad s_{34}^1=-dg_2b,$$
$$s_{44}^1=-b^2g_2-\beta\rho$$

and the matrix

$$S_1[h,g_1,g_2,\beta,\varsigma]=$$

$$= \begin{bmatrix} s_{11}^1 & s_{12}^1 & s_{13}^1 & s_{14}^1 \\ s_{12}^1 & s_{22}^1 & s_{23}^1 & s_{24}^1 \\ s_{13}^1 & s_{23}^1 & s_{33}^1 & s_{34}^1 \\ s_{14}^1 & s_{24}^1 & s_{34}^1 & s_{44}^1 \end{bmatrix}.$$

Our first result is the theorem on absolute stability for the considered system (1).

Theorem 1 *Suppose that there exist constants*

$$h > 0,\ g_1 > 0,\ g_2 > 0,\ \beta > 0,\ \varsigma > 0,$$

such that the matrix $S_1[h, g_1, g_2, \beta, \varsigma]$ *is positive definite. Then the system (1) is absolutely stable.*

Proof of this theorem follows directly from the corresponding Theorem 1 [8-10].

As follows from the Sylvester criterion [15,16] the necessary and sufficient condition for positive definite of the matrix $S_1[h, g_1, g_2, \beta, \varsigma]$ is its main diagonal elements positivity, i.e.

$$\Delta_1^1 = s_{11}^1 > 0, \qquad (4)$$

$$\Delta_2^1 = s_{11}^1 s_{22}^1 - (s_{12}^1)^2 > 0, \qquad (5)$$

$$\Delta_3^1 = \begin{vmatrix} s_{11}^1 & s_{12}^1 & s_{13}^1 \\ s_{12}^1 & s_{22}^1 & s_{23}^1 \\ s_{13}^1 & s_{23}^1 & s_{33}^1 \end{vmatrix} > 0, \qquad (6)$$

$$\Delta_4^1 = \begin{vmatrix} s_{11}^1 & s_{12}^1 & s_{13}^1 & s_{14}^1 \\ s_{12}^1 & s_{22}^1 & s_{23}^1 & s_{24}^1 \\ s_{13}^1 & s_{23}^1 & s_{33}^1 & s_{34}^1 \\ s_{14}^1 & s_{24}^1 & s_{34}^1 & s_{44}^1 \end{vmatrix} > 0 \qquad (7)$$

After investigating inequalities (4) - (7) we make conclusion about absolute stability of the system (1).

Let us consider another approach which based on a study of block matrices. We will need some auxiliary results from the theory of matrices.

Lemma 1 [16] *Let* A *be a regular* $n \times n$ *matrix,* B *be an* $n \times q$ *matrix, and* C *be a* $q \times q$ *regular matrix.. Let a Hermitian matrix* S *be represented as*

$$S = \begin{bmatrix} A & B \\ B^* & C \end{bmatrix}$$

Then the matrix S *is positive definite if and only if the matrices* A *and* $C - B^* A^{-1} B$ *are positive definite.*

Lemma 2 [15, Frobenius formula] *Let* A *be a regular* $n \times n$ *matrix,* B *be an* $n \times q$ *matrix,* C *be a* $q \times n$ *matrix* D *be a* $q \times q$ *matrix, and the matrix*

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

be regular. Then the matrix is regular $R = D - CA^{-1}B$ *and*

$$M^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BR^{-1}CA^{-1} & -A^{-1}BR^{-1} \\ -R^{-1}CA^{-1} & R^{-1} \end{bmatrix}.$$

Now we can formulate next stability conditions.

Theorem 2 *Suppose that there exist constants*

$$h > 0,\ g_1 > 0,\ g_2 > 0,\ \beta > 0,\ \varsigma > 0,$$

such that the inequalities (4), (5) hold and next matrix

$$\tilde{S}_1[h, g_1, g_2, \beta, \varsigma, \nu] = \begin{bmatrix} s_{33}^1 & s_{34}^1 \\ s_{34}^1 & s_{44}^1 \end{bmatrix} - \frac{1}{s_{11}^1 s_{22}^1 - (s_{12}^1)^2} \begin{bmatrix} s_{13}^1 & s_{14}^1 \\ s_{14}^1 & s_{24}^1 \end{bmatrix} \times \begin{bmatrix} s_{22}^1 & -s_{12}^1 \\ -s_{12}^1 & s_{11}^1 \end{bmatrix} \times \begin{bmatrix} s_{13}^1 & s_{14}^1 \\ s_{14}^1 & s_{24}^1 \end{bmatrix} \qquad (8)$$

is positive definite Then the system (1) is absolutely stable.

Proof. Let be matrix $S_1[h, g_1, g_2, \beta, \varsigma]$ presented in next block form

$$S_1[h, g_1, g_2, \beta, \varsigma] = \begin{bmatrix} S_{11}^1 & S_{12}^1 \\ (S_{12}^1)^T & S_{22}^1 \end{bmatrix},$$

where

$$S_{11}^1 = \begin{bmatrix} s_{11}^1 & s_{12}^1 \\ s_{12}^1 & s_{22}^1 \end{bmatrix},$$

$$S_{12}^1 = \begin{bmatrix} s_{13}^1 & s_{14}^1 \\ s_{23}^1 & s_{24}^1 \end{bmatrix},$$

$$S_{22}^1 = \begin{bmatrix} s_{33}^1 & s_{34}^1 \\ s_{34}^1 & s_{44}^1 \end{bmatrix}.$$

Then according to Lemma 1, for positive definite of matrix $S_1[h, g_1, g_2, \beta, \varsigma]$ necessary and sufficient that matrices

$$S_{11}^1, \quad S_{22}^1 - (S_{12}^1)^T (S_{11}^1)^{-1} S_{12}^1$$

will be positive definite too. In other words, need to be true inequalities (4), (5) and matrix (8) be a positive definite.

The crucial assumption in Theorem 2 is the assumption of positive definiteness of the matrix $S_1[h, g_1, g_2, \beta, \varsigma]$. If we cannot find suitable constants $h > 0$, $g_1 > 0$, $g_2 > 0$, $\beta > 0$, $\varsigma > 0$ to ensure positive definiteness, or such constants do not exist, Theorem 2 is not applicable. In such case, we can modify the control function in system (1) by adding a linear combination of the state function at moment t and $t-\tau$ we will consider modified system

$$\frac{d}{dt}[x(t) - dx(t-\tau)] = a_1 x(t) + a_2 x(t-\tau) +$$
$$+ bf(\sigma(t) + u(t),$$
$$\frac{d}{dt}\sigma(t) = cx(t) - \rho f \sigma(t) + v(t), \qquad (9)$$

where

$$u(t) = c_1 x(t) + c_2 x(t-\tau),$$
$$v(t) = c_3 x(t),$$

c_1, c_2, c_3 are suitable constants.

Let us rewrite system (9) in next form

$$\frac{d}{dt}[x(t) - dx(t-\tau)] = (a_1 + c_1) x(t) +$$
$$+ (a_2 + c_2) x(t-\tau) + bf(\sigma(t)),$$
$$\frac{d}{dt}\sigma(t) = (c + c_3) x(t) - \rho f(\sigma(t)). \qquad (10)$$

In this case matrix of total derivative takes from functional along solution will have next form

$$S_2[h, g_1, g_2, c_1, c_2, c_3, \beta, \varsigma] =$$
$$= \begin{bmatrix} s_{11}^2 & s_{12}^2 & s_{13}^2 & s_{14}^2 \\ s_{12}^2 & s_{22}^2 & s_{23}^2 & s_{24}^2 \\ s_{13}^2 & s_{23}^2 & s_{33}^1 & s_{34}^1 \\ s_{14}^2 & s_{24}^2 & s_{34}^1 & s_{44}^1 \end{bmatrix},$$

where

$$s_{11}^2 = s_{11}^1 - 2hc_1 - (2ac_1 + c_1^2) g_2,$$
$$s_{12}^2 = s_{12}^1 - (hc_2 + a_1 c_2 + a_2 c_1 + c_1 c_2) g_2,$$
$$s_{13}^2 = s_{13}^1 - c_1 g_2 d, \quad s_{14}^2 = s_{14}^1 - c_1 g_2 b - \frac{1}{2}\beta c_3,$$
$$s_{22}^2 = s_{22}^1 - (2a_2 c_2 + c_2^2) g_2,$$
$$s_{23}^2 = s_{23}^1 - (c_2 a_1 + c_1 a_2 + c_2 c_1) g_2,$$
$$s_{24}^2 = s_{24}^1 - a_2 g_1 b.$$

Finding of the stabilization conditions (option 1).

As follows from the Sylvester criterion [14,15] the necessary and sufficient condition for positive definite of the matrix $S_2[h, g_1, g_2, c_1, c_2, c_3, \beta, \varsigma]$ is its main diagonal elements positivity, i.e.

$$\Delta_1^2 = s_{11}^1 - 2hc_1 - (2ac_1 + c_1^2) g_2 > 0, \qquad (11)$$

$$\Delta_2^2 = \begin{vmatrix} s_{11}^2 & s_{12}^2 \\ s_{12}^2 & s_{22}^2 \end{vmatrix} = [s_{11}^1 - 2hc_1 - (2ac_1 + c_1^2) g_2] \times$$
$$\times [s_{22}^1 - (2a_2 c_2 + c_2^2)] -$$
$$-[s_{12}^1 - (hc_2 + a_1 c_2 + a_2 c_1 + c_1 c_2)] > 0, \quad (12)$$

$$\Delta_3^2 = \begin{vmatrix} s_{11}^2 & s_{12}^2 & s_{13}^2 \\ s_{12}^2 & s_{22}^2 & s_{23}^2 \\ s_{13}^2 & s_{23}^2 & s_{33}^2 \end{vmatrix} > 0, \qquad (13)$$

$$\Delta_4^2 = \begin{vmatrix} s_{11}^2 & s_{12}^2 & s_{13}^2 & s_{14}^2 \\ s_{12}^2 & s_{22}^2 & s_{23}^2 & s_{24}^2 \\ s_{13}^2 & s_{23}^2 & s_{33}^1 & s_{34}^1 \\ s_{14}^2 & s_{24}^2 & s_{34}^1 & s_{44}^1 \end{vmatrix} > 0. \qquad (14)$$

And stabilization task consist from finding parameters c_1, c_2, c_3, such that the system (10) will be absolute stable. Set of such parameters is defined by inequalities (10) - (14).

Finding of the stabilization conditions (option 2).

Using the result of Theorem 2, we will give next formulation of positive definiteness. Before, let rewrite matrix $S_2[h, g_1, g_2, c_1, c_2, c_3, \beta, \varsigma]$ at next block form

$$S_2[h, g_1, g_2, c_1, c_2, c_3, \beta, \varsigma] = \begin{bmatrix} S_{11}^2 & S_{12}^2 \\ (S_{12}^2)^T & S_{22}^2 \end{bmatrix},$$

where

$$S_{11}^2 = \begin{bmatrix} s_{11}^2 & s_{12}^2 \\ s_{12}^2 & s_{22}^2 \end{bmatrix},$$

$$S_{12}^2 = \begin{bmatrix} s_{13}^2 & s_{14}^2 \\ s_{23}^2 & s_{24}^2 \end{bmatrix},$$

$$S_{22}^2 = \begin{bmatrix} s_{33}^1 & s_{34}^1 \\ s_{34}^1 & s_{44}^1 \end{bmatrix}.$$

As follows from condition of positive definiteness of block matrices (Lemma 1), for positive definiteness of matrix $S_2[h, g_1, g_2, c_1, c_2, c_3, \beta, \varsigma]$ necessary and sufficient that the matrices

$$S_{11}^2, \quad S_{22}^2 - (S_{12}^2)^T (S_{11}^2)^{-1} S_{12}^2$$

will be positive definite too.

3 Conclusions

In this brief, we deal with the stabilization problem for a class of nonlinear control systems with state deviating argument of neutral type. Based on direct Lyapunov method (Lyapunov-Krasovskii approach) several stabilization criteria have been given in terms of a set of matrix algebraic inequalities. A new sufficient condition, which guarantees the close-loop system is absolutely stable, is presented. In recent years, the absolute stability problem of nonlinear control systems with aftereffect and, in particular, the stabization problem of the same system are a topic of great practical importance which has attracted a lot of interests. Author presented some of the same results at scientific conferences [20-22], and it aroused a great interest.The results can be extended to the case of multidimensional neutral type nonlinear control process.

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