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Загальний метод для виведення еволюційних рівнянь та моделювання нелінійних хвиль у шаруватих активних середовищах з об'ємними та поверхневими нелінійностями

Розвинутий загальний метод для виведення нелінійних хвильових еволюційних рівнянь для шаруватих структур зі слабкою нелінійністю. Цей метод є дуже ефективним та придатним для електромагнітних хвиль в середовищах з нелінійностями різної фізичної природи, поверхневими та об'ємними, та просторовою дисперсією: в біанізотропних (кіральних) метаматеріалах, гіротропних середовищах, плазмі, включаючи космічну, двовимірному електронному газі (наприклад графені) та ін. нелінійних та активних середовищах в різних частотних діапазонах, від мікрохвильового до оптичного.

Ключові слова: *нелінійні хвилі, метаматеріали, шаруваті середовища, поверхні розділу*

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General method for the derivation of the evolution equations and modeling nonlinear waves in active layered structures with surface and volume nonlinearities

The general method for the derivation of the nonlinear evolution equations for the layered structures with weak nonlinearitie is developed. This method is very effective and applicable for electromagnetic waves in media with nonlinearitiers of different physical nature, both volume and surface and spatial dispersion: bi-anisotropic (chiral) metamaterials, gyrotropic media, plasma including space plasma, 2D electron gas (graphene etc.) and other nonlinear and active media at different frequency ranges, from GHz to optics.

Key Words: *nonlinear waves, metamaterials, layered media, interfaces*

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Introduction

The general method for the derivation of the nonlinear evolution equations in layered structures (NEELS) with weak nonlinearities is very effective and applicable for waves in nonlineaer media of different physical nature. Surface nonlinearities and nonlinear boundary conditions are accounted for. Nonlinearity in the auxiliary boundary conditions (ABC), connected with the spatial dispersion, is implemented. In the present paper, the development and new results of the application of the method NEELS for different media are presented and compared to the results obtained earlier [1-4].

Nonlinear evolution equations for the waves in the layered bi-anisotropic structures with the volume and surface nonlinearities. Formulation of the problem

Let us describe the details of the method for quite general problem of nonlinear waves in the layered bi-anisotropic structures [6, 5]. Consider the system shown in Fig. 1. Suppose that a layered structure has waveguiding properties, and linear eigenmodes corresponding to wavenumber $\vec{k}_0$ and frequency ω_0 are determined. These modes are characterized by the functions $(\vec{f}_E, \vec{f}_H)$ describing the distributions of the electric and magnetic fields in the direction (Z), normal to the interface between layers; linear dispersion relation $D(\omega_0, k_{0y}, k_{0x}) = 0$ is known, where Y and X are the directions of the wave propagation in the layered structure and transverse direction, respectively (see Fig.1). Suppose that for the metamaterial layer, linear "homogenized" material equations are written in the form

$$\vec{D} = \hat{\varepsilon}_{eff}\vec{E} + \hat{\alpha}_{eff}\vec{H}\ ,\ \vec{B} = \hat{\beta}_{eff}\vec{E} + \hat{\mu}_{eff}\vec{H} \qquad (1)$$

where $\vec{E}, \vec{H}$ and $\vec{D}, \vec{B}$ are electric and magnetic fields and inductions, respectively,

$\hat{\varepsilon}_{eff}, \hat{\mu}_{eff}, \hat{\alpha}_{eff}$ and $\hat{\beta}_{eff}$ are tensors of electric permittivity, magnetic permeability, magnetic to electric and electric to magnetic coupling, respectively. Denote linear fields (solution of linear problem for some eigenmode of the structure shown in Fig. 1) by index "l"; in the Fourier presentation,

$$\begin{pmatrix} \vec{E}_l \\ \vec{H}_l \end{pmatrix} = A_0 \begin{pmatrix} \vec{f}_e \\ \vec{f}_h \end{pmatrix} e^{i(\omega_0 t - \vec{k}_0 \vec{r})} \qquad (2)$$

where $\vec{r}$ is the coordinates in the XY plane, A_0 is "the amplitude of input wave" which is used for the proper normalization of the nonlinear coefficient.

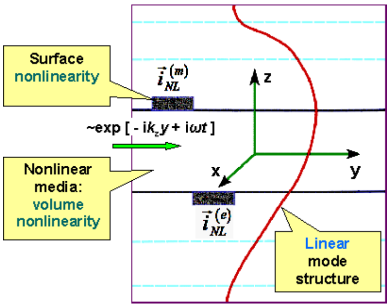

Fig. 1. Layered nonlinear metamaterial waveguiding structure. The same coordinate system is used also for nonlinear dielectric with the spatial dispersion. For the nonlinear gyrotropic (ferrite) layer rotated system of coordinate is used where the following replacement is done: $Y \to Z$, $Z \to X$, $X \to Y$ with respect to the system shown in Fig. 1, and bias magnetic field is directed along Z. The thickness of the layer is equal to L.

Functions $\vec{f}_{e,h}$ describe the dependences of the linear fields on Z coordinate; they are obtained using (linear) boundary conditions and taking into account linear dispersion. Therefore $\vec{f}_{e.h} = \vec{f}_{e.h}(\omega_0, \vec{k}_0, z)$ and $|\vec{f}_{e,h}| \to 0$, when $z \to \pm\infty$ (see Fig. 1). Suppose that the field of spectrally narrow wave packet is presented in the form

$$\begin{pmatrix} \vec{E}_1 \\ \vec{H}_1 \end{pmatrix} = A_1(t, \vec{r}) \begin{pmatrix} \vec{E}_{1l} \\ \vec{H}_{1l} \end{pmatrix} \qquad (3)$$

where A_1 is slowly varying amplitude. Therefore we neglect in the first approximation with the change of the field of the main harmonic in the transverse direction, what is valid in the case of small nonlinearity. Spectral narrowness of the wave packet is determined by the conditions $\Delta\omega << \omega_0$, $\Delta k << k_0$, where $\Delta\omega$, Δk are the widths of frequency and wavenumber spectrums, respectively. Suppose that dyadics included into (1) could be presented in the form $\hat{\varepsilon}_{eff} = \hat{\varepsilon} + \hat{\varepsilon}_{act}$, $\hat{\mu}_{eff} = \hat{\mu} + \hat{\mu}_{act}$, , $|\hat{\mu}| >> |\hat{\mu}_{act}|$, $\hat{\alpha}_{eff} = \hat{\alpha} + \hat{\alpha}_{act}$, $\hat{\beta}_{eff} = \hat{\beta} + \hat{\beta}_{act}$, where $|\hat{\varepsilon}| >> |\hat{\varepsilon}_{act}|, |\hat{\mu}| >> |\hat{\mu}_{act}|, |\hat{\alpha}| >> |\alpha_{act}|$, $|\hat{\beta}| >> |\hat{\beta}_{act}|$. Index "act" corresponds to the (small) part of the tensors responsible for dissipative losses or gain of electromagnetic waves. The parts $\hat{\varepsilon}, \hat{\mu}, \hat{\alpha}, \hat{\beta}$ of the corresponding tensors provide the energy conservation, in other words [6, 5], $\hat{\varepsilon}^+ = \hat{\varepsilon}, \hat{\mu}^+ = \hat{\mu}, \hat{\beta} = \hat{\alpha}^+$, where upper index"+" denotes Hermitian conjugation. Suppose we know also volume nonlinear electric and magnetic polarizations, $\vec{P}_{NL}$ and $\vec{M}_{NL}$ (14e).

Nonlinear evolution equations for an envelope amplitude accounting for spatial dispersion.

Using the procedure similar to that described in [2-4], we get finally the following evolution equation in the *integral* form:

$$c^{-1} w |A_{01}|^2 \frac{\partial A_1}{\partial t} + |A_{01}|^2 \, div(\vec{P} A_1) + [\text{Higher-Order Terms}] = -ik_0 q |A_0|^2 A_1 - \frac{4\pi}{c}(\vec{H}_{1l}^* \frac{\partial \vec{M}_{NL}}{\partial t} + \vec{E}_{1l}^* \frac{\partial \vec{P}_{NL}}{\partial t}) \qquad (4)$$

Here:

$$w = [\vec{f}_H^* \frac{\partial}{\partial\omega}(\omega\hat{\mu})\vec{f}_H + \vec{f}_E^* \frac{\partial}{\partial\omega}(\omega\hat{\varepsilon})\vec{f}_E + \vec{f}_H^* \frac{\partial}{\partial\omega}(\omega\hat{\alpha}^+)\vec{f}_E + \vec{f}_E^* \frac{\partial}{\partial\omega}(\omega\hat{\alpha})\vec{f}_H]$$

$$\vec{P} = \{[\vec{f}_E^* \times \vec{f}_H] + [\vec{f}_E \times \vec{f}_H^*]\} - k_0\{[(\vec{f}_H^* \frac{\partial\hat{\mu}}{\partial\vec{k}} \vec{f}_H + \vec{f}_E^* \frac{\partial\hat{\varepsilon}}{\partial\vec{k}} \vec{f}_E) + (\vec{f}_H^* \frac{\partial\hat{\alpha}}{\partial\vec{k}} \vec{f}_E + \vec{f}_E^* \frac{\partial\hat{\alpha}}{\partial\vec{k}} \vec{f}_H)]\} \qquad (5)$$

$$q = (\vec{f}_H^* \hat{\mu}_{act} \vec{f}_H - \vec{f}_H \hat{\mu}_{act}^* \vec{f}_H^*) + (\vec{f}_E^* \hat{\varepsilon}_{act} \vec{f}_E - \vec{f}_E \hat{\varepsilon}_{act}^* \vec{f}_E^*) + (\vec{f}_E^* \hat{\alpha}_{act} \vec{f}_H - \vec{f}_H \hat{\beta}_{act}^* \vec{f}_E^*) + (\vec{f}_H^* \hat{\beta}_{act} \vec{f}_E - \vec{f}_E \hat{\alpha}_{act}^* \vec{f}_H^*)$$

$$\frac{\partial A_1}{\partial t}|A_{01}|^2+[|V_{gy}|\frac{\partial}{\partial y}+\frac{i}{2}D_y\frac{\partial^2}{\partial y^2}+\frac{i}{2}D_x\frac{\partial^2}{\partial x^2}]|A_{01}|^2A_1+i\omega\frac{Q}{W_0}|A_{01}|^2A_1=$$
$$-\frac{4\pi}{W_0}\int_{-\infty}^{\infty}(\vec{H}^*_{1l}\frac{\partial\vec{P}_{NL}}{\partial t}+\vec{E}^*_{1l}\frac{\partial\vec{M}_{NL}}{\partial t})dz-\frac{4\pi}{W_0}\sum_{n=1,2}[E^*_{1lx}i_x^{(e)}+E^*_{1ly}i_y^{(e)}+H^*_{1ly}i_y^{(m)}+H^*_{1lx}i_x^{(m)}]_{z=z_{0n}} \tag{6}$$

$$i_{x,y}^{(e)}|_{z=\pm L/2}=\int_{\pm L/2-\delta}^{\pm L/2}\frac{\partial P_{SNLx,y}}{\partial t}dz,\ \vec{n}\times\Delta\vec{E}_1=-(4\pi/c)\vec{i}_{tg}^{(m)}$$

$$i_{x,y}^{(m)}|_{z=\pm L/2}=\int_{\pm L/2-\delta}^{\pm L/2}\frac{\partial M_{SNLx,y}}{\partial t}dz,\ \vec{n}\times\Delta\vec{H}_1=(4\pi/c)\vec{i}_{tg}^{(e)}$$

$\vec{n},\Delta\vec{E}_1,\Delta\vec{H}_1$ are the normal to the corresponding interface and the “boundary jump” of the corresponding electric and magnetic fields, respectively, δ - thickness of the near-interface layer which includes “surface nonlinearity”, $P_{SNLx,y}$ and $M_{SNLx,y}$ are corresponding (or equivalent) components of surface nonlinear electric and magnetic polarizations, $i_{x,y}^{(e.m)}|_{z=\pm L/2}$ are equivalent to $P_{SNLx,y}$ and $M_{SNLx,y}$, respectively, nonlinear electric and magnetic surface currents; $W_0=\int_{-\infty}^{\infty}wdz$, $Q=\int_{-\infty}^{\infty}qdz$, and we used the relation $\Pi_y=\int_{-\infty}^{\infty}P_ydz=V_{gy}W_0$; V_{gy} is the component of group velocity along the direction of propagation, and the energy is supposed to propagates in a positive Y direction; $D_{x,y}$ are the coefficients of linear dispersion and diffraction, respectively [7, 2].

$$\begin{pmatrix}\vec{E}_1\\ \vec{H}_1\end{pmatrix}=A_1(t,\vec{r})\begin{pmatrix}\vec{E}_{1l}\\ \vec{H}_{1l}\end{pmatrix} \tag{7}$$

In all nonlinear terms in Eq. (6), the values proportional to the $\exp[i(\omega_0t-\vec{k}_0\vec{r})]$ are revealed, because, among possible nonlinear effects, an effect of self-interaction is of interest here.

Magnetized gyrotropic and dielectric structures with the spatial dispersion and nonlinear auxiliary boundary conditions

We present two examples of the media with nonlinearities in ABC, connected with the presence of the spatial dispersion [9-11, 7]. WE outline here semi-phenomenological approach for the evaluation of a contribution of corresponding surface nonlinearity into the total nonlinearity of the layered structure. The general form of the evolution equation in the parabolic approximation has the form [2], where, in distinction to [7], the term corresponding to the volume and with surface nonlinearities $F_{NLV,NVS}$ are revealed.

$$\partial U_1/\partial t+V_{g1}\partial U_1/\partial z+(i/2)(\partial^2\omega/\partial k^2{}_z)(\partial^2U_1/\partial z^2)+(i/2)(\partial^2\omega/\partial k^2{}_y)(\partial^2U_1/\partial y^2)=F_{NLV}+F_{NLS} \tag{8}$$

Now the waves propagate along axis Z, which lies in the plane of a layer, while normal to the layer is directed along axis X (see also caption to the Fig. 1). Bias magnetic field is directed along axis Z therefore backward volume magnetostatic waves (BVMSW) propagate parallel to the bias field in ferrite films [8, 2]. U_1 is the amplitude of magnetostatic potential [2]. The following relation is obtained in [9]:

$$\vec{\Pi}_-\vec{n}_--\vec{\Pi}_+\vec{n}_+=[n_k(\frac{\partial F}{\partial\frac{\partial\vec{M}}{\partial x_k}})(\frac{\partial\vec{M}}{\partial t})^*_{LIN}]_{exch} \tag{9}$$

where $\vec{\Pi}_\pm$ are the energy flows from the both sides of the interface, index “LIN” means a value corresponding to the system shown in Fig. 1 in the absence of nonlinearity. In the presence of nonlinearity we can use phenomenologically Eq. (9) where we need to put

$$(\frac{\partial F}{\partial\frac{\partial\vec{M}}{\partial x_k}})=(\frac{\partial F}{\partial\frac{\partial\vec{M}}{\partial x_k}})_{LIN}+(\frac{\partial F}{\partial\frac{\partial\vec{M}}{\partial x_k}})_{NL} \tag{10}$$

Function F in Eqs. (9), (10) is the potential energy of a ferromagnetic [9], and in fact, its part describing exchange interaction is included into RHS of (10), which includes, generally speaking, linear and nonlinear part (with the indexes “LIN” and “NL”, respectively). In the nonlinear part of (10), again, an effect of self-interaction and respectively, the field of the main harmonic is revealed in the nonlinear terms, as described in the previous section. Index “exch” in the relation (9) is used to emphasize that Eq. (8) includes the exchange terms. Here F has the form [9, formula (3.3.2)]. Suppose that constant of surface anisotropy is β_S . Let us write the nonlinear exchange ABC in the form [8]

$$(\pm\alpha\ \partial m_{x1} / \partial x - \beta_0 m_{x1} + M_0^{-1} R_{1NL\pm})_{x=\pm L/2} = 0 \quad (11)$$

where $M_0^{-1} R_{1NL\pm}$ describes the contribution of the nonlinearity into boundary condition (11), and in the presence of only one (signal) wave, one can get [8]:

$$R_{1NL\pm} = -\beta_0 (m_{x1} m_{z1})^{(1)} \pm \alpha(-m_{x1} \frac{\partial m_{z1}}{\partial x} + m_{z1} \frac{\partial m_{x1}}{\partial x})^{(1)} \quad (12)$$

Accounting for (9), it is possible to note that the last (nonlinear) term in (11) corresponds to the second (nonlinear) term in the RHS from (10). As a result one becomes able to evaluate the nonlinear term in the RHS of (9). This phenomenological approach yields, with the accuracy up to a certain value of order unity, the nonlinear term F_{NLS} from the RHS in Eq. (8). The exact formula for this term [2] is written as

$$F_{NLS} = -(1/W_V)(4\pi\gamma / \omega_M) i\omega\ [(m_{1lx}{}^* R_{1NL\pm})_{x=+L/2} + (m_{1lx}{}^* R_{1NL\pm})_{x=-L/2}] \quad (13)$$

Let us now evaluate, using the outlined phenomenological approach, the surface nonlinear term for the waves in a layered nonlinear spatially dispersive dielectric. Polaritons in spatially dispersive dielectrics, such as exciton dielectrics [10] or ferroelectrics [11] are described by the 2nd order differential equation in time, such as

$$m\frac{\partial^2 P_i}{\partial t^2} = -m\omega_t^2 P_i + C[(\frac{\partial P_i}{\partial y})^2 + (\frac{\partial P_i}{\partial z})^2] + E_i - \gamma\frac{\partial P_i}{\partial t} \equiv -\frac{\delta F}{\delta P_i} - -\gamma\frac{\partial P_i}{\partial t} \quad (14)$$

Here $\vec{P}, m, \omega_t, C, \vec{E}, \gamma, F$ are polarization, effective mass, effective esxciton resonance frequency, proper constant, external electric field, coefficient of dissipative losses and effective potential energy of the unit mass, respectively, and a media is supposed to be lossless. Effective potential energy in (14) has the form

$$F = \frac{1}{2}\sum_i [m\omega_t^2 P_i^2 + C(\nabla P_i)^2] - \vec{E}\vec{P} \quad (15)$$

Note that in the RHS of the identity in (14), variational derivative of F is included: It is possible to derive the following conservation law corresponding to the Eqs. (14):

$$\frac{\partial}{\partial t}(w_0 + F) + \frac{\partial}{\partial x_k}\Pi_{SDk} = -\gamma(\frac{\partial \vec{P}}{\partial t})^2 \quad (16)$$

where effective kinetic energy w_0 and energy flux connected to spatial dispersion $\vec{\Pi}_{SD}$ are

$$w_0 = \frac{1}{2} m(\frac{\partial \vec{P}}{\partial t})^2,\ \Pi_{SDk} = -\frac{\partial P_i}{\partial t}\frac{\partial F}{\partial \frac{\partial P_i}{\partial x_k}} \quad (17)$$

Finally, using the relations (9), (10), where the electric polarization replaces magnetic one, yields

$$[\vec{\Pi}_{-}\vec{n}_{-} - \vec{\Pi}_{+}\vec{n}_{+}]_{NL} = [n_k(\frac{\partial F}{\partial \frac{\partial \vec{P}}{\partial x_k}})_{NL}(\frac{\partial \vec{P}}{\partial t})^*_{LIN}] = [NL_S(\frac{\partial \vec{P}}{\partial t})^*_{LIN}]_{z=L/2} \quad (18)$$

Corresponding to (18) contribution of the surface spatial dispersion dielectric (exciton/ferroelectric) nonlinear effect into the total nonlinearity in the relation describing NEELM is described by the term $W_0^{-1}[NL_S(\frac{\partial \vec{P}}{\partial t})^*_{LIN}]_{z=L/2}$, which should be added into RHS of the Eq. (6), altogether with corresponding term connected to interface $z = -L/2$.

The contribution of surface and volume nonlinearities in the formation of surface plasmon nonlinear structures and giant generation of resonant second harmonic

Let us introduce the variation of the plasma concentration, velocity, electric and magnetic fields, respectively, $\tilde{n}_1, \vec{v}_1, \vec{E}_1, \vec{H}_1$ and $\tilde{n}_2^*, \vec{v}_2^*, \vec{E}_2^*, \vec{H}_2^*$. Here values with indices "1" correspond to surface nonlinear plasmon and are proportional to the slowly varying nonlinear amplitude, and values with index "2" describes corresponding linear wave in the same system, but without any nonlinearity. Upper index "*" means complex conjugation. Corresponding equations of motion in nonlinear plasma have the form

$$\begin{aligned} &\frac{\partial \tilde{n}_1}{\partial t} + div\left(n\vec{v}_1\right) = -div\left(\tilde{n}\tilde{\vec{v}}\right) \\ &\frac{\partial \vec{v}_1}{\partial t} + \frac{e}{m}\vec{E}_1 = -\left(\vec{v}_1\vec{\nabla}\right)\vec{v}_1 - \frac{e}{m}\left[\vec{v}_{1^x}\vec{H}_1\right] \\ &curl\ \vec{H}_1 = \frac{1}{c}\frac{\partial E_1}{\partial t} + \frac{4\pi}{c}\left(-en_0\vec{v}_1 - e\tilde{n}\vec{v}\right) \\ &curl\ \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{H}_1}{\partial t} \end{aligned} \tag{19}$$

The similar to (19) equations (but without nonlinear terms) should be written for the linear components with indices "2". Then the procedure [2-4] similar to the derivation of the energy conservation law [10, 9, 7], is applied to the nonlinear surface plasmons. To account for the surface nonlinearity, the nonlinear motion of surface charges (with surface concentration n_S) at $z=0$ should be treated. The equation of continuity is modified due to the presence of surface charges as follows:

$$\frac{\partial v_1}{\partial t} + nv_{1z}\Big|_{z=-l+0} + \frac{\partial}{\partial x}\left(n_S v_{1x}\right)\Big|_{z=-l+0} = 0 \tag{20}$$

We account for the boundary conditions at the interface ($z=0$) "plasma-dielectric", the presence of free carriers, and, respectively, surface charge and surface current j_{surf}, namely

$$\begin{aligned} &H_{1y}\Big|_{z=+0} - H_{1y}\Big|_{z=-0} = -\frac{4\pi}{c} j_{xsurf}, \\ &j_{xsurf} = -en_S\vec{v}_x\Big|_{z=+0} \end{aligned} \tag{21}$$

Using (21), one can get

$$\begin{aligned} &\left[\vec{E}_1^* \times \vec{H}_2\right]_{z;z=+0} - \left[\vec{E}_2^* \times \vec{H}_1\right]_{z;z=-0} = \\ &\frac{4\pi}{c} E_{2x}^* en_S v_x\Big|_{z=+0} \end{aligned} \tag{22}$$

In equation (22), denotations like $[...]_{z;z=\pm 0}$ mean Z-component of the vector […], taken at $z=\pm 0$. Finally, the integral NEELM for surface plasmons is obtained in the form:

$$\begin{aligned} &\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\left\{\frac{1}{4\pi}\left(\vec{E}_1\vec{E}_2^* + \vec{H}_1\vec{H}_2^*\right) + mn_0\vec{v}_1\vec{v}_2^*\right\}dz + \frac{c}{4\pi}\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\left\{\left[\vec{E}_1 \times \vec{H}_2^*\right]_x + \left[\vec{E}_2^* \times \vec{H}_1\right]_x\right\}dz = \\ &= eE_{2x}^* n_S v_x\Big|_{z=+0} - m\int_0^{\infty} n_0\vec{v}_2^*\left(\vec{v}\vec{\nabla}\right)\vec{v}dz - \frac{e}{c}\int_0^{\infty} n_0\vec{v}_2^*\left[\vec{v} \times \vec{H}\right]dz + e\int_0^{\infty} E_2^*\left(\tilde{n}\vec{v}\right)dz \end{aligned} \tag{23}$$

The first term in RHS in Eq. (23) appears due to surface nonlinearity, caused by nonlinear motion of surface free charge. The next three terms are the parts of volume nonlinearity and correspond to substantional nonlinearity, nonlinear Lorenz force and concentration nonlinearity, respectively. Detail calculations showing some very interesting nonlinear effects have been presented in [4]. It was shown in particular that the contribution of the surface nonlinearity into the giant generation of the resonant second harmonic appeared to be three times larger than the volume one.

Application of the method for the electromagnetic waves in the ionosphere and seismoionospheric phenomena

Consider, for the sake of simplicity, TM mode with the components H_y, E_z, E_x, where the electromagnetic wave propagates along the X axis, Z is directed vertically upward (as before) and we consider 2D problem, $\partial/\partial y = 0$. Using the method of NEELS, we come to the relation [12]:

$$\frac{\partial}{\partial t}\Phi = \int_{X_1}^{X_2}\Delta k_x(x)dx = -\frac{\omega}{16\pi V_g}\int_{X_1}^{X_2}\left[\frac{\int_{Z_1}^{Z_2}(\vec{F}_E^{\,*}\Delta\hat{\varepsilon}\vec{F}_E)dz}{\int_0^{Z_{\max}}(\vec{F}_E^{\,*}\frac{\partial}{\partial\omega}(\hat{\varepsilon}\omega)\vec{F}_E + \vec{F}_H^{\,*}\vec{F}_H^*)dz}\right]dx \tag{24}$$

In (24), $\omega = 2\pi f; f, V_g$ are frequency and group velocity of the electromagnetic wave, respectively; are the perturbations (due to the seismogenic processes) of the tensor of dielectric permittivity, X component of the wavenumber of the electromagnetic wave and integral complex change of the electromagnetic wave phase, respectively; X_1, X_2, Z_1, Z_2 are the coordinates of the region where the perturbation of the media parameters takes place, $\Delta X = X_2 - X_1$, $\Delta Z = Z_2 - Z_1$ are the characteristic dimensions of the region of perturbation in the X, Z directions, respectively; $Z_{\max}$ is the characteristic width of the equivalent waveguide for the electromagnetic waves; $\vec{F}_{E,H}$ are the functions which describe the polarization of the electric and magnetic field components of the electromagnetic wave, respectively. The relation (3) can be used for the estimation of the change of phase and amplitude of electromagnetic waves in the "Earth-Ionosphere" waveguide before strong earthquakes (see details concerning this physical phenomena, f.e. in [13]).

Application of the method NEELS for the modeling spatio-temporal solitons and nonlinear wave structures in the controllable and active nonlinear gyrotropic and metamaterial waveguides: from GHz to optics

The following dimensionless higher-order evolution equations for the electromagnetic controllable spatio-temporal wave structures can be obtained, using the method NEELS:

$$\begin{aligned} & i\frac{\partial \psi}{\partial Z} - \frac{1}{2} sgn(\beta_2)\frac{\partial^2 \psi}{\partial T^2} - i\delta_3 \frac{\partial^3 \psi}{\partial T^3} + \frac{1}{2} D\frac{\partial^2 \psi}{\partial y^2} + \mathrm{K}\frac{\partial^2}{\partial y^2}(|\psi|^2 \psi) \\ & + sgn(\bar{\chi}^{(3)})\left(|\psi|^2 \psi + iS\frac{\partial}{\partial T}\left(|\psi|^2 \psi\right) - \tau_R \psi \frac{\partial}{\partial T}\left(|\psi|^2\right)\right) + i\gamma_0 \psi - \nu\psi = 0 \end{aligned} \tag{25}$$

In (25), ψ is normalized amplitude of the corresponding component of electromagnetic waveguiding mode (in particular an amplitude of E_y component of the TM mode, $\beta_2, \delta_3, \bar{\chi}^{(3)}, S, \tau_R$ are the normalized coefficients of dispersion, third order linear dispersion, cubic nonlinearity, self-steepening (nonlinear dispersion) and the time characterizing nonlinear Raman effect, respectively; $D, \gamma_0, \nu, \mathrm{K}$ are the coefficients of linear diffraction, linear losses/gain(for an active media) , transverse magnetooptic effect and nonlinear diffraction, respectively. Using Eq. (25), the magnetooptic control of solitons in metamaterials was demonstrated in [14]. The set of important new effects for optical active nonlinear metamaterial [15] waveguiding structures including magnetooptic and diffraction-management stabilization of spatio-temporal (2+1) solitons/bullets [16] will be published elsewhere.

As it was shown in [1] (see also [2] concerning the details of the applications of the method NEELS to the magnetostatic waves in GLs/ferrite films), application of the method NEELS for the coupling of BVMSW in longitudinally-magnetized gyromagnetic layered structure/ferrite film leads to the system of the normalized equations for the parametric coupling of counterpropagating signal and idle pulses having equal carrier frequencies and equal by absolute values carrier wavenumbers:

$$\begin{aligned} & \frac{\partial U_1}{\partial t} + V_g \frac{\partial U_1}{\partial z} + i\left(\frac{D}{2}\right)\frac{\partial^2 U_1}{\partial z^2} + iS\frac{\partial^2 U_1}{\partial y^2} + iN(|U_1|^2 + 2|U_2|^2)U_1 + \omega_r U_1 + Vh_{pmp}(z,t)U_2^* = 0 \ , \\ & \frac{\partial U_2}{\partial t} - V_g \frac{\partial U_2}{\partial z} + i\left(\frac{D}{2}\right)\frac{\partial^2 U_1}{\partial z^2} + iS\frac{\partial^2 U_1}{\partial y^2} + iN(|U_2|^2 + 2|U_1|^2)U_2 + \omega_r U_2 + Vh_{pmp}(z,t)U_1^* = 0 \ . \end{aligned} \tag{26}$$

Here $U_{1,2}, D, S, N, Vh_{pmp}, \omega_r$ are normalized amplitudes of the signal and idle pulses, coefficient of diffraction and dispersion and cubic nonlinearity, and the amplitude of pumping wave having double frequency (respectively to the carrier pulse frequency). Application of the method NEELS yields all the coefficients in (26), in particular $V \approx i\gamma\sigma_0 / (\omega_H / \omega_M)[\sigma_0^{\ 2} + (\mu - 1)^2]$, where $\omega_{H,M}$ are characteristic frequencies and μ, σ_0 diagonal and non-diagonal elements of the tensor $\hat{\mu}$ [8, 9, 2, 1] of a magnetized (saturated) ferrite, respectively (see also [2] concerning the values of other coefficients in (26)). New types of structures, in particular "knife-shaped bullets" in narrow ferrite films are obtained.

The set of other new interesting effects of the structure formation in this active nonlinear system with parametric amplification will be reported in a separate paper. The system of equation similar to (4), (26) have been derived using the method NEELS but not presented explicitly in the conference paper [5] for the parametric pumping of counterpropagating electromagnetic signal and idle pulses in bianisotropic waveguide with active nonlinear bianisotropic metamolecules (in the form of planar Ω-particles). The method of corresponding nonlinear homogenization of the bi-anisotropic metamaterial had been proposed in [5]. Now we present such a system, first:

$$\frac{\partial U_{1,2}}{\partial t} \pm V_{g1,2}\frac{\partial U_{1,2}}{\partial t} + i\frac{V_{g1,2}}{2k_{1,2}}\frac{\partial^2 U_{1,2}}{\partial z^2} + \frac{i}{2}\frac{\partial^2 \omega}{\partial k_y^2}\frac{\partial^2 U_{1,2}}{\partial y^2} + \omega_{r1,2}U_{1,2} -$$
$$\frac{i\omega}{W_{01,2}}\int_{-\infty}^{\infty}(\vec{f}^{\,*}_{E1,2}\vec{P}_{A1,2} + \vec{f}^{\,*}_{H1,2}\vec{M}_{A1,2})dx = i\omega Q_0 U_3 U^*_{2,1};\; W_0 = \frac{c}{4\pi V_{g1}}\int_{-\infty}^{\infty}\mathrm{Re}(\vec{f}_{E1}\times\vec{f}^{\,*}_{H1})_y\,dx \tag{27}$$

In (27), Q_0, U_3 are the parametric coupling coefficient and an amplitude of pumping field, respectively. Indexes "1, 2" correspond to the signal and incident pulses, respectively. A possibility of quasisoliton parametric amplification in the bianisotropic metamaterial waveguide had been shown using the system (27).

Nonlinear transmission of electromagnetic waves through multilayered dielectric-graphene-dielectric… metamaterials

Using the NEELS method, it is possible to derive nonlinear evolution equations for the electromagnetic waves propagating normally to the multilayered dielectric-graphene-dielectric… metamaterial structures, where the nonlinearity is associated with the graphene layers. In particular nonlinearity in the case is connected with the conducting surface current i_s in graphene [27]:

$$i_s = \frac{ev_F p_F}{\pi}(1+\Psi^2)^{-1/2}\Psi,\; \Psi = eA/(\hbar p_F);$$
$$p_F = (\pi n_{20})^{1/2};\; E = \partial A/\partial t \tag{28}$$

Here $v_F \approx 10^8\ cm/s$ is the characteristic Fermi velocity of electrons in the graphene, $\hbar p_F$ is the Fermi electron momentum; A is the vector-potential of electromagnetic wave. The analog of the effective mass is $m^* = \hbar p_F / v_F \sim (0.01-0.03)m_e$. Nonlinear conductive current (28) can be used for the derivation of the NEELS under the transverse wave propagation as was described above, yielding the evolution equation [17]

$$\frac{\partial C}{\partial t} + \frac{ic^2}{2\omega\varepsilon^{(1)}}\left(\frac{\partial^2 C}{\partial z^2} + \Delta_\perp C\right) + + \frac{i}{2\omega\varepsilon^{(1)}}\left(\omega^2\varepsilon - \sum_j \omega_{pg}{}^2 l_n \delta(z-z_j)(1+\frac{i\nu}{\omega})\right)C +$$
$$\frac{i\omega_{pg}{}^2 l_n \omega}{2\omega^2\varepsilon^{(1)}}\sum_j\left(1 - Q_g^{-1/2}\left(1 - \frac{1}{8}\frac{e^2|C|^2}{(m^* v_F\omega)^2}Q_g^{-1}\right)\right)\times\delta(z-z_j)C = 0;\; Q_g = 1 + \frac{e^2|C|^2}{2(m^* v_F\omega)^2}. \tag{29}$$

In (29), $\varepsilon^{(l)} = \varepsilon + (\omega/2)(d\varepsilon/d\omega)$ is the dielectric permittivity with the correction following from the temporal dispersion. The collision frequency ν of electrons in the grapheme layers is taken into account here. The value of ν is of about $((3\cdot10^{11} - 3\cdot10^{12})\ s^{-1}$. Eq. (29) describes the focusing of nonlinear electromagnetic THz pulses in the multilayered structure both in transverse and longitudinal directions. The new effect of the nonlinear transformation to the "predominantly transmission" regime was presented in [17].

Conclusions

We developed and presented the new method NEELM for the derivation of the nonlinear equations for envelope amplitudes of wave packets in layered systems included nonlinear gyrotropic layers and both volume and surface nonlinearities, media with spatial dispersion, from GHz to optic ranges. The proposed method is formulated in very general form. We believe that NEELM is the most effective contemporary methods for the derivation of the evolution equations for the waves in the multi-layered systems which include materials and nonlinearities of different physical nature, such as metamaterials, plasma, ionosphere, gyrotropic nonlinear media, multilayered dielectric-graphene-dielectric and bianisotropic/chiral metamaterials etc. The qualitatively new features in the behavior of the nonlinear pulse propagation in the layered systems are revealed. Higher-order nonlinear effects, retardation and surface nonlinearities including an effect caused by higher nonlinear modes [2] are incorporated into the proposed method.

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