12. De Oliveira, W. A. Creation from organizational project management: a case study in a government agency [Text] / W. A. De Oliveira, C. F. De Muylder Malue // Journal of Information Systems and Technology Management. – 2012. – Vol. 9, Issue 3. – P. 497–514.
13. Lytvynov, V. V. Formuvannia i pidvyshchennia kultury informatsiinoi bezpeky orhanizatsii [Text] / V. V. Lytvynov, O. V. Trunova, M. M. Voitsekhovska // Stvorennia ta modernizatsiia ozbroiennia i viiskovoi tekhniky v suchasnykh umovakh, 2016. – P. 163–164.
14. Bushuyev, S. D. Formuvannya innovatsiynykh metodiv ta modeley upravlinnya proektamy na osnovi konverhentsiyi [Text] / S. D. Bushuyev, M. S. Dorosh // Upravlinnya rozvytkom skladnykh system. – 2015. – Vol. 23. – P. 30–37.
15. Shatihin, L. G. Strukturnyie matritsyi i ih primenenie dlya issledovaniya sistem [Text] / L. G. Shatihin. – Moscow: Mashinostroenie, 1991. – 256 p.

UDC 621:681.65
DOI: 10.15587/1729-4061.2016.85204

SYNTHESIS OF INTEGRAL QUALITY INDEX OF PARAMETRIC SYSTEM STATE IN CONDITIONS OF SITUATIONAL UNCERTAINTY

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Запропоновано ступінь невизначеності стану параметричної системи чисельно оцінювати середньою кількістю інформації на виході досліджуваної системи, використовуючи модифіковану метрику Шеннона. На основі інформаційно-ентропійного підходу синтезовано інтегральний показник якості стану системи, отримані загальна і часткова аналітичні форми його представлення. Оцінено втрати інформації про поточний стан параметричної системи в умовах впливу зовнішніх збурень і розширення спектру внутрішньосистемних збурень, включаючи також багатопараметричний вектор адаптивного управління

Ключові слова: інформаційно-ентропійний підхід, інтегральний показник, зовнішні збурення, внутрішньосистемні збурення, ситуаційна невизначеність, інформаційні втрати

Предложено степень неопределенности состояния параметрической системы численно оценивать средним количеством информации на выходе исследуемой системы, используя модифицированную метрику Шеннона. На основе информационно-энтропийного подхода синтезирован интегральный показатель качества состояния системы, получены общая и частная аналитические формы его представления. Оценены потери информации о текущем состоянии параметрической системы в условиях воздействия внешних возмущений и расширения спектра внутрисистемных возмущений, включая и многопараметрический вектор адаптивного управления

Ключевые слова: информационно-энтропийный подход, интегральный показатель, внешние возмущения, внутрисистемные возмущения, ситуационная неопределенность, информационные потери

1. Introduction

Evaluating the quality of parametric systems' condition involves the necessity to solve multi-criteria (vector) optimization problems. The most difficult are multi-criteria problems to be solved in terms of situational (a priori) uncertainty; they belong to the class of ill-posed problems as viewed by Hadamard and Tikhonov [1–4]. In these problems, minor variations of the observed sample implementations $u_i(t)$, where $i=1, 2,..., \eta$, lead to unintended results.

Situational uncertainty in assessing a parametric system is determined by the effect of unplanned destabilizing factors of its environment – external uncertainty, and the existence of a variety of resource constraints – internal uncertainty [5–8]. Being inherent in any real system, unplanned destabilizing factors and inevitable resource constraints at the physical level are manifested in the form of random external and internal disturbances. With such a determination, the degree of situational uncertainty of a parametric system state will be different at any time.

2. Literature review and problem statement

An objective evaluation of the system optimality is based on an index that integrates a set of properties and characteristics to determine whether the parametric system meets the requirements for its use [6, 9–12]. Selection of the integral index involves establishing a criterion to assess the quality status of the system. According to the basic assumptions of the theory of system analysis [5, 11, 13, 14], an integrated or global (general) index and a criterion for assessing the quality of any technical system must be a function of all most important characteristics of the system; in the physical sense, it should be rational and easy, apt to display the quality of the system performance while solving specified tasks [9, 12, 14, 15].

Contradictory terms and conditions of forming the integral index to assess the parametric system quality is complicated by the fact that most real processes are characterized by a continuous change of parameters that determine the criteria of optimality. In such situations, the scope of decision making strategies is infinite, which limits the application of vector optimization methods to the scalar level, with the introduction of the global quality index [12, 14, 15]. The argument for such a conclusion derives directly from the analysis of complex systems of different physical nature that, despite the difference, have a common feature – the existence of a significant uncertainty in the optimized parameters. This is especially true of energy supply systems [16–19], industrial sites [20, 21], as well as transport and communication systems [22]. For example, in [16], it is postulated that the state evaluation (SE) in the electric power system includes uncertainties associated with the presence of random errors in measuring the parameters, structural inaccuracies in the transmission lines, etc. In particular, the accuracy of the system states evaluation can affect the arrangement of reference scales. The researchers analyse the effects of uncertainties and suggest an algorithm of selecting a reference scale that ensures maximum accuracy of the system evaluation. The operational principles of this type of algorithms, aimed at reducing the effects of uncertainty, should underpin advanced technology solutions, including control, communication or interconnection, and powerful computing capabilities. The authors of [17] believe that these components are among the factors that affect the reduction of uncertainties caused by the nature of alternative energy sources. The latter, being the most promising power sources in the systems of electric power production and distribution, are among the objects that are the most vulnerable in terms of a multilevel uncertainty. A powerful tool of evaluating and controlling such systems is stochastic optimization, whereas the systems themselves can be viewed as intelligent power systems of the next generation [17].

Study [18] highlights the impact of uncertainty of the system parameters in energy planning that are caused, in particular, by a climate change and energy security settings. It emphasizes that most models that are used to assess the quality of power supply processes are either too complicated or require expensive computing, especially given the uncertainties that are inherent in such systems. To solve problems of strategic planning in the energy field under uncertainties, the authors of this study suggest using mixed integer linear programming (MILP).

A reliable model that provides a possibility of controlling risks under uncertainty a day ahead is suggested in [19]. The proposed model, uncertainty is represented as a range of values with varying probability scenarios. The proposed method does not require intensive and complex calculations; moreover, it prevents losing complete information. This allows selecting an optimal strategy for dispatching thermal units and wind turbines as well as organizes a proper stock and arranges a flexible program to respond to the demand to minimize the power prediction error. Thus, the optimal strategy provides the best compromise between the maximum benefit and minimal risks of the system failure: there must be some integral quality criterion to evaluate the parameters of a technically complex power system. Similar approaches are considered when describing industrial systems. Thus, speaking of such systems' quality indices, the authors of [20] focus on the technical product systematic sense – PSS, which includes both a standalone technical product and a combination of products and services such as machine tools. The authors propose a customer-oriented approach to assess the PSS quality in the machine tool industry since it provides a systematic analysis of the existing quality criteria for products and services in the machine tool industry. These criteria are established by expert assessments provided by manufacturers of machine tools, customer surveys, and specialist literature analysis. This allows determining the relationship between the product and the service quality, as well as identifying key performance indices and the PSS benchmark quality. A similar combined view of establishing quality evaluation criteria for parametric systems is suggested in [21], which considers industrial production systems. In particular, the system describing parameters that produce elements of uncertainty, in the authors' view, should account for the complex structure of products, variety of products, and unqualified human resources. The study describes an approach that is aimed at selecting a suitable system of the enterprise resource planning (ERP) on the basis of a fuzzy hierarchy analysis and fuzzy expansion of decision-making with the help of a multi-criteria approach.

Addressing the assessment of transport systems, the authors of [22] find the problem solution in implementing the concept of intelligent transport systems (ITS). Such systems include navigation devices and motion sensors that allow the system elements both to access information in real time and to form a feedback loop. The study especially emphasizes the specifics of describing such systems, including discrepancies in the description of various information and various sources of error – the heterogeneity of data sources as well as information gathering and description. All this leads to deviations from pre-set modes of the transport system operation.

The evaluation of optimal systems based on a parameter that determines their compatibility with the requirements for their application is valuably researched in studies on drafting technical systems. This is primarily essential due to the fact that the existing objective contradictions between technologists and designers about the certainty of optimum design can not be ignored at the stage of assembling devices according to their functional purposes. Thus, study [23] emphasizes the fact that uncertainties arising in the process of assembling mechanical systems should be accounted to improve the assembly accuracy. To solve the above problem, it is suggested to use an assembly quality adaptive control system (A_QACS). Its structure takes into account technological peculiarities: identification of the assembly components, monitoring units, data collection and quality assessment units based on the technology of adaptive optimization. The authors note that owing to the latter the proposed system provides a practical and effective quality control of the assembly of mechanical systems. However, the study does not consider parametric reliability factors that underlay this stage of manufacturing. Instead, the emphasis on reliability is made in [24], which focuses on a method of optimizing complex structures by the reliability criteria with regard to uncertainty caused by insufficient input data. The last factor of uncertainty in the system assessment is associated with resource limitations. The authors suggest that this kind of uncertainty might be eliminated in the context of precise evaluation of the system by means of using the reference function for the values of fuzzy variables. This approach, as stated by the authors, provides the possibility to synthesize an optimal design. A similar approach to the functioning phase of a complex (in terms of control) technological object is considered in [25], which describes a synthesis of a speed-optimal regulator that would allow reaching the specified quality parameters of technological systems under uncertainty. The latter is related to the inability to measure the input variables that describe the state of the system, as well as the limited data sampling. The proposed regulator, as noted in the study, also provides a solution of the control problem that would be optimal in its final state. A reliable two-stage regulator that ensures minimized operating costs of the energy system is described in [26]. A characteristic feature of the solutions that are proposed in this paper is such an adjustment of the regulator at which the adjustment parameter provides a reliable solution from the point of view of minimizing uncertainty in the operation of the solar energy generation heat accumulation system. The efficiency of evaluating the proposed strategy is based on the simulation system Monte Carlo. However, it is emphasized that the proposed strategy produces results that are similar to those of the model based on intelligent control strategies – the relative deviation between the two strategies is less than 2 %.

All this reveals the presence of conflicting requirements for assessing the state of a parametric system under uncertainty and an objective problem of the system related to its inability to meet the requirements for its functionality. These circumstances largely impede an integrated assessment of the parametric system and make it impossible to formalize the quality criteria indicator for systems with sets of parameters and partial criteria. In addition, the global index of the system quality as a sum of partial indices with corresponding weight coefficients may be irrelevant as a fault of one partial index can be compensated by benefits of another one [10, 11].

The context of overcoming the problems actualizes an idea of interpreting the situational uncertainty of a parametric system as an information model in the modified Shannon's metric representation [27–29, 31, 32]. In this presentation, the procedure of using the information and entropy approach to synthesizing the integral index of quality becomes practically important. The latter reflects the degree of meeting the requirements for partial indices of quality, allows evaluating the loss of information about the quality of the system with any parametric dimension, and thus provides evidence of the global criterion index [2, 10, 13, 18, 30].

3. The purpose and objectives of the study

The purpose of the study is to use an information and entropy approach to synthesizing the integral index of quality and using it for evaluating the state of a parametric system under situational uncertainty.

To achieve this goal, it is essential to do the following tasks:

– to synthesize an integral quality index and to obtain the total and partial analytical forms of its presentation;

– to assess the loss of information about the qualitative status of a parametric system under external and internal destabilizing factors;

– to investigate the practical application of the integral quality index for evaluating the information capabilities of a parametric system under expansion of the internal disturbances range.

4. Synthesis of the integral quality index and the information entropy assessment of a system with a random parametric dimension

4. 1. Formulation of the problem and input data of the research

Broadly speaking, the information model of the parametric system state under external and internal disturbances is characterized by Shannon's a priori and a posteriori entropies H(*) before and after evaluating the η-dimensional process $U^T=[u_1(t),..., u(t), u_\eta(t)]$, which contains information about the system. Their difference shows the average amount of information at the output of the parametric system under study. While presenting a generalized criterion of uncertainty of the state of any technical system, entropy is a valid function that depends on the density of probability $p(\mathbf{U})$. Among the collectively known functions of the distribution $p(\mathbf{U})$ with the same variance σ^2, the Gaussian distribution has a maximum entropy [5, 6, 29].

By this logic, the global index of the parametric system quality can be represented by the modified Shannon information and entropy metric, in which the average amount of the obtained information on the state of the parametric system is actually determined by the value of the discontinued uncertainty [7, 28, 29, 31]:

$$I\left[f(\mathbf{V}),f(\mathbf{U})\right]=H\left[f(\mathbf{U})\right]-H\left[f(\mathbf{U})/f(\mathbf{V})\right], \tag{1}$$

where $f(\mathbf{U})$ and $f(\mathbf{V})$ are the results of the algorithm of f(*)-processed components, respectively, of the input vector process $\mathbf{U}$, (so-called vector of the system) and the vector process $\mathbf{V}$ that contains useful (true) state information about the system, a priori (unconditional) entropy $H[f(\mathbf{U})]$, and a posteriori (conditional) entropy $H[f(\mathbf{U})/f(\mathbf{V})]$.

According to the information model, the state of a parametric system and its environment is characterized by a complex of useful processes and external disturbances specified by the Gaussian M-dimensional vector $\mathbf{X}(M,1)$ with a zero mean $E(\mathbf{X})$ and the correlation matrix

$$\mathbf{Q}_X = E(\mathbf{X}\mathbf{X}^T) = \text{diag}\left(\sigma_1^2,...\sigma_i^2,...\sigma_M^2\right).$$

Here E(*) is a statistical averaging operation, σ_i^2 is a variance of the i-scalar random process $x_i(t)\in\mathbf{X}$, $i\in\overline{1,M}$; and T is the transposition index.

4. 2. A synthesis of the integral index of the system quality

In the vector-matrix form, the process at the input of the parametric system is described by a linear transformation of the vector **X**(M, 1) into an N-dimensional vector:

$$\mathbf{U} = \mathbf{K}\mathbf{X} + \mathbf{n}, \tag{2}$$

where **K**=**K**(N,M) is a rectangular (N,M)-dimensional matrix that defines the transformation of the vector **X** into N independent inputs of the system; **n**=**n**(N,1) means interconnect inner disturbances represented by an (N×1)-dimensional Gaussian column vector with the zero mean E(**n**), the one-correlation matrix E(**X n**^{T})=**0**(M, N), and the correlation matrix $\mathbf{A}_n = E(\mathbf{n}\,\mathbf{n}^T) = \sigma_0^2\mathbf{I}(N,N)$. Here **I**(*) is a single matrix and 0(*) is a zero matrix; σ_0^2 is a variance of the interconnect disturbances.

Combining the Gaussian processes **X** and **n** into an advanced vector **Z** of the dimension

$$\mathbf{Z}^T = \mathbf{Z}^T(1,M+N) = \left[\mathbf{X}(1,M) \quad \mathbf{n}(1,N)\right],$$

let us represent vector-matrix combination (2) as follows: **U=GZ**. This transformation matrix **G**=**G**(N,M+N) of the vector **Z** is equal to **G**=[K(N,M) **I**(N,N)]. By definition, the characteristic function of the Gaussian process **Z** with the zero mean E(**Z**) and the $\mathbf{Q}_Z$ matrix has the following form:

$$F_Z(i\ ) = E\left[\exp(j\boldsymbol{\mu}^T\mathbf{Z})\right] = \exp(-0{,}5\boldsymbol{\mu}^T\mathbf{Q}_Z\boldsymbol{\mu}), \tag{3}$$

where **μ** is the vector parameter function.

Hence, the characteristic function of the **U** vector in equation (2) takes the form:

$$F_U(\boldsymbol{\mu}) = E\left[\exp(j\boldsymbol{\mu}^T\mathbf{U})\right] = E\left[\exp(j\boldsymbol{\mu}^T\mathbf{G}\mathbf{Z})\right].$$

After performing appropriate transformations, we obtain the following analytical expression:

$$F_U(\boldsymbol{\mu}) = F_Z(\mathbf{G}^T\boldsymbol{\mu}) = \exp(-0{,}5\boldsymbol{\mu}^T\mathbf{G}\mathbf{Q}_Z\mathbf{G}^T\boldsymbol{\mu}). \tag{4}$$

According to (4), the linear combination of the initial Gaussian vector processes at the N-dimensional input to the parametric system (2) will have a Gaussian distribution with the zero mean and a correlation matrix of the following form:

$$\mathbf{A} = \mathbf{G}\,\mathbf{Q}_Z\mathbf{G}^T = \mathbf{K}\mathbf{Q}_X\,\mathbf{K}^T + \sigma_0^2\mathbf{I}. \tag{5}$$

Based on the initial specification of the components of the vector **X** and the structure of the transformation matrix **K**, correlation matrix (5) can be represented by the sum of the matrices of the useful information process $\mathbf{A}_V$, external disturbances $\mathbf{A}_j$, and interconnect disturbances $\mathbf{A}_n = \sigma_0^2\mathbf{I}$, namely:

$$\mathbf{A} = \mathbf{A}_V + \mathbf{A}_j + \mathbf{A}_n. \tag{6}$$

The solving of the multitask problem involves a transformation of the vector of observations f(**U**) during which it is possible to reach an extreme of the given optimality criterion J(f(**U**)). For a broad class of information problems, the transformation f(**U**) is implemented by the pattern of the weighted summation [5]:

$$f(\mathbf{U}) = \mathbf{W}^T\mathbf{U} = U_\Sigma, \tag{7}$$

where **W** is a multi-parameter vector of control, which is formed in the technical system by the surveillance results on the η-dimensional process **U**.

In this case, the characteristic function of the random process U_Σ=f(**U**) at the output of the N-dimensional system will be related to some scalar argument β by the following relationship:

$$F_\Sigma(\beta) = E\left\{\exp\left[j\beta f(\mathbf{U})\right]\right\} = E\left[\exp(j\beta\mathbf{W}^T\mathbf{U})\right]. \tag{8}$$

By defining a=β**W**, dependence (8) can be represented as $F_\Sigma(\beta)=F_U(a)$. The definition of the characteristic function of the Gaussian process **U** suggests that $F_U(a)=\exp(-0.5a^TAa)$. Therefore,

$$F_U(\beta\mathbf{W}) = \exp(-0{,}5\,\beta^2\mathbf{W}^T\mathbf{A}\mathbf{W}).$$

Given that the variance of the scalar random process at the output of the multi-parameter system is

$$\sigma_\Sigma^2 = \sigma_\Sigma^2(\mathbf{W}) = \mathbf{W}^T\mathbf{A}\mathbf{W},$$

characteristic function (8) of the output process U_Σ can be represented as follows:

$$F_\Sigma(\beta) = \exp\left(-0{,}5\,\beta^2\sigma_\Sigma^2\right). \tag{9}$$

In summarizing the presented arguments, it can be concluded that the hypothesis on the Gaussian distribution of the vector process **U** at the output of the researched system also facilitates observing the scalar Gaussian process U_Σ with the zero mean and the variance

$$\sigma_\Sigma^2 = E\left[f^2(\mathbf{U})\right] = \mathbf{W}^T\mathbf{A}\mathbf{W},$$

and the law of its distribution is described with the function

$$p(U_\Sigma) = (\sqrt{2\pi}\sigma_\Sigma)^{-1}\exp(-0{,}5U_\Sigma^2/\sigma_\Sigma^2).$$

The latter is a necessary and sufficient condition for a potential entropic measure of the random process U_Σ at the output of the N-dimensional parametric system:

$$H(U_\Sigma) = H\left[f(\mathbf{U})\right] = -\int_{-\infty}^{\infty} p(U_\Sigma)\lg\left[p(U_\Sigma)\right]dU_\Sigma = 0{,}5\lg\left(2\pi e\mathbf{W}^T\mathbf{A}\mathbf{W}\right). \tag{10}$$

If in (10) the correlation matrix **A** is replaced by the sum of matrices (6), the result is

$$H(U_\Sigma) = 0{,}5\lg\left[\mathbf{W}^T(\mathbf{A}_V + \mathbf{A}_j + \mathbf{A}_n)\mathbf{W}\right] + C, \tag{11}$$

where C=0.5lg(2pe) is a constant factor.

Equation (11) implies that the entropy of the information process at the output of any technical system depends on the value of the control vector **W**. Based on the assumption of the ergodicity and independence of the processes observed and in view of the transformation (7) and its projection onto the plane of the beneficial processes $f(\mathbf{V})=\mathbf{W}^{T}V=V_{\Sigma}$, expression (1) can be rewritten as

$$I\left(V_{\Sigma},U_{\Sigma}\right)=H(U_{\Sigma})-H(U_{\Sigma}/V_{\Sigma}), \tag{12}$$

where $H(U_{\Sigma}/V_{\Sigma})$ is a posteriori entropy of the output process of the parametric system, which is equal to the entropy of the interconnect disturbances

$$H(U_{\Sigma}/V_{\Sigma})=0{,}5\lg\left[\mathbf{W}^{T}(\mathbf{A}_{j}+\mathbf{A}_{n})\mathbf{W}\right]+C. \tag{13}$$

If in expression (12) the decreased and the subtrahend are replaced by their values (11) and (13), and a number of changes are made, the average number of information discrepancies between the useful process V_{Σ} and the process U_{Σ} at the output of the system can be determined as follows

$$I\left(V_{\Sigma},U_{\Sigma}\right)=0{,}5\lg\left\{\mathbf{W}^{T}\mathbf{A}\,\mathbf{W}\left[\mathbf{W}^{T}(\mathbf{A}_{j}+\mathbf{A}_{n})\mathbf{W}\right]^{-1}\right\}. \tag{14}$$

Taking into account (6), analytical expression (14) is reduced to the following:

$$I\left(V_{\Sigma},U_{\Sigma}\right)=$$
$$=0{,}5\lg\left\{1+\mathbf{W}^{T}\mathbf{A}_{V}\mathbf{W}\left[\mathbf{W}^{T}(\mathbf{A}_{j}+\mathbf{A}_{n})\mathbf{W}\right]^{-1}\right\}, \tag{15}$$

where $\mathbf{W}^{T}\mathbf{A}_{v}\mathbf{W}[\mathbf{W}^{T}(\mathbf{A}_{j}+\mathbf{A}_{n})\mathbf{W}]^{-1}$ is a relative parameter of the degree of exceeding the useful information process over the processes of interconnect and external perturbations at the output of the parametric system.

Analytical expression (15), reduced to the level of interconnect disturbances, eventually takes the form

$$I\left(V_{\Sigma},U_{\Sigma}\right)=0{,}5\lg\left\{1+\xi_{V}(\mathbf{W})\left[1+\xi_{j}(\mathbf{W})\right]^{-1}\right\}, \tag{16}$$

where $\xi_{v}(\mathbf{W})$ and $\xi_{v}(\mathbf{W})$ are relative parameters that determine, respectively, the degree of exceeding the value of the useful process and the external disturbances over the processes of interconnect perturbations at the output of the parametric system.

Analytical expression (16) can help determine the theoretical limit of the integrated assessment of the average amount of information as an indicator of the current state of the technical system of any arbitrary parametric dimension, provided that the interconnect process with a fixed dispersion value has the highest entropy. The latter is adequate to the maximum loss of information at the output of the parametric system. In terms of the substantive features, the synthesized indicator $I(V_{\Sigma}, U_{\Sigma})$ meets all primary requirements for the global (generalized) indicator of the criterion of evaluating the status quality of a technical system [7, 33].

The analysis of the nature of the practical problems that are associated with evaluating the status quality of parametric systems shows the possibility of using the resulting rate criterion under circumstances where the multi-parameter control vector **W** takes a unit value of $\mathbf{W}=\mathbf{I}(N,1)$. In this case, analytical expression (16) takes the partial form

$$I\left(V_{\Sigma},U_{\Sigma}\right)=0{,}5\lg\left(1+N\frac{\xi_{V}(\mathbf{W})}{1+\xi_{j}(\mathbf{W})}\right). \tag{17}$$

When the status quality of a technical system is evaluated in the context of the Wiener problem solving, the multi-parameter vector of control takes the following form: $\mathbf{W}=A^{-1}\mathbf{r}$. Here $\mathbf{r}=E(U_{0}\mathbf{U})$ is the vector of the cross-correlation between the scalar process of the objective function (standard) U_{0} and the status vector of the system **U**. In this case, the formula of the criterion factor for the status quality of the parametric system (15) obtains another partial perspective [7, 31]:

$$I\left(V_{\Sigma},U_{\Sigma}\right)=0{,}5\lg\left[1+\frac{\mathbf{r}^{T}\mathbf{A}^{-1}\mathbf{A}_{V}\mathbf{A}^{-1}\mathbf{r}}{\mathbf{r}^{T}\mathbf{A}^{-1}(\mathbf{A}_{j}+\mathbf{A}_{n})\mathbf{A}^{-1}\mathbf{r}}\right]. \tag{18}$$

Due to the logic of practical implementation, the integral quality factor (16) and the analytical expressions of its partial forms (17) and (18) represent the function of measuring the uncertainty degree of the parametric system status, its dependence on the structural characteristics of the system, and a set of matrix parameters that reflect the processes of the actual status of the system $\mathbf{A}_{v}$, external disturbances $\mathbf{A}_{j}$, and interconnect perturbations $\mathbf{A}_{n}$. The contents of ratios (16)–(18) are the "substance" of the mathematical model, i. e. a formal image of a statistical description of the evaluation procedure concerning the information status of a parametric system with the highest degree of approximation to the actually existing environment.

5. Discussion of the results: assessment of information losses and system opportunities under the influence of external disturbances and interconnect perturbations

5. 1. Evaluation of information losses on the parametric system status with fixed dimensions and under the influence of external disturbances

In terms of hardware and software, the modelling is done in the environment of the MATLAB complex. The results of the simulation are shown in Fig. 1.

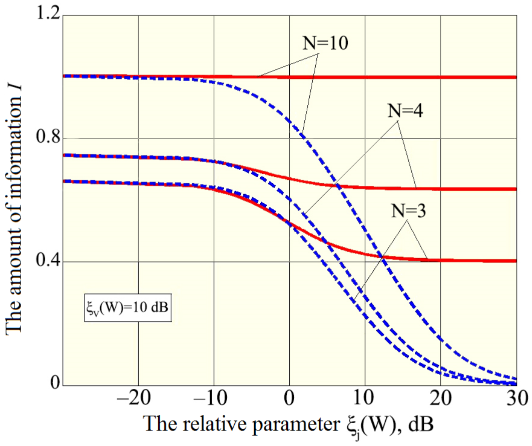

Fig. 1. The dynamics of information losses on the qualitative status of an N-dimensional parametric system subject to the influence of external disturbances

The presented graphic images of the two families of curves reflect changes in the amount of information at the output of the researched system $I(V_\Sigma, U_\Sigma)$ depending on the excess value of external disturbances over the intersystem disturbances $\xi j(\mathbf{W})$ at the input of the system if the parameter $\xi_V(\mathbf{W})$=10 dB is fixed and the dimensions of the system are N=3; 4; 10. The dashed lines show the curves $I(V_\Sigma, U_\Sigma)= =\varphi[\xi_J(\mathbf{W})]$ that, according to (17), illustrate information losses in the system in the absence of an adaptive management regime. The solid lines show the curves $I(V_\Sigma, U_\Sigma)= =\psi[\xi_J(\mathbf{W})]$ that, according to (18), illustrate information losses if there is an adaptive management regime.

The results of evaluating information entropy by analysing the series of curves (Fig. 1) show that the level of information loss is significantly reduced by introducing elements of adaptive control over the parametric system characteristics if their mode of operation corresponds to the dynamics of external destabilizing factors. A somewhat optimal status or behaviour of a technical system can be achieved at a priori uncertainty, which means that the level of these losses can be reduced to a minimum by increasing the dimension of the system N. However, this solution leads to the system redundancy (N>>M) and to an increased defect of the correlational observation matrix **A**.

In conclusion, it should be noted that external system uncertainties can be overcome by implementing the potential possibilities of introduced elements of adaptive management; it is guaranteed by the presence of extremely compelling interconnect disturbances. Based on this assumption, unstable information estimates on the quality status of a parametric system by other criteria are excluded from consideration [7, 11, 13, 33, 34]. Meanwhile, it is not denied that the obtained criterion estimates are sensitive to the level of interconnect disturbances of the system multi-parameter control vector **W**.

5. 2. Using an integral indicator for evaluating information capabilities of a system under interconnect disturbances

It is advisable that information capabilities of a parametric system in extending the range of interconnect disturbances be researched by a variational-parametric method [1–3, 6, 11]. Deviations in the system status quality parameters with elements of adaptive management from the optimal value are associated primarily with random variations $\Delta\mathbf{W}$ of the parametric vector **W**. These variations most adequately reflect the inner uncertainty of the state of any technical system regardless of the characteristics of a specific physical problem. An analytical generalization of the inner uncertainty of the state of a parametric system can be represented by an excited parametric vector of the following type

$$\hat{\mathbf{W}} = F(\Delta\mathbf{W}) = \mathbf{W} + \Delta\mathbf{W}, \qquad (19)$$

where the variation $\Delta\mathbf{W}=\Delta\mathbf{W}(N,1)$ is a random vector of interconnect disturbances with the distribution $p(\Delta\mathbf{W})$, which is characterized by the zero mean $E(\Delta\mathbf{W})$, the relative variance

$$\sigma_W^2 = \|\Delta\mathbf{W}\|^2 \Big/ \left(N\|\mathbf{W}\|^2\right),$$

and the correlation matrix

$$E\,(\Delta\mathbf{W}\Delta\mathbf{W}^T) = \sigma_W^2 \|\mathbf{W}\|^2 \cdot \mathbf{I}(N,N).$$

Here ||*|| is the vector norm.

Given the accepted notation, the variance of the random process U_Σ at the output of the system under inner uncertainty is equal to

$$\hat{\sigma}_\Sigma^2 = \iint_\infty \left[F(\Delta\mathbf{W})\right]^T \mathbf{U}\mathbf{U}^T \left[F(\Delta\mathbf{W})\right] p(\mathbf{U},\Delta\mathbf{W})\, d\mathbf{U}\, d(\Delta\mathbf{W}), \qquad (20)$$

where $p(\mathbf{U}, \Delta\mathbf{W})$ is the joint probability density of the random processes U and $\Delta\mathbf{W}$.

Let us integrate expression (20) while using the introduced notation of the statistical points of the vector $\Delta\mathbf{W}$ and the factorized idea

$$p(\mathbf{U},\Delta\mathbf{W}) = p(\mathbf{U})p(\Delta\mathbf{W}),$$

resulting from the natural condition of the statistical independence of the random vectors **U** and $\Delta\mathbf{W}$. The result of the integrating operation is

$$\hat{\sigma}_\Sigma^2 = \mathbf{W}^T\mathbf{A}\mathbf{W} + \sigma_W^2 \|\mathbf{W}\|^2 \cdot \mathrm{tr}\,\mathbf{A}, \qquad (21)$$

where tr**A** is the matrix trace.

With the independence of the vector processes, expression (21) helps determine the entropy of disturbance (noise) at the output of the parametric system with the excited parametric vector (19) as a posteriori entropy

$$H(\hat{U}_\Sigma / \hat{V}_\Sigma) = 0{,}5\lg\left[\,\mathbf{W}^T(\mathbf{A}_j + \mathbf{A}_n + \sigma_W^2 \cdot \mathrm{tr}\,\mathbf{A}\cdot\mathbf{I})\,\mathbf{W}\right] + C. \qquad (22)$$

The difference between entropies (22) and (13) shows that if there is some inner uncertainty the entropy of perturbations (noise) at the output of the parametric system with the excited parametric vector of the adaptive control exceeds the same indicator of the quality of an unperturbed system by the value of

$$\Delta H = 0{,}5\lg\left[1 + \sigma_W^2 \cdot \mathrm{tr}\,\mathbf{A}\cdot\|\mathbf{W}\|^2 (\mathbf{W}^T\mathbf{A}\mathbf{W})^{-1}\right]. \qquad (23)$$

From the information point of view, this is equivalent to providing the N-dimensional input in a parametric system with a perturbed process in the form of some “white” noise pattern, with its level depending on the parametric variance of interconnect perturbations of the system vector σ_W^2.

For the Wiener solution, the information amount at the output of the researched system with elements of adaptive management and an excited parametric vector (19) is determined by the expression:

$$I\left(\hat{V}_\Sigma, \hat{U}_\Sigma\right) = H(\hat{U}_\Sigma) - H(\hat{U}_\Sigma / \hat{V}_\Sigma) =$$
$$= 0{,}5\lg\left[1 + \frac{\mathbf{r}^T\mathbf{A}^{-1}\mathbf{A}_V\,\mathbf{A}^{-1}\,\mathbf{r}}{\mathbf{r}^T\mathbf{A}^{-1}(\mathbf{A}_j + \mathbf{A}_n + \sigma_W^2\,\mathrm{tr}\,\mathbf{A}\cdot\mathbf{I})\,\mathbf{A}^{-1}\mathbf{r}}\right]. \qquad (24)$$

In analytical expression (24), the excess of the trace of the correlation matrix of observations above the interconnect disturbance variance σ_0^2 is as follows:

$$\mathrm{tr}\,\mathbf{A}/\sigma_0^2 = N(\xi_V(\mathbf{W}) + \xi_j(\mathbf{W}) + 1).$$

The degree of this excess determines the sensitivity of both the parametric system and its integrated status indicator to the level of dispersion of interconnect disturbances, which are defined as the values of σ_0^2 and σ_W^2. Therefore, if there are inner perturbations, the growth of the parameters of an external obstacle $\xi_j(\mathbf{W})$, when $\xi_j(\mathbf{W}) >> \xi_v(\mathbf{W})$ and $\sigma_W^2 = \text{const}$, reduces the amount of information $I(\hat{V}_\Sigma, \hat{U}_\Sigma)$. In the perturbation theory [6, 33, 34], this phenomenon is explained by an increased sensitivity of the Hermitian matrix with a bad conditionality and a large defect as to random variations of its elements.

The amount of information at the output of N-dimensional parametric systems with adaptive control elements and the relative parameter $\xi_j(\mathbf{W})$ in terms of intra-parametric perturbations of the control vector are related to the dependence

$$I(\hat{V}_\Sigma, \hat{U}_\Sigma) = \gamma(\xi_j(\mathbf{W}).$$

Fig. 2 shows families of graphic curves that reflect this dependence at fixed values of the parameter $\xi_v(\mathbf{W})$=10 dB, the system dimension N=3; 4; 10, and the interconnect perturbation variance of the parametric control vector $\sigma_W^2 = 10^{-2}$.

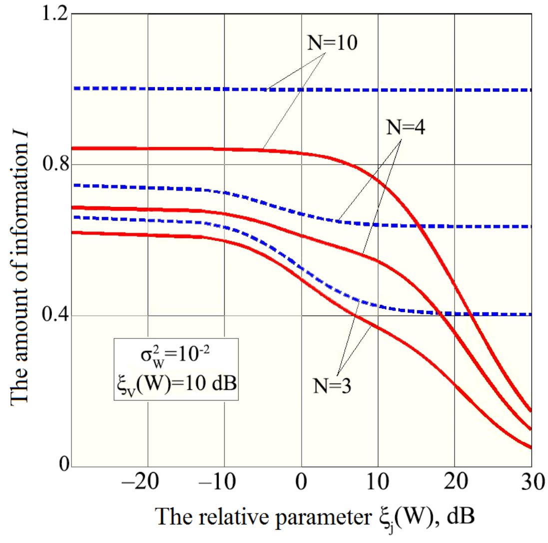

Fig. 2. The dynamics of information losses on the qualitative status of an N-dimensional parametric system subject to interconnect disturbances of the parametric control vector σ

The family of curves that are graphically marked by solid lines are obtained under interconnect disturbances of the parametric vector ($\sigma_W^2 \neq 0$), and the dashed lines indicate their absence ($\sigma_W^2 = 0$).

A comparison of the dynamics of the graphic curves in Fig. 2 reveals that slight external disturbances $\xi_j(\mathbf{W}) \leq$ ≤ -10 dB do not virtually change the information amount at the output of the parametric system with elements of adaptive management, but there are information losses relative to a situation when $\sigma_W^2 = 0$. In case of some inner uncertainty in the system, information losses increase significantly if $\xi_j(\mathbf{W}) \geq 10$ dB and if there is a dimensional increase of the parametric system. It is also noteworthy that a loss of information about the qualitative state of a parametric system significantly increases if there is an increased dispersion of interconnect perturbations of the parametric control vector σ_W^2. Consequently, the slope of the curves that are marked as solid lines in Fig. 2 increases substantially.

6. Conclusions

1. In the study, the situational uncertainty of the state of a parametric system is interpreted as an information model in a modified Shannon metric representation, and an information entropy approach is used to synthesize the integral indicator of quality, taking into account the total and partial analytical forms of determining it. The forms function to represent a measure of the status uncertainty of a system with any parametric dimension to assess:

– first, the theoretical limit of the average amount of information at the output of the researched system with any parametric dimension;

– secondly, the maximum loss of information at the output of a parametric system, provided that the interconnect process with a fixed dispersion value has the highest entropy.

2. The results of the entropy evaluation illustrate a significant reduction of information losses concerning the parametric system state in case of implementing adaptive management that is consistent, in time and space, with the dynamics of external destabilizing factors. The implementation of the potential capabilities of the adaptive management regime is guaranteed by the presence of extremely compelling interconnect disturbances.

3. The sensitivity of the multi-parameter vector of adaptive management to interconnect disturbances increases with an increased defect of the correlation matrix of observations.

References

1. Tihonov, A. N. Metody resheniya nekorrektnyh zadach. 2-e izd. [Text] / A. N. Tihonov, V. Ya. Arsenin. – Moscow: Nauka: Glavnaya redakciya fiziko-matematicheskoj literatury, 1979. – 285 p.
2. Vasin, V. V. Nekorrektnye zadachi s apriornoj informaciej [Text] / V. V. Vasin, A. L. Ageev. – Ekaterinburg: UIF «Nauka», 1993. – 262 p.
3. Greshilov, A. A. Nekorrektnye zadachi cifrovoj obrabotki informacii i signalov [Text] / A. A. Greshilov. – Moscow: Radio i svyaz', 1984. – 160 p.
4. Leonov, A. S. Obobshchenie metoda maksimal'noj ehntropii dlya resheniya nekorrektnyh zadach [Text] / A. S. Leonov // Sibirskij matematicheskij zhurnal. – 2000. – Vol. 41, Issue 4. – P. 863–872.
5. Repin, V. G. Statisticheskij sintez pri apriornoj neopredelyonnosti i adaptaciya informacionnyh sistem [Text] / V. G. Repin, G. P. Tartakovskij. – Moscow: Sov. radio, 1977. – 404 p.
6. Petrov, V. V. Informacionnaya teoriya slozhnyh sistem, funkcioniruyushchih v usloviyah nepolnoj informacii. V kn.: Itogi nauki i tekhniki. «Tekhnicheskaya kibernetika». T. 13 [Text] / V. V. Petrov, V. M. Ageev, A. V. Zaporozhec, A. V. Kort'ev, V. M. Kostyukov, S. B. Medvedev, I. N. Polyakov. – Moscow:VINITI, 1980. – P. 121–137.

7. Skachkov, V. V. Ehntropijnyj podhod k issledovaniyu informacionnyh vozmozhnostej adaptivnoj radiotekhnicheskoj sistemy pri vnutrisistemnoj neopredelennosti [Text] / V. V. Skachkov, V. V. Chepkij, G. D. Bratchenko, A. N. Efimchikov // Izvestiya vuzov. Radioehlektronika. – 2015. – Vol. 58, Issue 6. – P. 3–12.
8. Kurzhanskij, A. B. Upravlenie i nablyudenie v usloviyah neopredelennosti [Text] / A. B. Kurzhanskij. – Moscow: Glavnaya redakciya fiziko-matematicheskoj literatury izdatel'stva «Nauka», 2006. – 511 p.
9. Ventcel', E. S. Vvedenie v issledovanie operacij [Text] / E. S. Ventcel'. – Moscow: Sovetskoe radio, 1964. – 390 p.
10. Skorik, B. I. K voprosu o teorii informacionnogo vzaimodejstviya slozhnyh tekhnicheskih sistem [Text] / B. I. Skorik, A. B. Skorik, A. A. Zverev // Sistemi upravlinnya, navigacії ta zv'yazku. – 2009. – Vol. 4 (12). – P. 116–119.
11. Chumakov, N. M. Ocenka ehffektivnosti slozhnyh tekhnicheskih ustrojstv [Text] / N. M. Chumakov, E. I. Serebryanyj. – Moscow: Sov. radio, 1980. – 192 p.
12. Podinovskij, V. V. Optimizaciya po posledovatel'no primenyaemym kriteriyam [Text] / V. V. Podinovskij, V. M. Gavrilov. – Moscow: Sov. radio, 1975. – 192 p.
13. Kini, R. L. Prinyatie reshenij pri mnogih kriteriyah: predpochteniya i zameshcheniya [Text] / R. L. Kini, H. Rajfa. – Moscow: Radio i svyaz', 1981. – 560 p.
14. Gig, Dzh. Van. Prikladnaya obshchaya teoriya sistem [Text] / Dzh. Van Gig. – Moscow: Mir, 1981. – 336 p.
15. Dulesov, A. S. Svojstva ehntropii tekhnicheskoj sistemy [Text] / A. S. Dulesov, M. Yu. Semenova, V. I. Hrustalev // Fundamental'nye issledovaniya. – 2011. – Vol. 8. – P. 631–636.
16. Caro, E. Uncertainty analysis of power system state estimates and reference bus selection [Text] / E. Caro // Electric Power Systems Research. – 2016. – Vol. 136. – P. 322–330. doi: 10.1016/j.epsr.2016.03.032
17. Alqurashi, A. Treatment of uncertainty for next generation power systems: State-of-the-art in stochastic optimization [Text] / A. Alqurashi, A. H. Etemadi, A. Khodaei // Electric Power Systems Research. – 2016. – Vol. 141. – P. 233–245. doi: 10.1016/j.epsr.2016.08.009
18. Moret, S. Strategic Energy Planning under Uncertainty: a Mixed-Integer Linear Programming Modeling Framework for Large-Scale Energy Systems [Text] / S. Moret, M. Bierlaire, F. Mar chal // 26th European Symposium on Computer Aided Process Engineering, 2016. – P. 1899–1904. doi: 10.1016/b978-0-444-63428-3.50321-0
19. Ji, L. Inexact stochastic risk-aversion optimal day-ahead dispatch model for electricity system management with wind power under uncertainty [Text] / L. Ji, G.-H. Huang, L.-C. Huang, Y.-L. Xie, D.-X. Niu // Energy. – 2016. – Vol. 109. – P. 920–932. doi: 10.1016/j.energy.2016.05.018
20. Mert, G. Quality Assessment of Technical Product-service Systems in the Machine Tool Industry [Text] / G. Mert, S. Waltemode, J. C. Aurich // Procedia CIRP. – 2014. – Vol. 16. – P. 253–258. doi: 10.1016/j.procir.2014.06.002
21. Cebeci, U. Fuzzy AHP-based decision support system for selecting ERP systems in textile industry by using balanced scorecard [Text] / U. Cebeci // Expert Systems with Applications. – 2009. – Vol. 36, Issue 5. – P. 8900–8909. doi: 10.1016/j.eswa.2008.11.046
22. Litescu, S. C. The effect of information uncertainty in road transportation systems [Text] / S. C. Litescu, V. Viswanathan, H. Aydt, A. Knoll // Journal of Computational Science. – 2016. – Vol. 16. – P. 170–176. doi: 10.1016/j.jocs.2016.04.017
23. Wang, X. Research on assembly quality adaptive control system for complex mechanical products assembly process under uncertainty [Text] / X. Wang, M. Liu, M. Ge, L. Ling, C. Liu // Computers in Industry. – 2015. – Vol. 74. – P. 43–57. doi: 10.1016/j.compind.2015.09.001
24. Du, L. Possibility-Based Design Optimization Method for Design Problems With Both Statistical and Fuzzy Input Data [Text] / L. Du, K. K. Choi, B. D. Youn, D. Gorsich // Journal of Mechanical Design. – 2006. – Vol. 128, Issue 4. – P. 928–935. doi: 10.1115/1.2204972
25. Demin, D. A. Synthesis of optimal temperature regulator of electroarc holding furnace bath [Text] / D. A. Demin // Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu. – 2012. – Vol. 6. – P. 52–58.
26. Wang, C. Robust scheduling of building energy system under uncertainty [Text] / C. Wang, B. Jiao, L. Guo, Z. Tian, J. Niu, S. Li // Applied Energy. – 2016. – Vol. 167. – P. 366–376. doi: 10.1016/j.apenergy.2015.09.070
27. Shennon, K. Raboty po teorii informacii i kibernetike [Text] / K. Shennon. – Moscow: In. lit., 1963. – 827 p.
28. Manko, G. I. Informacionnyj podhod k ocenke neopredelennosti upravlyayushchih vozdejstvij [Text] / G. I. Manko, I. G. Kayun // Voprosy himii i himicheskoj tekhnologii. – 2008. – Vol. 1. – P. 180–182.
29. Vil'son, A. Dzh. Ehntropijnye metody modelirovaniya slozhnyh sistem [Text] / A. Dzh. Vil'son. – Moscow: Glavnaya redakciya fiziko-matematicheskoj literatury izdatel'stva «Nauka», 1978. – 248 p.
30. Novichihina, T. I. Ehntropijnyj podhod kak neot"emlemyj atribut polucheniya informacionnyh znanij [Text] / T. I. Novichihina // Mir nauki, kul'tury, obrazovaniya. – 2009. – Vol. 6 (18). – P. 168–173.
31. Mazur, M. Kachestvennaya teoriya informacii [Text] / M. Mazur. – Moscow: Mir, 1974. – 238 p.
32. Skachkov, V. V. Ehntropijnoe ocenivanie vliyaniya vnutrisistemnyh vozmushchenij na informacionnye vozmozhnosti adaptivnoj radiotekhnicheskoj sistemy. In 4 volumes. Vol. 4 [Text] / V. V. Skachkov, V. V. Chepkij, A. N. Efimchikov, G. D. Bratchenko, V. I. Pavlovich // Integrirovannye informacionnye radioehlektronnye sistemy i tekhnologii, 2014. – P. 55–59.
33. Cheremisin, O. P. Ehffektivnost‘ adaptivnogo algoritma s regulyarizaciej vyborochnoj korrelyacionnoj matricy [Text] / O. P. Cheremisin // Radiotekhnika i ehlektronika. – 1982. – Vol. 27, Issue 10. – P. 1933–1942.
34. Gorban‘, I. I. Fenomen statisticheskoj ustojchivosti [Text] / I. I. Gorban‘. – Kyiv: Naukova dumka, 2014. – 444 p.