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STATISTICAL SIMULATION OF RANDOM FIELD ON 2D AREA WITH WHITTLE-MATERN TYPECORRELATION FUNCTION IN THE GEOPHYSICAL PROBLEM OF ENVIRONMENT MONITORING

(Рекомендовано членом редакційної колегії д-ром фіз.-мат. наук, проф. С.А. Вижвою)

Due to the increasing number of natural and technogenic disasters the development of geological environment monitoring system is actual one using modern mathematical tools and information technology. The local monitoring of potentially dangerous objects is an important part of the overall environment monitoring system.

The complex geophysical research was conducted on Rivne NPP area. The monitoring observations radioisotope study of soil density and humidity near the perimeter of buildings is of the greatest interest among these.

In this case a problem occurred to supplement simulated data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells.

This problem was solved in this work by statistical simulation method that provides the ability to display values (the random field of a research object on a plane) in any point of the monitoring area. The chalk strata averaged density at the industrial area was simulated using the built model and the involvement optimal in the mean square sense Whittle-Matern type correlation function.

In this paper the method is used and the model and procedure were developed with enough adequate data for Whittle-Matern type correlation function.

The model and algorithm were developed and examples of karst-suffusion phenomena statistical simulation were given in the problem of density chalk strata monitoring at the Rivne NPP area. The statistical model of averaged density chalk strata distribution was built in the plane and statistical simulation algorithm was developed using Whittle-Matern type correlation function on the basis of spectral decomposition. The research subject realizations were obtained with required detail and regularity at the observation grid based on the developed software. Statistical analysis of the numerical simulation results was done and tested for its adequacy.

Keywords: statistical simulation, Whittle-Matern type correlation function, spectral decomposition, conditional maps.

Introduction. Due to the increasing number of dangerous natural and technogenic disasters the development of geological environment monitoring system is actual one using modern mathematical tools and information technology. The local monitoring of potentially dangerous objects is an important part of the overall environment monitoring system. When monitoring such objects, a lot of problems were raised, for example, such as the lack of some data in the database, or insufficient quantity or necessity to supplement the database without conducting additional research.

The Department of Geophysics at Institute of Geology and involved experts from Mechanics and Mathematics Faculty of Kyiv National Taras Shevchenko University in recent years developed theoretical and methodological application basics of statistical simulation in the development of geological environment monitoring.

Theoretical aspects of capacity use of statistical simulation to solving problems in the work of Geophysics considered in (*Yadrenko, 1983*; *Grikh (Vyzhva) et al., 1993*; *Vyzhva, 2003*; *2011*). Practical testing on real data density chalky strata on the territory of the Rivne NPP was carried out for the fields on the plane – in the (*Vyzhva et al., 2004*), by using only Bessel correlation function and Cauchy correlation function (*Vyzhva et al., 2014*; *2017*).

In this paper, the statistical simulation method is proposed to be used with the model and procedure involving enough adequate in the mean square sense data Whittle-Matren type correlation function (*Gneiting et al., 2010*).

Note, that methods of random fields statistical simulation used in geosciences problems were considered in such works as: (*Chiles and Delfiner, 2012*; *Gneiting, 1997; Gneiting et al., 2010*; *Mantoglov and Wilson, 1981*; *Wackernagel, 2003*; *Prigarin, 2005*; *Vyzhva et al., 2010*; *2014*; *2017*) and other.

Problem of karst-suffusion phenomena monitoring at Rivne NPP area. The complex geophysical research was conducted on Rivne NPP area. The radioisotope study of soil density and humidity near the perimeter of buildings is of the greatest interest among these monitoring observations. The soil density was determined by gamma-gamma well logging, soil humidity – by neutron-neutron logging.

In this case (*Vyzhva et al., 2017*) a problem occurred to supplement simulated data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells. Schematic representation of the measurement results at the object that was investigated, and the well locations are shown on fig. 1. These data are obviously not enough to represent the overall picture of the chalk strata, where due to the aggressive water action the karst-suffusion processes were significantly intensified.

This problem was solved in works: (*Vyzhva et al., 2004*; *2014*; *2017*) by statistical simulation method that provides the ability to display values (random field on a plane) in any point of the monitoring area. The chalk strata averaged density at the industrial area was simulated using the built model and the involvement of the Bessel type correlation function (*Vyzhva et al., 2004*) and Cauchy correlation function (*Vyzhva et al., 2004*; *2014*; *2017*).

This paper continues development of methods for statistical simulation, involvingoptimal in the mean square sense Whittle-Materntypecorrelation function that is well-known in geostatisticworks (*Chiles and Delfiner, 2012*; *Gneiting, 1997; Gneiting et al., 2010*).

This operation was done for data array of density chalk strata in 1984–2002 years' for 29 wells at Rivne NPP industrial area and depth is 28 m below the surface.

The method of solving the problem. Data of density chalky strata was divided (*Vyzhva et al., 2017*) into deterministic and random components. Deterministic function can be selected by the method of approaching the minimum curve (separation of the trend). The difference between the map of input density values and the trend is a homogeneous isotropic random field in most cases. With the assumption that the input data is a random field $\eta(\bar{x})$, then we express them through a random component $\xi(\bar{x})$ (so-called "noise" random field) and trend $f(\bar{x})$ as a deterministic function as follows: $\eta(\bar{x}) = f(\bar{x}) + \xi(\bar{x})$.

Thus, the problem has been reduced to simulation of random component $\xi(\bar{x})$, which in most cases is a homogeneous and isotropic.

We consider the same approach as in works: (*Grikh (Vyzhva) et al., 1993*; *Vyzhva et al., 2004*; *Vyzhva, 2011*). We use the method of statistical simulation of random fields, which are homogenous and isotropic, based on their spectral decomposition. By means of the obtained values of realizations, this technique allows finding the perfect image of these isotropic fields in the whole observation interval.

It is necessary to make the statistical analysis to build the model and procedure of statistical data simulation at observation area. If the verified data has distribution density with approximately Gaussian type, then procedure can be used, which is developed in (*Grikh (Vyzhva) et al., 1993*; *Vyzhva et al., 2004*; *Vyzhva, 2011*), to generate on the computer realizations of the simulated data by means of standard normal random variable sequences.

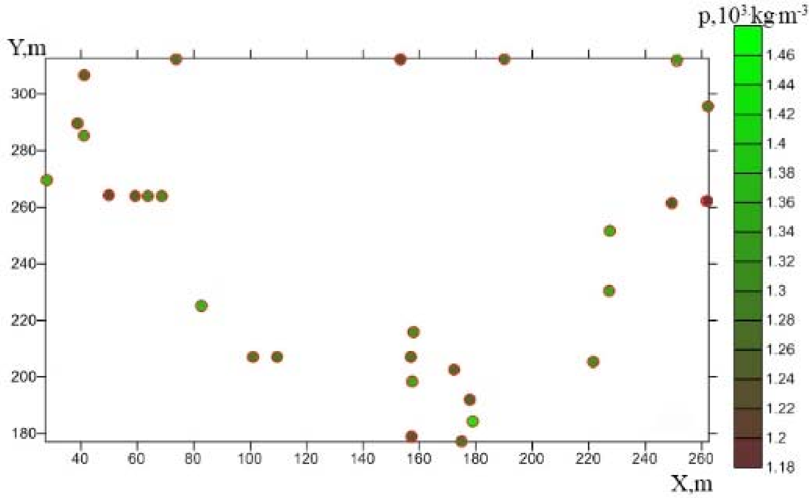


Fig. 1. Observation points and chalk strata averaged density at industrial area of Rivne NPP

At first the distribution is determined. The preliminary statistical analysis of data shows that the distribution histogram of chalky strata density at the Rivne industrial area (29 boreholes) approximately has Gaussian distribution (fig. 2).

The use of authors' techniques of statistical simulation implies preliminary statistical data processing to determine its statistical characteristics: the mathematical expectation and the correlation function. If the hypothesis of Gaussian distribution of the investigated field is confirmed, then the mathematical expectation and the correlation function completely define this field and give us the opportunity to build the adequate statistical model, which is based on spectral decomposition of random functions. The principles of constructing the models and procedures are described below.

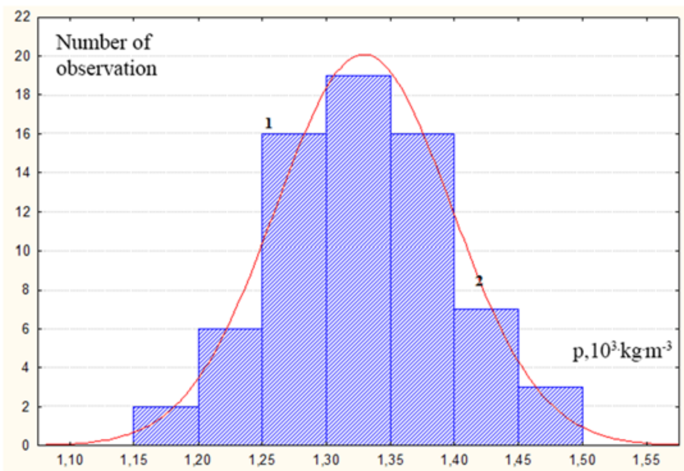


Fig. 2. Histogram of the chalky strata density (averaged data for all years of observation).
1 – the number of observations in a separate range of density; 2 – theoretical Gaussian curve

Then statistical models were chosen for the data correlation function $B(\rho)$ (ρ – the distance between vectors $x, y \in R^2$ $\left(x = (r_1, \theta_1), y = (r_2, \theta_2)\right)$, for distribution of chalky strata density in the flat observation area. This function is defined by comparing the mean square approximation of the empirical and theoretical variograms. As result, the input data was most adequately described by means of 4 types of correlation functions:the holeeffectcorrelation function (1) at the value of parameter $c = 1,4$, the Bessel correlation function (2) at the value of parameter $a = 5$, the Cauchy correlation function (3) at the value of parameter $a = 1$ and the Whittle-Matern type correlation function (4) at the value of parameter $c = 0{,}00333$

$$B(\rho) = \exp(c\rho)\cos(c\rho), \quad c = 1{,}4; \qquad (1)$$

$$B(\rho) = J_0(a\rho), \quad a = 5; \qquad (2)$$

where $J_k(x)$ is the Bessel function of the first kind of order $k = 0$.

$$B(\rho) = \frac{a^4}{\left(a^2 + \rho^2\right)^2}, \quad a = 1, \qquad (3)$$

$$B(\rho) = \frac{2^{1-\frac{3}{2}}}{\Gamma\left(\frac{3}{2}\right)}(c\rho)^{\frac{3}{2}} K_{\frac{3}{2}}(c\rho), \quad c = 0{,}00333; \qquad (4)$$

where $\Gamma(t)$ is a Gamma function, $K_{3/2}(z)$ is a modified Hankel function of order3/2 and c is parameter.

Variograms of input chalky strata density data was built by using the R software and geoR package They corresponding to the: holeeffect (1) correlation function (the mean square approximation is 0,000742); Bessel (2) correlation function (the mean square approximation is 0,0008599); Cauchy (3) correlation function (the mean square approximation is 0,002816); Whittle-Matern type correlation function (4) (the mean square approximation is 0,000311). Variograms plots were presented at figure 3, according to all types of correlation functions for the random component of investigation data.

Note that the generalized Whittle-Matern typemodel of correlation function is:

$$B_\nu(\rho) = \frac{2^{1-\nu}}{\Gamma(\nu)}(c\rho)^\nu K_\nu(c\rho), \quad c > 0, \ \nu \geq 0. \qquad (5)$$

where $K_\nu(z)$ is a modified Hankel function of order ν and *c* is parameter.

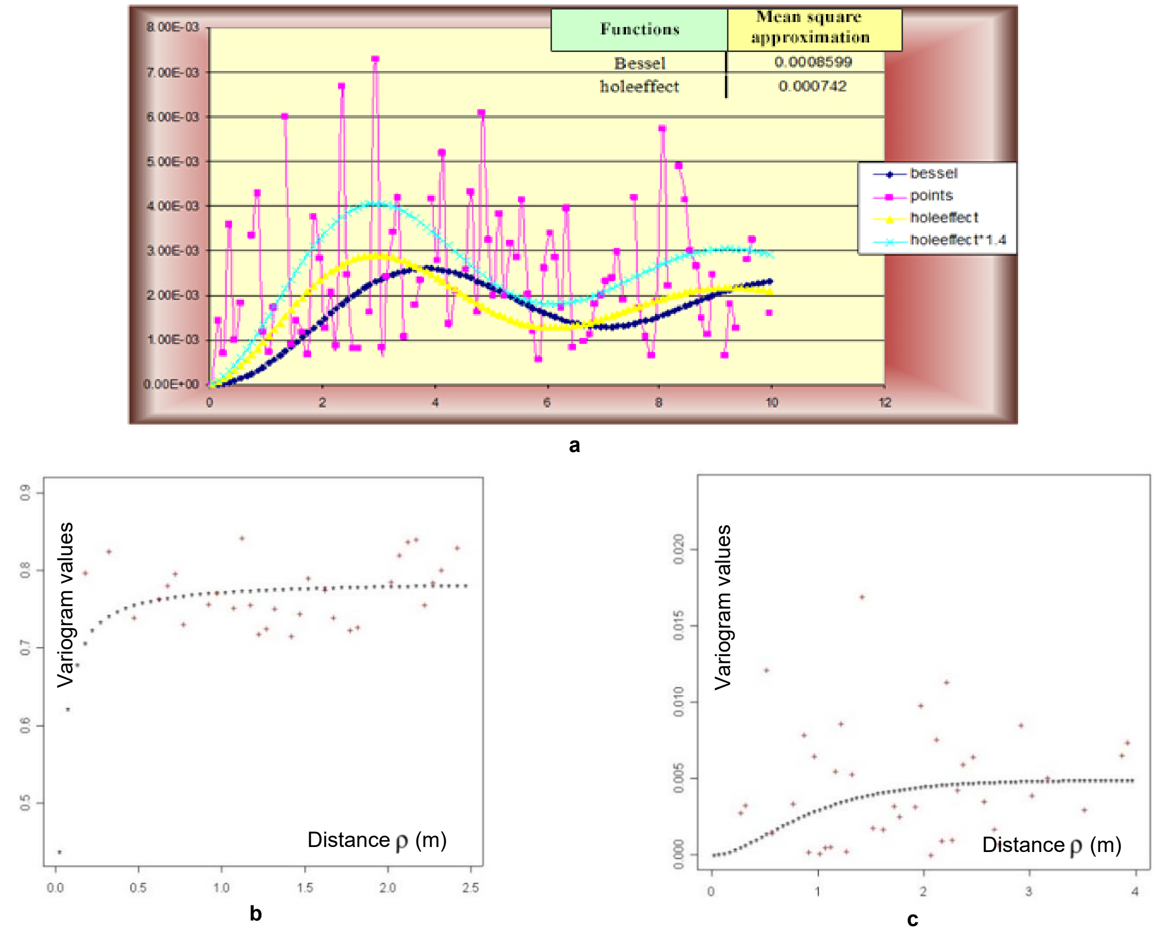


Fig. 3. Variograms of input data of the chalky strata, that corresponding to the:
a – the holeeffect (1) correlation function; the Bessel (2) correlation function; b – the Cauchy (3) correlation function; c – the Whittle-Matern type (4) correlation function

The generalized Whittle-Matern typemodel (4) was studied by (*Gneiting, 1997; Gneiting et al., 2010*) who considered the Whittle-Matern typefunction at the values of the parameter ν = 1/2, 3/2, 5/2, 7/2.

Graphic representations of the Whittle-Matern typefunction at different values of parameters ν = 0,065, 0,1, 0,25, 0,5, 1, 2, 4 and 25 and at $c = 1$ are presented in fig. 4 (*Abrahamsen, 1997*).

Let us find the spectral density, corresponding to the Whittle-Matern typecorrelation function (4), by using the formula on page 449 (*Watson, 1949*). This formula is mentioned below as integral Veber-Shafheitlin type:

$$\int_0^\infty K_\mu(ax) J_\nu(bx) x^{\mu+\nu+1} dx = (2a)^\mu (2b)^\nu \frac{\Gamma(\mu+\nu+1)}{\left(a^2+b^2\right)^{\mu+\nu+1}}, \qquad (6)$$

$$\mathrm{Re}(a) > I(b), \mathrm{Re}(\mu) < \mathrm{Re}(2\nu+\mu+2).$$

where $\Gamma(t)$ is a Gamma function.

Then such spectral density is calculated by conducting the following formula:

$$f(\lambda) = \lambda \int_0^\infty x J_0(\lambda x) B(x) dx = \\ = \lambda\sqrt{2/\pi}\int_0^\infty x^{5/2} J_0(\lambda x) K_{\frac{3}{2}}(cx)\,dx, \quad (7)$$

where $B(\rho)$ is correlation function

Thus, this spectral density we obtain by using (6) and we have :

$$f(\lambda) = \lambda\sqrt{2/\pi}\frac{(2c)^{\frac{3}{2}}\Gamma\left(\frac{5}{2}\right)}{\left(c^2+\lambda^2\right)^{5/2}} = \lambda\frac{3c^{\frac{3}{2}}}{\left(c^2+\lambda^2\right)^{5/2}}, \quad (8)$$

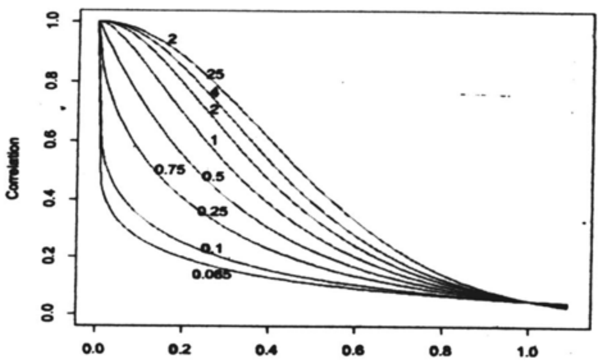


Fig. 4. The Whittle-Matern type function (3) at parameter values c = 1 and ν = 0,065, 0,1, 0.25, 0,5, 1, 2, 4 and 25 and exponential function $B(\rho) = ae^{-a\rho}$ at parameter values a = 0,75, 2

The spectral coefficients, that corresponding to such Whittle-Matern typecorrelation function (4), are determined by calculating the integral:

$$b_k(r) = 2\int_0^\infty J_k^2(r\lambda)\, f(\lambda) d\lambda = \\ = 6c^{\frac{3}{2}}\int_0^\infty J_k^2(r\lambda)\frac{\lambda}{\left(c^2+\lambda^2\right)^{\frac{5}{2}}}d\lambda, k = 0,1,2.... \quad (9)$$

These spectral coefficients are calculated by Mathematica software.

The model of random field on 2D areawith the Whittle-Matern typecorrelation function and the numerical simulation procedure. We generated the realizations of random field on 2D area with the Whittle-Matern typecorrelation function (3) at the values of parameter $c = 0,00333$. The statistical simulation was performed by the technique of spectral decomposition and finding of spectral coefficients.

From the spectral theory (*Vyzhva, 2011*) it is follows that the model of random fields on a plane with such correlation functions is a sum of:

$$\xi_N(r,\varphi) = \sum_{k=0}^{N}\sqrt{\nu_k}\left[\zeta_{k,1}(r)\cos(k\varphi) + \zeta_{k,2}(r)\sin(k\varphi)\right], \quad (10)$$

where: $\nu_k = \begin{cases} 1, k = 0; \\ 2, k > 0, \end{cases}$

- r and φ ($r \in R_+$, $\phi \in [0, 2\pi]$) are polar coordinates of the pointx on the plane (includes observation area), and the distance ρ between the points $x_1 = (r_1, \varphi_1)$ and $x_2 = (r_2, \varphi_2)$ is equals $\rho = \sqrt{r_1^2 + r_2^2 - 2r_1\, r_2 \cos(\phi_1 - \phi_2)}$;
- N is an integer number (the number of the summands in the model), the value of N is determined by the prescribed small number ε (approximation accuracy) by the inequality from paper (*Grikh (Vyzhva) et al., 1993*), which is the estimate of the mean square approximation of random field $\xi(r,\varphi)$ by partial sums $\xi_N(r,\varphi)$;
- $b_k(r)$, $(k = 0,\ 1,\ 2,...N)$ are the spectral coefficients in form of(9), which corresponding to the Whittle- Matern typecorrelation function (4);
- $\{\ \zeta_{k,i}(r), i = 1,2; k = 0, 1, 2,..., N\ \}$ are thesequences of standard normal random variables, such that satisfying the following conditions:

1) $\mathrm{M}\,\varsigma_{k,i}(r) = 0, i = 1,2; k = 0, 1, 2,..., N$; (11)

2) $\mathrm{M}\,\varsigma_{k,l}(r)\,\varsigma_{k',l'}(r) = \delta_l^{l'}\ \delta_k^{k'}\ b_k(r), l = 1,2; k = 0,1,2,..., N.$ (12)

The procedure of numerical simulation the realizations of the data field random component, by means of the abovementioned model (10), was conducted by using the Spectr 2.1 software, which is described in (*Vyzhva et al., 2004*).

The value of number N for the constructed model is determined by the inequality, which is the estimate of the mean square approximation of random field $\xi(r,\varphi)$ by partial sums $\xi_N(r,\varphi)$. This number N corresponds to the prescribed small number ε **(**approximation accuracy). The mentioned inequality was obtained in work (*Grikh (Vyzhva) et al., 1993*) and in form of:

$$\mathrm{M}\left[\xi(r,\varphi) - \xi_N(r,\varphi)\right]^2 \le \frac{1}{\pi N}\left(\frac{1}{2}r\mu_1 + r^2\mu_2\right), \quad (13)$$

where $\mu_k = \int_0^\infty \lambda^k f(\lambda) d\lambda, k = 1,2.$

Define dependence number N on r and ε in the case of Whittle-Matern typecorrelation function (4). It is necessary to calculate the values of $\mu_k, \kappa = 1,2$ for the inequality (13), by using the density of distribution (8).

Then the calculated values hold:

$$\mu_1 = 3c^{\frac{3}{2}}\int_0^\infty \frac{x^2}{(c^2+x^2)^{5/2}}dx = 3c,$$

$$\mu_2 = 3c^{\frac{3}{2}}\int_0^\infty \frac{x^3}{(c^2+x^2)^{5/2}} = 6c^{\frac{3}{2}}\sqrt{c} = 6c^2. \quad (14)$$

Consequently, the estimate of the mean square approximation of the random field $\xi(r,\varphi)$ with Whittle-Matern typecorrelation function (4) by the partial sums $\xi_N(r,\varphi)$ has the following representation:

$$N(r,\varepsilon) \geq \frac{1}{\pi\varepsilon}\left(\frac{1}{2}r\mu_1 + r^2\mu_2\right) = \frac{3}{\pi\varepsilon}\left(\frac{1}{2}cr + 2c^2r^2\right). \quad (15)$$

The statistical simulation procedure of Gaussian homogeneous isotropic random field $\xi(r,\varphi)$ on the plane with Whittle-Matern typecorrelation function (4) was built by means of the model (10) and the estimate (15). This random field is determined by its statistical characteristics: the mathematical expectation and the Whittle-Matern typecorrelation function $B(\rho)$ (4) at the value of parameter $c = 0,00333$.

Procedure.

1) The positive integer number N is determined corresponding to the prescribed accuracy ε and by using inequality (15), where r is a radius of the point on the plane, in which the realization of the random field $\xi(r,\varphi)$ is generated. The integer number N equals *8* by using the prescribed accuracy $\varepsilon = 5\times10^{-2}$ and values of parameters $\nu = 3/2,\ c = 0,00333$.

2) We calculated the spectral coefficients by formula (9) at the value of parameter $c = 0,00333$:

$$b_k(r) = 6c^{\frac{3}{2}}\int_0^{\infty} J_k^2(r\lambda)\frac{\lambda}{\left(c^2+\lambda^2\right)^{\frac{5}{2}}}d\lambda, k = 0,1,2...8.$$

3) We generate values of the normal random variables sequences $\left\{\zeta_{k,i}(r), i = 1,2; k = 0,1,2,..., N(r,\varepsilon)\right\}$, such that satisfying the f conditions(11) and (12);

4) We calculate the realization of the random field $\xi(r,\varphi)$ in 100 points $(r_i,\varphi_j), i = 1,...,10; j = 0,...,9, \in R^2$ and evaluated the expression (10) by substituting in it values which were found in the previous steps, $r_i = 181,32 + 20\times i, i = 1,...,10;$ $\varphi_j = \varphi_0 + j\times\frac{2\pi}{10},\ j = 0,...,9$.

5) The statistical estimate of the correlation function is obtained by the realizations of the random $\xi(r,\varphi)$. This estimate compares with a given Whittle-Matern type(4)correlation function at $c = 0,000333$ and provides the statistical analysis the adequacy of realization.

Note that the procedure can be applied to random fields with another type of distribution. Then the sequences ofrandom variables $\left\{\zeta_{k,i}(r), i = 1,2; k = 0,1,2,..., N(r,\varepsilon)\right\}$ should be distributed by corresponding type of distribution.

The original Spectr software, based on the results of the statistical data processing and the mentioned procedure for the simulation values of such data realization in the two-dimensional case, was developed in Python, where selected Whittle-Matern typecorrelation function (4) was used.

The results, which were obtained by the simulating procedure, are displayed in figure 5. Figure 5, a presents an example of constructed chalky strata density map according to observations data boreholes (averaged data over the years to 29 boreholes at 28 m) by Surfer software. Using available data the accuracy of this construction can not provide a reliable characteristic of the chalky strata status, because the number of measurement results is not sufficient.

Fig. 5, b presents the contours of equal chalky strata density values that based on simulated data including values of the anchor boreholes by means of calculating the spectral coefficients. Additionally, the output data (100 simulated values in intervals between the observation points of this level) can have more reliable approximation that enables more informed decisions about the status of chalky strata and determines places for testing and additional research.

The following fig. 6 presents the plot of the variogram of the separated random data component of chalky strata density according to the Whittle-Matern typecorrelation functions (4) (fig. 6, a) and plot of the variogram of the simulated random data component according to the Whittle-Matern typecorrelation functions (4) (fig. 6, b).

The results present that the chosen model of the data is enough adequate. The developed Spectr2_1 software works with sufficient accuracy.

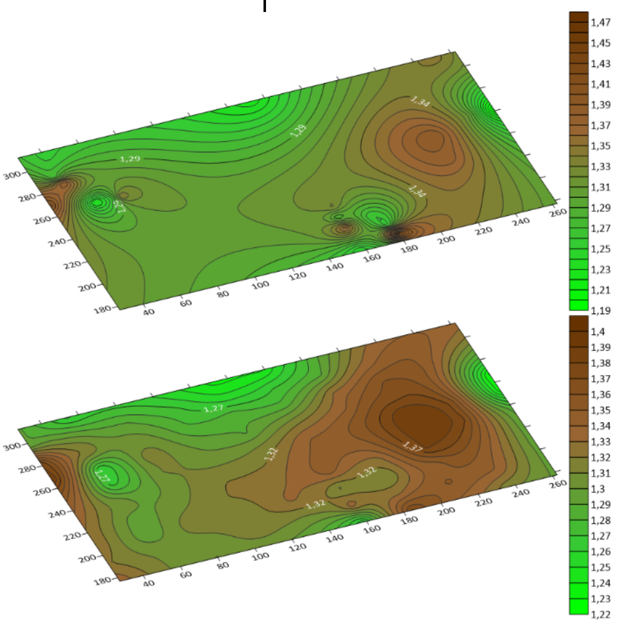
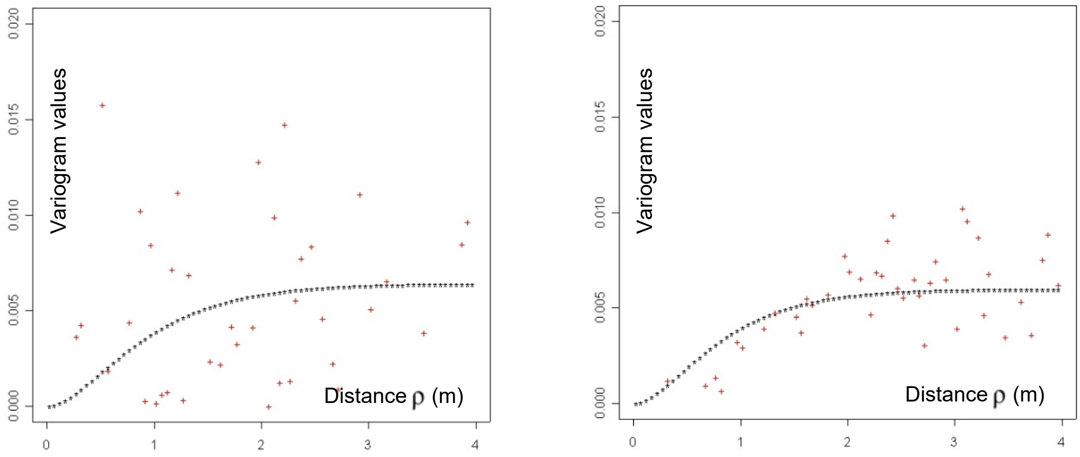


Fig. 5. The distribution of chalky strata density is on the industrial area of Rivne nuclear power plant at a depth of 28 m. from the surface, according to (a) the averaged data of 29 observational boreholes over 1984–2004 years., for (b) the simulated data that based on the values in secure boreholes by spectral coefficients the Whittle-Matern type



a b

Fig. 6. The variogram of separated (a) and of simulated (b) random data component of the chalky strata density, corresponding to Whittle-Matern type correlation function at values of parameters $\nu = 3/2$ and $c = 0,00333$

Conclusions. The theory, techniques and procedure of statistical simulation of random fields on the planeby usingoptimal in the mean square sense the Whittle-Matern type correlation functioncan significantly increase the effectiveness of monitoring observations on the territory of potentially dangerous objects. This makes it possible to simulate the values in the area between regime observation grids and abroad, adequately describe real geological processes.

The method of statistical simulation of random fields with the Whittle-Matern correlation functions allows complementing data with a given accuracy. It can also be used to detect abnormal areas.

There are several other fields of statistical simulation methods application in geosciences. Among them primary are soil science and environmental magnetism (*Menshov et al., 2015*).

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СТАТИСТИЧНЕ МОДЕЛЮВАННЯ ВИПАДКОВОГО ПОЛЯ НА ПЛОСКІЙ ОБЛАСТІ З КОРЕЛЯЦІЙНОЮ ФУНКЦІЄЮ ТИПУ УІТТЛІ–МАТЕРНА В ГЕОФІЗИЧНІЙ ЗАДАЧІ МОНІТОРИНГУ ДОВКІЛЛЯ

У зв'язку з ростом кількості природно-техногенних катастроф актуальною є розробка систем моніторингу за станом геологічного середовища з використанням сучасного математичного апарата та інформаційних технологій. У загальній системі моніторингу довкілля важливою складовою є локальний моніторинг територій розташування потенційно небезпечних об'єктів.

На території розміщення Рівненської АЕС проводився комплекс геофізичних досліджень. Серед таких моніторингових спостережень найбільший інтерес являють собою радіоізотопні дослідження густини та вологості ґрунтів по периметру збудованих споруд. При цьому виникла проблема доповнення моделюванням даних, які отримано в результаті контролю зміни густини крейдяної товщі на території досліджуваного проммайданчика з використанням радіоізотопних методів по сітці, що включала 29 свердловин. Таку проблему було розв'язано в роботі методом статистичного моделювання, який надає можливість відображати явище (випадкове поле об'єкта дослідження на площині) у будь-якій точці зони спостереження. При цьому моделювалися усереднені значення густини крейдяної товщі на території проммайданчика з використанням побудованої моделі та залученням оптимальної у середньому квадратичному наближенні кореляційної функції типу Уіттлі–Матерна.

Розроблено алгоритм і приклад статистичного моделювання карстово-суфозійних явищ у задачі моніторингу густини крейдяної товщі на території Рівненської АЕС. За спектральним розкладом побудовано статистичну модель розподілу усередненої густини крейдяної товщі на площині та розроблено алгоритм статистичного моделювання з використанням функції типу Уіттлі–Матерна. На базі розробленого програмного забезпечення отримано реалізації предмета дослідження на сітці спостережень необхідної детальності та регулярності. Проведено статистичний аналіз результатів чисельного моделювання та їхню перевірку на адекватність.

Ключові слова: статистичне моделювання, кореляційна функція типу Уіттлі–Матерна, спектральний розклад, кондиційність карт.

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СТАТИСТИЧЕСКОЕ МОДЕЛИРОВАНИЕ СЛУЧАЙНОГО ПОЛЯ НА ПЛОСКОЙ ОБЛАСТИ С КОРРЕЛЯЦИОННОЙ ФУНКЦИЕЙ ТИПА УИТТЛИ–МАТЕРНА В ГЕОФИЗИЧЕСКОЙ ЗАДАЧЕ МОНИТОРИНГА ОКРУЖАЮЩЕЙ СРЕДЫ

В связи с ростом количества природно-техногенных катастроф актуальной является задача разработки систем мониторинга состояния геологической среды с использованием современного математического аппарата и информационных технологий. В общей системе мониторинга окружающей среды важной составляющей является локальный мониторинг территорий размещения потенциально опасных объектов.

На территории расположения Ровенской АЭС проводился комплекс геофизических исследований. Среди таких мониторинговых исследований наибольший интерес представляют радиоизотопные исследования плотности и влажности грунтов по периметру построенных сооружений. При этом возникла необходимость дополнения данных путем моделирования, которые изначально были получены при контроле изменений плотности меловой толщи на территории исследуемой промплощадки с использованием радиоизотопных методов по сетке, которая включала 29 скважин. Эта проблема была решена в работе методом статистического моделирования, который даёт возможность отображать явление (случайное поле объекта исследования на плоскости) в любой точке области наблюдения. При этом моделировались усреднённые значения плотности меловой толщи на территории промплощадки с использованием построенной модели с привлечением корреляционной функции типа Уиттли–Матерна.

Авторами разработан алгоритм и приведен пример статистического моделирования карстово-суффозионных явлений в задаче мониторинга плотности меловой толщи на территории Ровенской АЭС. По спектральному разложению построена статистическая модель распределения плотности меловой толщи на плоскости и разработан алгоритм статистического моделирования с использованием корреляционной функции типа Уиттли–Матерна. На базе разработанного программного обеспечения получены реализации предмета исследования на сетке наблюдений необходимой детальности и регулярности. Проведен статистический анализ результатов численного моделирования и их проверка на адекватность.

Ключевые слова: статистическое моделирование, корреляционная функция типа Уиттли–Матерна, спектральное разложение, кондиционность карт.