UDK 629.4

CALCULATION METHODS OF SUSPENSION STIFFNESS DETERMINATION

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АНАЛІЗ РОЗРАХУНКОВИХ МЕТОДІВ ВИЗНАЧЕННЯ ЖОРСТКОСТІ РЕСОР

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The paper deals with the calculation of a suspension stiffness and the different methods for their calculation. The vertical stiffness of the springs was calculated using the ANSYS program. The results were compared with calculated values afterwards. The lateral stiffness was evaluated in a similar manner. Analytical method by Gross, Wahl, Budrick, Timoshenko and Ponomarieva was used for comparison with numerical values. The ANSYS simulation was performed for calculating the vertical stiffness of the triple springs. The obtained data will be used as an input for the design of coil springs which will be implemented in a model of a vehicle with a tilting car body, for which the comfort values during transition in curve will eventually be determined.

Keywords: *suspensions, stiffness, calculation methods.*

Spring makes an important part of complex mechanical systems [2, 8]. By choosing a proper shape and material, it is able to accumulate deformation energy [5, 9]. In mechanical engineering, they serve mostly to cushion a part of a tool or to produce pressure [4, 10, 11]. Because of these properties, coil springs are used in construction of rail vehicle bogies as well.

The deformation work of the spring represents the accumulated energy, which can be expressed in the form:

$$dA = Fdy = Cydy\text{, or } dA = Md\varphi = C\varphi d\varphi. \quad (1)$$

From which the deformation work is defined as:

$$A = \int_0^y Cydy = \frac{1}{2}Cy^2 = \frac{1}{2}Fy \text{ or } A = \int_0^y Cd\varphi = \frac{1}{2}C\varphi^2 = \frac{1}{2}M\varphi. \quad (2)$$

The force or moment that will cause the unit displacement or rotation of the spring rate:

$$k = \frac{dF}{dy}\text{, resp. } k = \frac{dM}{d\varphi}. \quad (3)$$

Stiffness of the springs with linear characteristic is constant:

$$k = \frac{F}{y} \text{ or } k = \frac{M}{\varphi}. \quad (4)$$

The first natural frequency serves as a sort of suspension quality indicator and can be approximately determined using the formula:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \cong \frac{0.5}{\sqrt{z_{stat}}}, \quad (5)$$

where f is the natural frequence, Hz;

k is the total stiffness of the vertical suspension, kN.m^{-1};

m is the vehicle car body mass including the sprung parts of the bogie, t;

z_{stat} is the static press of the vertical suspension under the weight of the car body, m.

Default parameters of a coil spring are the mean diameter of the spring D, diameter of the wire d, amount of active threads n, amount of closing threads n', free length of a spring H_0, gradient of the helix s [6, 12]. The free length of a spring is must be chosen in such manner, so that even by its maximal press z_{max} (constrained by the buffers) the threads would not come to contact themselves, but a clearence of at least 10 to 15 % diameter of the spring wire [7].

Free lenght of a spring H_0:

$$H_0 = \left(n + n'\right)d + z_{max} + n\left(from\, 0.1\, to\, 0.15\right)d. \quad (6)$$

Determination of vertical spring stiffness

Coil springs represent the most proper steel suspension element in the suspension system of rail vehicles [1]. They are favourable in the means of dimensions and mass. They are used in systems along with dampers, because they do not have the ability to absorb the energy of oscillating motion of parts [3].

For the coil spring stiffness calculation, the formula (7) applies, which was used for analytical determination of individual vertical stiffness values. These were compared with values taken from ANSYS afterwards (Tab. 2):

$$k = \frac{Gd^4}{64R^3 n}, \text{ N.mm}^{-1}, \tag{7}$$

where n is the amount of the active threads;

G is the shear modulus.

In the following Tab. 1, basic parameters of coil springs used in the bogie model are displayed.

Geometrical model of individual springs was created in CATIA and imported to ANSYS afterwards. As a boundary condition, a vertical displacement (along z-axis) value of 50 mm was set. The manner of end coils placing is displayed in Fig. 1. After carrying out simulations and displaying results for individual springs A (outer), B (middle) and C (inner), we found the value of vertical force acting on the spring. From the rate of acting force and spring compression we determined the result value of vertical spring stiffness.

Table 1

Parameters of the coil springs

Spring	A	B	C
d *[mm]*	35	25	20
D *[mm]*	280	210	150
n *[-]*	6	7	8
s *[mm]*	56.67	50.00	44.38
n_0 *[-]*	0.75	0.75	0.75
h *[mm]*	375	375	375
F_0 *[N]*	5707.10	3021.10	2983.70
E *[MPa]*	$2.06\text{x}10^5$	$2.06\text{x}10^5$	$2.06\text{x}10^5$
G *[MPa]*	$8.15\text{x}10^4$	$8.15\text{x}10^4$	$8.15\text{x}10^4$

Table 2

Coil springs vertical stiffness comparison

Spring	Analytically determined stiffness - k_z *[N.mm^{-1}]*	Stiffness determined in ANSYS – k_y *[N.mm^{-1}]*	Difference *[%]*
A	103.17	114.50	9.90
B	55.45	59.74	7.19
C	54.57	60.89	10.37

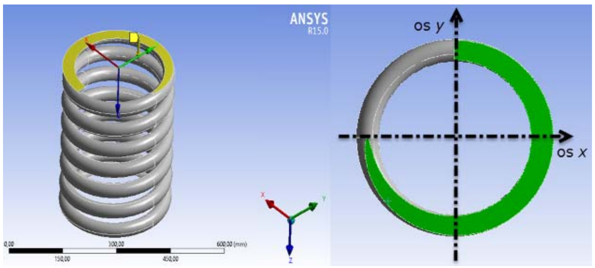

Fig. 1. Model of loaded coilspring A in ANSYS

Determination of lateral spring stiffness

For the analytical calculation of lateral stiffness, many formulas from various authors were derived, but they are only approximate and do not apply in general, because they do not regard all the affecting factors. For the analytical determination of the spring stiffness, formulas by Gross, by Wahl, by Budrick and by Tymoshenko and Ponomarev have been used [12].

Calculation by Gross:

Stiffness of a spring under load in lateral direction:

$$k_y = \frac{1}{\frac{1}{F_0}\cdot\left[\frac{2}{\alpha}\cdot tg\left(\alpha\cdot\frac{h}{2}\right)-h\right]+\frac{h}{k_s}}, \text{ N.mm}^{-1}, \tag{8}$$

where

$$\alpha = \sqrt{\frac{F_0}{k_0\left(1-\frac{F_0}{k_s}\right)}} \text{ is a constant,} \tag{9}$$

where G is shear modulus, Pa,

E is Young's modulus, Pa,

F_0 is vertical force, N,

k_0 is Bending stiffness, N.mm^{-1},

k_s is Shear stiffness, N.mm^{-1}.

Bending stiffness:

$$k_0 = \frac{h}{\pi\cdot n\cdot\frac{D}{2}\cdot\left(\frac{1}{E\cdot I_1}+\frac{1}{G\cdot I_p}\right)}, \text{ N.mm}^{-1}. \tag{10}$$

where E is spring length, mm,

D is wire diameter, mm,

π is mathematical constant, -,

I_l is moment of inertia, kg.m^2,

I_p is polar moment of area, kg.m^2.

Shear stiffness:

$$k_s = \frac{E\cdot h\cdot I_1}{\pi\cdot n\cdot\left(\frac{D}{2}\right)^3}, \text{ N}\cdot\text{mm}^{-1}. \tag{11}$$

Moment of inertia:

$$I_1 = \frac{\pi\cdot d^4}{64}, \text{ kg}\cdot\text{m}^2. \tag{12}$$

Polar moment of area:

$$I_P = \frac{\pi\cdot d^4}{32}, \text{ kg}\cdot\text{m}^2. \tag{13}$$

Calculation by Wahl:

Stiffness of a spring under load in lateral direction:

$$k_y = \frac{2.6 \cdot k_z}{1+0.77\cdot\beta^2}\cdot\left(1-\frac{F_0}{U\cdot h_0\cdot k_z}\right)\ \mathrm{N{\cdot}mm^{-1}} \qquad (14)$$

where:

$$\beta = \frac{h}{D},\ \mathrm{N{\cdot}mm^{-1}} \qquad (15)$$

Coefficient *U* is defined in Tab. 3.

Table 3

Dependence of the coefficient *U* on *β*

β	1.5	2	2.5	3	3.5	4	4.5	5
U	0.69	0.63	0.53	0.39	0.27	0.2	0.14	0.11

Calculation by Budrick:

Stiffness of a spring under load in lateral direction:

$$k_y = k_z\cdot\frac{G}{E}\cdot\left(1+\frac{2+\mu}{3}\cdot\beta^2\right),\ \mathrm{N{\cdot}mm^{-1}} \qquad (16)$$

where:

$$k_z = \frac{2}{\pi D n\left(\frac{h^2}{12}\left(\frac{1}{GI_p}+\frac{1}{EI_1}\right)+\frac{D^2}{4EI_1}\right)},\ \mathrm{N.mm^{-1}}. \qquad (17)$$

Calculation by Tymoshenko and Ponomarev:

Stiffness of a spring under load in lateral direction:

$$k_y = k_z\frac{D^2\cdot(1-\gamma)}{\frac{0.2936(h-\chi d)^3}{(h-1.5\cdot h\cdot d)}+0.381D^2},\ \mathrm{N{\cdot}mm^{-1}} \qquad (18)$$

where: γ.is variable quantity, -,

χ is constant, -,

for $\beta_0 < 2.62$:

$$\gamma = 0.375\cdot\frac{F_0}{k_z\cdot h}\cdot\beta\cdot\left(\beta-1.5\cdot\frac{d}{D}\right) \qquad (19)$$

for $\beta_0 > 2.62$:

$$\gamma = \frac{\frac{F_0}{k_z\cdot h}\cdot\beta}{0.813\cdot\left(\beta_0-\sqrt{\beta_0^2-6.87}\right)} \qquad (20)$$

In formula (18) there is non-dimensional variable quantity γ, which is dependant on slenderness ratio of a loaded spring. Its value can be calculated from formulas (19) and (20). The constant χ is an auxiliary quantity, which regards the manner of mounting the end coils of the springs (joint or rigid mounting). For the analysed springs, the constant is equal 0.5. We defined the values of lateral stiffness analytically, based on formulas according to individual methods (8 - 20). These values were afterwards compared with the values obtained from ANSYS. Input parameters of the individual springs are given in Tab. 4.

Table 4

Calculated and determined values of lateral stiffness

Method	A	B	C
ANSYS k_y *[N.mm^{-1}]*	172.12	38.28	35.47
ANSYS k_x *[N.mm^{-1}]*	142.99	50.36	25.60
Gross *[N.mm^{-1}]*	100.36	34.18	15.90
Wahl *[N.mm^{-1}]*	88.57	32.10	17.70
Budrick *[N.mm^{-1}]*	179.49	78,99	66.85
Timoshenko and Ponomarev *[N.mm^{-1}]*	109.40	37.13	17.60
Percentage of difference between results from individual methods and ANSYS			
G-A-k_y *[%]*	41.69	10.70	55.18
G-A-k_x *[%]*	29.81	32.13	37.89
W-A-k_y *[%]*	48.54	16.14	50.11
W-A-k_x *[%]*	38.06	36.26	**30.88**
B-A-k_y *[%]*	**4.28**	106.37	88.46
B-A-k_x *[%]*	25.53	56.85	161.13
TaP-A-k_y *[%]*	36.44	**2.99**	50.37
TaP-A-k_x *[%]*	23.49	26.26	31.23

Determination of the triple spring stiffness

The secondary suspension of the analysed bogie is consists of three coil springs (Fig. 2). Lateral forces in springs Fp have been calculated using the ANSYS program for 8 different mounting positions of the end coil (Fig. 3). The maximal vertical displacement was set to the value of 50 mm and the lateral displacement was in the range of 0 – 4 mm. In Fig. 4 we can see the characteristic of the triple spring lateral forces in dependency on lateral displacement. If we turn the spring system around the vertical axis (45 degrees) the characteristic is shifting. As can be seen in Fig. 4, the mounting positions of the end coil P0 and P180 appear to be most suitable (Fig. 3).

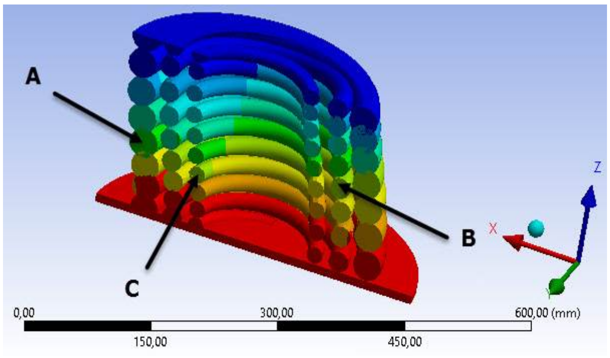

Fig. 2. Model of a loaded triple coil spring in ANSYS program

The resulting vertical stiffness of the triple spring is constant with a value of 221.88 N·mm^{-1} with an applied vertical force of 11 091.22 N. The calculated values of lateral forces Fp are in the following Tab. 5. We determined the lateral stiffness for the position of the end coils P180 (Tab. 6), its value is linear and rises with increasing lateral displacement.

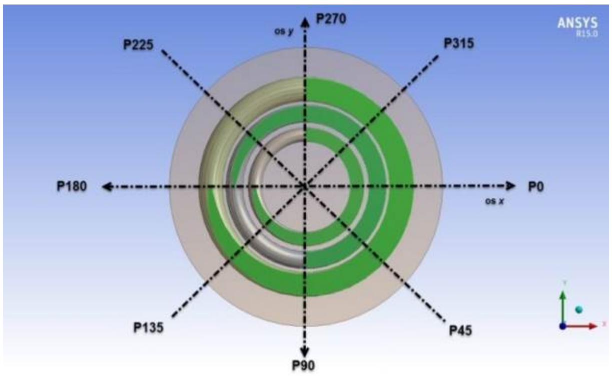

Fig. 3. Method of the placement of the ending coils of the springs

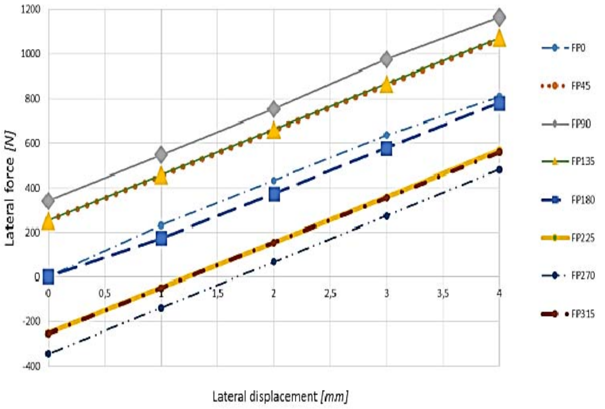

Fig. 4. Dependence of the lateral force on the lateral displacement

Table 5

Calculated values of the lateral forces

Lateral displa-cement [mm	4	3	2	1	0
F_{P0} [N]	809.21	636.54	434.30	232.00	2.99
F_{P45} [N]	1066.50	862.84	659.07	454.90	251.72
F_{P90} [N]	1165.30	975.62	753.84	547.72	342.89
F_{P135} [N]	1071.80	866.24	661.52	457.33	253.00
F_{P180} [N]	780.43	578.80	374.85	174.96	2.99
F_{P225} [N]	569.23	359.50	155.58	-47.52	-251.72
F_{P270} [N]	481,09	274.78	69.04	-136.86	-342.89
F_{P315} [N]	559.77	355.94	155.44	-48.64	-253.00

Table 6

Calculated values of the secondary suspension lateral stiffness in position P180

Lateral displacement *[mm]*	F_{P180} *[N]*	Lateral stiffness k_y *[N.mm^{-1}]*
4	780.43	195.11
3	578.80	192.93
2	374.85	187.43
1	174.96	174.96

Conclusion

Suspension is one of the most important parts of a bogie. In this article we focused on the secondary suspension, consisting of three coil springs. Stiffness of the individual springs was examined separately using analytical methods and numerical calculation.

When determing the vertical stiffness, the difference between calculated values and values obtained from ANSYS was about 10 % in average, which is sufficient for use in SIMPACK program. After determination of lateral stiffness using individual analytical methods and consequential comparison with values obtained using numerical method we discovered that the method by Tymoshenko and Ponomarev, where the percentage difference was the smallest, is the most suitable. From the above can be concluded, that not every analytical method is suitable for determination of lateral stiffness of coil springs.

The advantage of using numerical method for spring stiffness determination and using simulation program ANSYS is the possibility of solving problems as a whole and the parametrisation of the models. Lateral stiffness of a triple spring is dependent on the end coils mounting position. The results will be further used in simulation of the whole bogie and of its transition in curve using SIMPACK program.

Acknowledgement

The work was supported by the Scientific Grant Agency of the Ministry of Education of the Slovak Republic and the Slovak Academy of Sciences in project No. 1/0347/12: “Railway wheel tread profile wear research under the rail vehicle in operation conditions simulation on the test bench”, project No. 1/0383/12: “The rail vehicle running properties research with the help of a computer simulation.” and the project No. APVV-0842-11:

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Лоулова М., Суханек А., Хаусер В., Ноженко О.С. Кравченко К.О. Аналіз розрахункових методів визначення жорсткості ресор

У статті розглядається різні методи розрахунку ресор. Вертикальна жорсткість пружин розраховувалася з використанням програми ANSYS і іншими методами. Згодом результати порівнювалися. Бічна жорсткість оцінювалася аналогічним чином. Для порівняння з чисельними значеннями використовувалися аналітичні методи Гросса, Вагла, Будріка, Тимошенко і Пономарьової. Моделювання в програмному пакеті ANSYS проводилося для розрахунку вертикальної жорсткості потрійних пружин. Отримані дані можуть бути використані в якості вхідних даних при конструюванні циліндричних пружин. Дані пружини можуть бути встановлені в конструкціях трранспортного рухомого складу з керованим нахилом кузова в кривих. Для підвищення комфорту руху в кривих ділянках колії важливе значення відіграє конструкція пружин. Оцінка представлених методів розрахунку дозволить оцінити ефективність конкретної ресори.

Ключові слова: *ресори, жорсткість, розрахункові методи.*

Лоулова М., Суханек А., Хаусер В., Ноженко Е.С. Кравченко Е.А. Анализ расчётных методов определения жёсткости рессор

В статье рассматривается различные методы расчёта рессор. Вертикальная жесткость пружин рассчитывалась с использованием программы ANSYS и другими методами. Впоследствии результаты сравнивались. Боковая жесткость оценивалась аналогичным образом. Для сравнения с численными значениями использовались ана-

литические методы Гросса, Вагла, Будрика, Тимошенко и Пономарёвой. Моделирование в програмном пакете ANSYS проводилось для расчета вертикальной жесткости тройных пружин. Полученные данные могут быть использованы в качестве входных данных при конструировании цилиндрических пружин. Данные пружины могут быть установлены в конструкциях трранспортного подвижного состава с управляемым наклоном кузова в кривых. Для повышения комфорта движения в кривых участках пути важное значение оказывает конструкция пружин. Оценка представленных методов расчёта позволит оценить эффективность конкретной рессоры.

Ключевые слова: *рессоры, жёсткость, расчётные методы.*

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Стаття подана 14.03.2017