

*Робота присвячена дослідженню впливу нерівномірної подачі сипкої суміші на процес завантаження віброрешета. Встановлені закономірності товщини шару, поздовжньої та поперечної складових швидкості, щільності сипкої суміші та питомого завантаження на всій площі поверхні віброрешета при нерівномірній подачі на вході. Нерівномірність подачі задавалась опуклим, увігнутим та трикутним профілем початкової швидкості по ширині на вході решета.*

*При розглянутих профілях характеристики потоку змінюються однаково по довжині решета. По ширині решета характеристики потоку змінюються у відповідності з профілем початкової швидкості. При опуклому профілі – товщина постійна, поверхнева щільність та поздовжня складова швидкості найбільші вздовж поздовжньої осі решета та найменші біля бокових стінок, поперечна складова швидкості напрямлена від поздовжньої осі до бокових стінок. При увігнутому профілі – товщина постійна, поверхнева щільність та поздовжня складова швидкості найбільші вздовж бокових стінок, найменші вздовж поздовжньої осі, поперечна складова швидкості напрямлена від бокових стінок до осі. При трикутному профілі – товщина постійна, поверхнева щільність та поздовжня складова швидкості найбільші вздовж однієї бокової стінки і найменші біля протилежної, поперечна складова швидкості напрямлена до однієї бокової стінки.*

*При опуклому профілі початкової швидкості відбувається перевантаження поверхні вздовж поздовжньої осі решета, та недовантаження біля бокових стінок. При увігнутому профілі – поверхня перевантажена біля бокових стінок, а вздовж поздовжньої осі решета недовантажена. При трикутному профілі – поверхня перевантажена вздовж однієї бокової стінки, і недовантажена вздовж протилежної. Найбільші відхилення питомого завантаження мають місце біля вхідного перерізу потоку, найменші – біля вихідного.*

*Закономірності розподілу питомого завантаження решета є визначальними при проектуванні живильників і розподільників сипких сумішей та розрахунках режимів сепарування*

*Ключові слова: вібраційне решето, сипка суміш, питоме завантаження поверхні, нерівномірна подача суміші*

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# RESEARCH INTO THE PROCESS OF LOADING THE SURFACE OF A VIBROSIEVE WHEN A LOOSE MIXTURE IS FED UNEVENLY

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## 1. Introduction

Loading of grain separators, including feeding and distribution of the material processed on the surface of a sieve, is one of the factors that ensure quality and productivity of the separation process. Authors of most studies [1–8] on motion and separation of loose mixtures assume that inlet feeding is uniform and there are no investigations on changes in load of a sieve. However, deviation of specific load from the normative value leads to the occurrence of areas of overload or underload of the working surface. This reduces efficiency of separation. The unevenness of distribution of a loose mixture

over the working surface appears due to the uneven feeding of a mixture to the sieve inlet for width [9]. The feeding of the mixture changes due to instability of operation of transporting machines, uneven falling of the mixture from accumulation bins, errors of dosing and distribution devices, heterogeneity of physical and mechanical properties of the mixture and other production factors.

Experimental studies under production conditions established that load of working bodies of grain cleaning machines is uneven for width [9]. From the whole width, 70 % of material processed falls to the central area, which is 30 % of the entire width. 10 % of a grain mixture falls to the side parts of

a sieve, which makes up 50 % of the entire width. The maximum feeding for width does not coincide with the middle of a working body, and the largest deviation from the average feed rate for width reaches 20 %.

We determine load of the working surface by characteristics of a flow, such as thickness of a layer, velocity and density of a mixture. These characteristics have a significant effect on components of the separation process, such as passage of fine particles through the layer to the sieve and sift through its openings. Consequently, first of all, the study on the process of loading of vibrating sieve requires investigation on characteristics of a flow of a mixture.

Investigation of the process of loading of vibrating sieve at unevenness of feeding different in nature will make possible to determine areas of overload or underload of a working surface, taking into account causes of their occurrence. These characteristics are decisive for the design of feeders and distributors of loose mixtures and for calculations of separation modes. Therefore, the subject of the study is relevant.

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## 2. Literature review and problem statement

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Papers [10, 11] present results of investigations on the process of separation of granular materials on sieve classifiers by size. There is a mathematical model of separation of materials based on stochastic processes constructed and intensity of sifting of particles through openings determined. The model takes into account changes in load, an effect of side walls and multi-stageness of a mixture. It shows that the sifting process is sensitive to changes in load of a working surface. But it leaves issues related to distribution of material processed over the sieve area and an influence of the initial feeding of a mixture on distribution unresolved. The reason for this may be objective difficulties associated with complexity of the task, which depends on many random factors. The mentioned facts make the relevant studies inappropriate.

Authors of work [12] note the significant effect of uniform distribution of a mixture on the surface of a sieve on the separation process. The separation process itself has stages: a disordered mode, when the initial distribution of a loose mixture plays a significant role and determines duration of the separation process; a stationary mode, when distribution of a loose mixture is uniform at all points in a layer.

Authors of paper [13] established a nature of unstable motion of a mixture at the beginning of a sieve. They determined that the nature of relative motion of mixture particles may be unstable in the loading zone of a sieve and they can move along complex curvilinear trajectories. This affects uniformity of distribution of a mixture along the perimeter of the separating surface directly and leads to overload in some areas and underload on others. Thus, the quality of separation decreases. But authors simulated motion of a material point, they did not investigate motion of a grain mixture as a continuous medium. However, application of such a model gives significant errors in calculation of motion of a layer, because its thickness exceeds sizes of individual particles. Thus, the study is inappropriate.

An option to overcome the mentioned difficulties may be application of a new method for modeling. The method describes motion of a layer of a mixture by equations of motion of viscous liquid [14, 15]. The basis of this direction is an analogy in behavior of viscous liquid and loose media under conditions of vibrations [16]. It is precisely this approach used for the study on changes in velocity of a mixture flow along the

length of a sieve at harmonic pulsations of initial velocity at a sieve inlet, which causes uneven feeding, in paper [17].

Authors established that if feeding to a sieve is uneven, the length of the uneven motion area is less than the length of the working surface of a sieve, but if the frequency decreases and the amplitude of vibrations of mixture feeding increases, the length of the area of uneven motion increases. However, authors of the paper do not consider a change in the initial velocity for width of a sieve in the paper.

Authors of work [18] developed a theory of motion of a mixture on a sieve of a finite-width to take into account an influence of side walls of a sieve on the process. Application of the Bubnov-Galerkin method made it possible to describe a boundary effect near walls with high precision and to determine average velocity of a mixture and efficiency of a sieve. But they assumed that the transverse component of velocity is equal to zero, the longitudinal velocity component does not depend on length, and the thickness of the layer is averaged over the length of a sieve. The assumed simplifications do not make it possible to investigate distribution of load over the area of the working surface.

Authors of paper [19] developed a mathematical model for adjusting of the level of a mixture on the surface of a sieve and took into account external and internal disturbances of the controlled object to stabilize and evenly distribute the flow on the surface of a sieve. Adjustment of the level of a mixture depends on the velocity of passage of particles through openings of a sieve and a volume of material that enters a bin. However, the paper did not consider uneven feeding at the inlet to a sieve.

Work [20] suggests using layer rabblers for large material feeding at the sieve inlet to improve the loading process. Rabblers redistribute a grain mixture, which falls into the operation area of rabblers, and thus, they improve conditions of separation. A work [21] proposes to use a feeder, which consists of cylindrical shovel coils of different diameters, for uniform load of the separating body. Feeder shovels have the same length, their direction is contrariwise to rotation of a shaft, and their forming is executed by brachistochrone. The alternate placement of coils on a shaft makes possible to distribute a grain mixture evenly along the width. However, the proposed designs of new working bodies require a theoretical substantiation in conjunction with studies on kinematic characteristics of a flow over the entire surface area of a sieve.

Several researchers tried to change a shape of the surface of a sieve to improve loading [22]. They developed the curvilinear form of a flat sieve. It ensures uniform filling of its working surface with a material. Changes in the local load of a sieve occurred due to variable angle of inclination of its areas to the horizon. Reduction of the angle of inclination of the sieve area led to a decrease in the velocity of movement of material in this place and, accordingly, to an increase in a degree of its filling with material. Productivity of such a sieve increased according to research results. But such a sieve shape provides the same thickness of a layer along its length and does not regulate the specific material flow rate for width.

Thus, the study of the sieve loading process did not consider an influence of uneven feeding to a sieve input for width on the loading process and kinematic characteristics of a flow over the entire surface of a sieve, which explain reasons for overload or underload of areas of the surface. Therefore, there are reasons to believe that the lack of certainty in the process makes necessary to conduct research on an effect of unevenness of feeding of a sieve for width on the process of its loading.

**3. The aim and objectives of the study**

The objective of the study is to examine an influence of uneven, in terms of width, feeding of a loose mixture to vibrating sieve on the process of its loading. This will make it possible to determine the areas of overload and underload of a sieve, their location, the magnitude of deviations from the initial load under different conditions of feeding a mixture at the inlet.

We solved the following tasks to achieve the objective:

- determination of regularities in the thickness of a layer, longitudinal and transverse velocity components, density of a loose mixture over the entire area of the surface of a vibrating sieve;
- determination of an influence of uneven, in terms of width, feeding at the inlet to a vibrating sieve on load of the working surface.

**4. Materials and methods to study the spatial motion of a loose mixture on a vibrating sieve**

It is necessary to consider spatial motion of a mixture to determine regularities in the parameters of motion of a loose mixture over the entire area of a vibrating sieve. These regularities will determine the specific load of the working surface of a sieve.

The flow of mixture enters the sieve through  $CDD_1C_1$  inlet section of the grid state (Fig. 1). It is distributed over the working surface and goes through  $ABB_1A_1$  outlet section. The state is oscillating, it is inclined at  $\theta$  angle to the horizon and it has parallel side walls, which are perpendicular to  $ADD_1A_1$  surface of the sieve. We chose the coordinate system so that its origin lies in  $CDD_1C_1$  plane, the direction of  $0x$  axis is along the length of the sieve to  $ABB_1A_1$  inlet section, the direction of  $0y$  axis is along the width of the sieve, and the direction of  $0z$  axis is along the normal to the surface of the sieve.

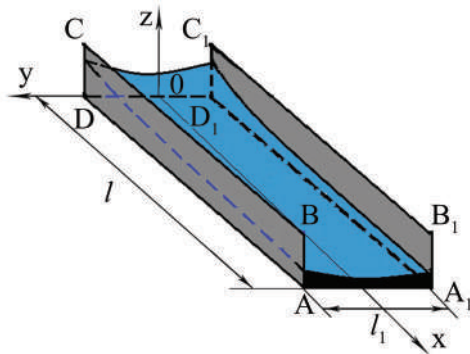


Fig. 1. Schematic of a mixture flow on the vibrating sieve

Authors of paper [23] developed a mathematical model of the spatial motion on a vibrating sieve with a finite width to study the flow dynamics of a loose mixture. They did not take into account the influence of sifting on the motion of a mixture, as the content of the passing fraction was negligible in the outlet mixture for sifting and sorting sieves. It was 2–3 % in some cases. An influence of vibrations on the medium showed itself in reduc-

tion of the internal friction at an increase of intensity of vibrations.

The equation of the dynamics of a loose mixture on a vibrating sieve takes the form:

$$\frac{\partial}{\partial t} \rho + \left( \frac{\partial}{\partial x} \rho \right) u + \rho \frac{\partial}{\partial x} u + \left( \frac{\partial}{\partial y} \rho \right) v + \rho \frac{\partial}{\partial y} v + \frac{\partial}{\partial z} (\rho w) = 0, \tag{1}$$

$$\rho \left( \frac{\partial}{\partial t} u + u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u + w \frac{\partial}{\partial z} u \right) - \frac{\partial}{\partial x} \sigma_{xx} - \frac{\partial}{\partial y} \sigma_{xy} - \frac{\partial}{\partial z} \sigma_{xz} - \rho g \sin(\theta) = 0, \tag{2}$$

$$\rho \left( \frac{\partial}{\partial t} v + u \frac{\partial}{\partial x} v + v \frac{\partial}{\partial y} v + w \frac{\partial}{\partial z} v \right) - \frac{\partial}{\partial x} \sigma_{xy} - \frac{\partial}{\partial y} \sigma_{yy} - \frac{\partial}{\partial z} \sigma_{yz} = 0, \tag{3}$$

$$\rho \left( \frac{\partial}{\partial t} w + u \frac{\partial}{\partial x} w + v \frac{\partial}{\partial y} w + w \frac{\partial}{\partial z} w \right) - \frac{\partial}{\partial x} \sigma_{xz} - \frac{\partial}{\partial y} \sigma_{yz} - \frac{\partial}{\partial z} \sigma_{zz} + \rho g \cos(\theta) = 0, \tag{4}$$

where  $u, v, w$  are the components of a velocity vector of the continuous medium;  $x, y, z$  are the coordinates of Cartesian coordinate system;  $\theta$  is the angle of inclination of a sieve;  $\rho = \gamma v$  is the density of the medium, taking into account gaps between particles;  $v$  is the volume density of particles;  $\gamma$  is the density of particles of a mixture;  $t$  is time.

Components of  $\dot{\sigma}$  stress tensor:

$$\dot{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}, \tag{5}$$

coincide with the matrix components of (3x3) order:

$$\begin{pmatrix} -p + 2\mu \frac{\partial u}{\partial x} & \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & -p + 2\mu \frac{\partial v}{\partial y} & \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) & \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) & -p + 2\mu \frac{\partial w}{\partial z} \end{pmatrix}, \tag{6}$$

where  $p$  is the pressure;  $\mu$  is the dynamic coefficient of shear viscosity.

The layer of the mixture takes the area bounded by the sieve  $\Sigma_0 (AA_1D_1D)$ , the side walls  $\Sigma_1 (A_1B_1C_1D)$  and  $\Sigma_2 (ABCD)$ , the inlet  $\Sigma_3 (AA_1B_1B)$  and the outlet  $\Sigma_4 (CC_1D_1D)$  sections, the free surface of the layer  $\Gamma$  (Fig. 2).

We calculate the thickness of the layer  $h = h(t, x, y, z)$  along the normal to the sieve to the free surface, the length of the sieve is  $l$ , the width is  $l_1$ . Then, we can determine the specified area by the following ratios:

$$\begin{aligned}
 V &= \{0 < x < l, -l_1/2 < y < l_1/2, 0 < z < h\}, \\
 \Sigma_0 &= \{0 < x < l, -l_1/2 < y < l_1/2, z = 0\}, \quad (z = 0), \\
 \Sigma_1 &= \{0 < x < l, y = -l_1/2, 0 < z < h\}, \quad (y = -l_1/2), \\
 \Sigma_2 &= \{0 < x < l, y = l_1/2, 0 < z < h\}, \quad (y = l_1/2), \\
 \Gamma &= \{0 < x < l, -l_1/2 < y < l_1/2, z = h\}, \quad (z = h), \\
 \Sigma_3 &= \{x = 0, -l_1/2 < y < l_1/2, 0 < z < h\}, \quad (x = 0), \\
 \Sigma_4 &= \{x = l, -l_1/2 < y < l_1/2, 0 < z < h\}, \quad (x = l). \quad (7)
 \end{aligned}$$

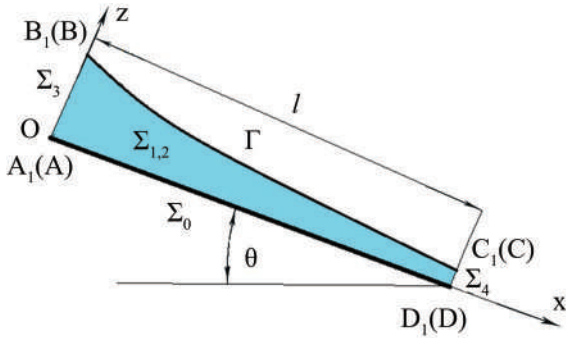


Fig. 2. Schematic of the location of surfaces, which bound the mixture flow

The boundary and initial conditions amplify the system of equations. The condition on the solid wall of the tray is a non-leakage condition.

$$v_n|_{\Sigma} = 0, \quad (8)$$

where  $v_n$  is the normal component of velocity of the mixture on the wall.

$p_{\tau}$  is the tangential stress on  $\Sigma$ :

$$\bar{p}_{\tau} = n_k \sigma_{ki} \tau_i \bar{e}_i = -C_s v_k \tau_k \tau_i \bar{e}_i, \quad (9)$$

where  $n = (n_1, n_2, n_3) = (n_x, n_y, n_z)$  is the single external by the ratio to  $V$  volume of normal to  $\Sigma$  surface;  $C_s$  is the phenomenological coefficient, similar to the Chezy coefficient;  $\tau = \{\tau_1, \tau_2, \tau_3\}$  is the arbitrary single tangent to  $\Sigma$  vector;  $e_i$  are the vectors of the basis of the Cartesian coordinate system.

There are distributions at the inlet cross section of the sieve:

$$\begin{aligned}
 \rho_0 &= \rho(t, 0, y, z), \quad u_0 = u(t, 0, y, z), \\
 v_0 &= v(t, 0, y, z), \quad w_0 = w(t, 0, y, z), \quad h_0 = h(t, 0, y). \quad (10)
 \end{aligned}$$

We select appropriate characteristics based on experimental data.

There are the corresponding conditions on the initial  $\Sigma_4$  section given:

$$\begin{aligned}
 \left. \frac{\partial u(t, x, y, z)}{\partial x} \right|_{x=l} &= 0, \quad \left. \frac{\partial v(t, x, y, z)}{\partial x} \right|_{x=l} = 0, \\
 \left. \frac{\partial w(t, x, y, z)}{\partial x} \right|_{x=l} &= 0. \quad (11)
 \end{aligned}$$

We establish a kinematic boundary condition on the free surface of « $\Gamma$ » grain layer:

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = w, \quad (12)$$

and two dynamic conditions: the first one follows from the mass conservation law, it expresses the continuity of the mass flow through « $\Gamma$ »:

$$\langle \rho(v_n - W) \rangle = 0, \quad (13)$$

where  $W$  is the normal component of the velocity of motion of the surface of the rupture of  $\Gamma$  medium;  $v_n$  is the normal component of the velocity of particles on  $\Gamma$ ; angle brackets mean a jump of a corresponding function on  $\Gamma$ .

Another dynamic condition is the continuity of stresses during the passage through « $\Gamma$ »:

$$-2 \left( \frac{\partial}{\partial x} h \right) \sigma_{xz} - 2 \left( \frac{\partial}{\partial y} h \right) \sigma_{yz} + \sigma_{zz} + P_0 = 0, \quad (14)$$

$$-\left( \frac{\partial}{\partial x} h \right) \sigma_{xx} - \left( \frac{\partial}{\partial y} h \right) \sigma_{xy} + \sigma_{xz} + \sigma_{zz} \frac{\partial}{\partial x} h = 0, \quad (15)$$

$$-\left( \frac{\partial}{\partial x} h \right) \sigma_{xy} - \left( \frac{\partial}{\partial y} h \right) \sigma_{yy} + \left( \frac{\partial}{\partial y} h \right) \sigma_{zz} + \sigma_{yz} = 0, \quad (16)$$

$$(z = h(t, x, y)),$$

where  $P_0$  is the pressure to the surface of the mixture layer caused by the air.

As the grain mixture acts as a viscous liquid under the influence of vibrations [6] and the depth of the layer is small compared to the linear dimensions in the plane of the flow, so we apply the theory of shallow water in the study into motion of a loose mixture [24]. The system of equations of the planned motion of the flow of the loose mixture on a vibrating sieve takes the form:

$$\frac{\partial}{\partial t} \gamma + u \frac{\partial}{\partial x} \gamma + v \frac{\partial}{\partial y} \gamma + \gamma \frac{\partial}{\partial x} u + \gamma \frac{\partial}{\partial y} v = 0, \quad (17)$$

$$\begin{aligned}
 &\frac{\partial}{\partial t} u + u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u + \frac{g \cos \theta}{2} \frac{\partial}{\partial x} h + \frac{hg \cos \theta}{2\gamma} \frac{\partial}{\partial x} \gamma - \\
 &- \frac{2\mu h}{\gamma} \frac{\partial^2}{\partial x^2} u - \frac{\mu h}{\gamma} \frac{\partial^2}{\partial y^2} u - \frac{2\mu}{\gamma} \frac{\partial}{\partial x} h \frac{\partial}{\partial x} u - \frac{\mu}{\gamma} \frac{\partial}{\partial y} h \frac{\partial}{\partial y} u - \\
 &- \frac{\mu}{\gamma} \frac{\partial}{\partial y} \left( h \frac{\partial}{\partial x} v \right) + \frac{C_s}{\gamma} u - g \sin \theta = 0, \quad (18)
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\partial}{\partial t} v + u \frac{\partial}{\partial x} v + v \frac{\partial}{\partial y} v + \frac{g \cos \theta}{2} \frac{\partial}{\partial y} h + \frac{hg \cos \theta}{2\gamma} \frac{\partial}{\partial y} \gamma - \\
 &- \frac{\mu h}{\gamma} \frac{\partial^2}{\partial x^2} v - \frac{2\mu h}{\gamma} \frac{\partial^2}{\partial y^2} v - \frac{\mu}{\gamma} \frac{\partial}{\partial x} h \frac{\partial}{\partial x} v - \frac{2\mu}{\gamma} \frac{\partial}{\partial y} h \frac{\partial}{\partial y} v - \\
 &- \frac{\mu}{\gamma} \frac{\partial}{\partial x} \left( h \frac{\partial}{\partial y} u \right) + \frac{C_s}{\gamma} v = 0. \quad (19)
 \end{aligned}$$

Three equations (17) to (19) contain four unknown functions  $h, \gamma, u, v$ . We accept the kinematic boundary con-

dition on the free surface of the layer to close this system of equations:

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = 0. \tag{20}$$

Equations of the dynamics of a thin layer show that the area of definition of unknown functions is  $\Sigma_0 = \{0 < x < l, -l_1/2 < y < l_1/2\}$ . The boundary of this area consists of lines:

$$\begin{aligned} L_1 &= \{0 < x < l, y = -l_1/2\}, \\ L_2 &= \{0 < x < l, y = l_1/2\}, \\ L_3 &= \{x = 0, -l_1/2 < y < l_1/2\}, \\ L_4 &= \{x = l, -l_1/2 < y < l_1/2\}. \end{aligned}$$

There are distributions given at  $L_3$  boundary:

$$\begin{aligned} h(t, 0, y) &= H^0(t, y), \quad \gamma(t, 0, y) = G^0(t, y), \\ u(t, 0, y) &= U^0(t, y), \quad v(t, 0, y) = V^0(t, y). \end{aligned} \tag{21}$$

There are the following conditions fulfilled on  $L_1, L_2$  lines:

$$v(t, x, -l_1/2) = 0, \quad \left. \frac{\partial u}{\partial y} \right|_{y=-l_1/2} - \frac{C_s}{\mu} u \Big|_{y=-l_1/2} = 0, \tag{22}$$

$$v(t, x, l_1/2) = 0, \quad \left. \frac{\partial u}{\partial y} \right|_{y=l_1/2} + \frac{C_s}{\mu} u \Big|_{y=l_1/2} = 0. \tag{23}$$

Authors of paper [25] numerically resolved equations by a finite-difference method. The essence of the method is that differential operators by spatial variables are replaced with finite-difference operators at nodes of the grid, and they must approximate the differentials applied finite-difference operators, which have the second order of accuracy.

We take into account changes, which add boundary conditions to coefficients of difference operators, in equations, which represent a system of ordinary differential equations relatively to a vector variable. The main members of dynamics equations, which affect velocity of convergence of approximate solutions to the exact ones, relate to summands of equations. Summands, which include mixed derivatives, relate to right parts. The equations are ready for use in a difference splitting method [26] in this form.

We use the following provisions to obtain a stationary solution of the problem: under constant external conditions, presence of dissipation in the task leads to stabilization of a loose mixture and its coming to a stationary state regardless of initial conditions. Then, the initial boundary-value task appears with unchanged boundary conditions, and we consider the stationary solution as asymptotic one.

The multidimensionality of the task leads to the need to solve systems, which contain a large number of equations. In addition, occurrence of numerical instability of the algorithm is possible at solution of the latter. The method for solution of equations consists in transition from differential equations

to the system of algebraic equations by replacement of the differentiation operator over time with the finite-difference operator. Subsequently, we use an implicit difference scheme, which corresponds to the Crank-Nicolson method with a double step in time. It has a second order of accuracy, both in time and by spatial variables.

Initial conditions:

$$\begin{aligned} h(0, x, y) &= H^{(0)}(y) \exp(-\kappa x / l), \\ \gamma(0, x, y) &= G^{(0)}(y) \exp(-\kappa x / l), \\ u(0, x, y) &= U^{(0)}(y) \exp(-\kappa x / l), \\ v(0, x, y) &= V^{(0)}(y) \exp(-\kappa x / l), \end{aligned} \tag{24}$$

contain  $\kappa$  parameter, which determines the velocity of the decrease of the corresponding value with the change of  $x$  (the initial data does not depend on  $x$  if  $\kappa=0$ ). Further, we assume that  $H^{(0)}, G^{(0)}, U^{(0)}$  values are stable, and we bind  $U^{(0)}$  with  $Q$  second volume expense by the relation  $Q = U^{(0)} l_1$ .  $V^{(0)}$  value defines the velocity component, which is transverse to the axis of the tray. We obtain its value in the form of functional dependence:

$$V^{(0)}(y) = V_0^0 \left[ \frac{64}{3} \left( \frac{y}{l_1} \right)^3 - \frac{16}{3} \frac{y}{l_1} \right]. \tag{25}$$

If  $V_0^0 > 0$ , the flow tends to the narrowing, and if  $V_0^0 < 0$  – to the expansion inside the tray.

The tray specific loading  $q = q(x, y)$ :

$$\begin{aligned} q(x, y) &= \lim_{\Delta x \rightarrow 0, \Delta y \rightarrow 0} \frac{1}{\Delta x \Delta y} \int_x^{x+\Delta x} \int_y^{y+\Delta y} \int_0^z \rho(x', y', z) u(x', y', z) dz dx' dy' = \\ &= \lim_{\Delta x \rightarrow 0, \Delta y \rightarrow 0} \frac{1}{\Delta x \Delta y} \int_x^{x+\Delta x} \int_y^{y+\Delta y} \gamma(x', y') u(x', y', z) dx' dy' = \gamma(x, y) u(x, y). \end{aligned} \tag{26}$$

Therefore,

$$q(x, y) = \gamma(x, y) u(x, y). \tag{27}$$

Thus, two characteristics of the flow determine the specific load on any section of the surface of a sieve. They are the surface density of a loose mixture and the velocity of motion.

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### 5. Results of studying the kinematic characteristics of a mixture flow and loading a sieve at uneven feed, in terms of width, at the inlet

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It is necessary to set initial conditions at the inlet of a vibrating sieve to determine an influence of uneven feed, in terms of width, on kinematic characteristics of the flow and load of the working surface. A profile of the initial velocity for the width of a sieve determines uneven feeding of a mixture at the inlet. Let us consider different velocity profiles and examine their influence on kinematic characteristics of a mixture flow and load of the surface over the



entire sieve area. We determine the equations of the initial velocity profiles, in terms of width, by the approximation of experimental values of the velocity at the inlet of a sieve.

We accept the following parameters of the process for numerical calculations: the density of the loose mixture is  $800 \text{ kg/m}^3$ ; the load at the sieve inlet is  $12,000 \text{ kg/h}$ ; the transverse component velocity of the mixture to the axis of the tray is  $V_0^0 = 0 \text{ m/s}$ ; the pressure on the surface of the mixture layer is  $P_0 = 20 \text{ kg/m} \cdot \text{s}^2$ ; the length of the sieve  $l = 1.5 \text{ m}$ ; the width of the sieve  $l_1 = 0.8 \text{ m}$ ; the angle of inclination of the sieve to the horizon  $\theta = 10 \text{ deg}$ ; the empirical coefficient  $k = 0$ ; the coefficient of shear viscosity  $\mu = 0.2 \text{ kg/m} \cdot \text{s}$ , the phenomenological coefficient similar to the Chezy coefficient  $C_z = 10 \text{ kg/m}^2 \cdot \text{s}$ .

A convex for width profile of the initial velocity at the sieve inlet is the most widespread for pouring out a grain mixture from bins. Because of friction of a mixture with side walls of a bin, which leads to a decrease in its speed on side areas in comparison to the central ones.

Fig. 3 shows characteristics of the flow of the mixture on the sieve with the convex, in terms of width, initial velocity profile:

- a* – thickness of the layer;
- b* – surface density of the mixture;
- c, d* – longitudinal and transverse components of the velocity;
- e* – specific loading.

We express the profile of the initial velocity of the mixture along the width of the sieve by the equation  $u(y) = 0,8 - 2,2y^2$ .

Opposite by nature, concave, in terms of width, profile of the velocity is possible if there are large impurities, a mixture with inhomogeneous physical-and-mechanical properties or deformation of guiding surfaces in a bin.

Fig. 4 shows characteristics of the flow of a mixture on the sieve with a concave profile, in terms of width, of the initial velocity:

- a* – thickness of the layer;
- b* – surface density of the mixture;
- c, d* – longitudinal and transverse components of the velocity;
- e* – specific loading.

We express the profile of the initial velocity of the mixture for width of the sieve by the equation  $u(y) = 0,6 + 2,5y^2$ .

In the case of incorrect adjustment of loading devices, an overturned valve of a bin, errors in the assembly of machine nodes, the profile of velocity of a mixture in the width of the sieve becomes a rectangular triangle, and the mixture concentrates near one side wall.

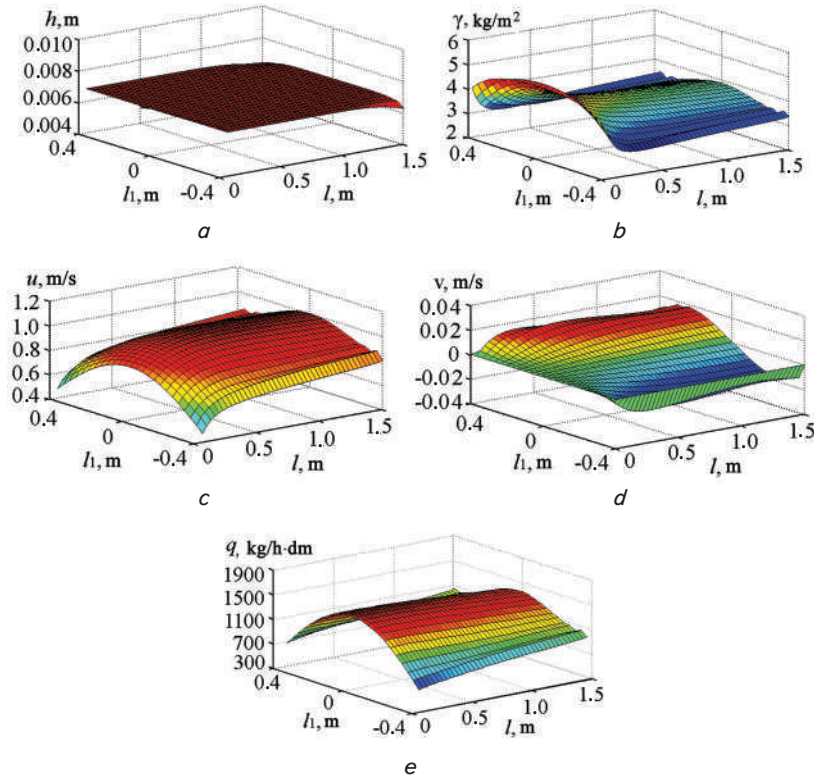


Fig. 3. Characteristics of the flow of the loose mixture with the convex for width profile of the initial velocity: *a* – thickness of the layer; *b* – surface density of the mixture; *c, d* – longitudinal and transverse velocity components; *e* – specific load

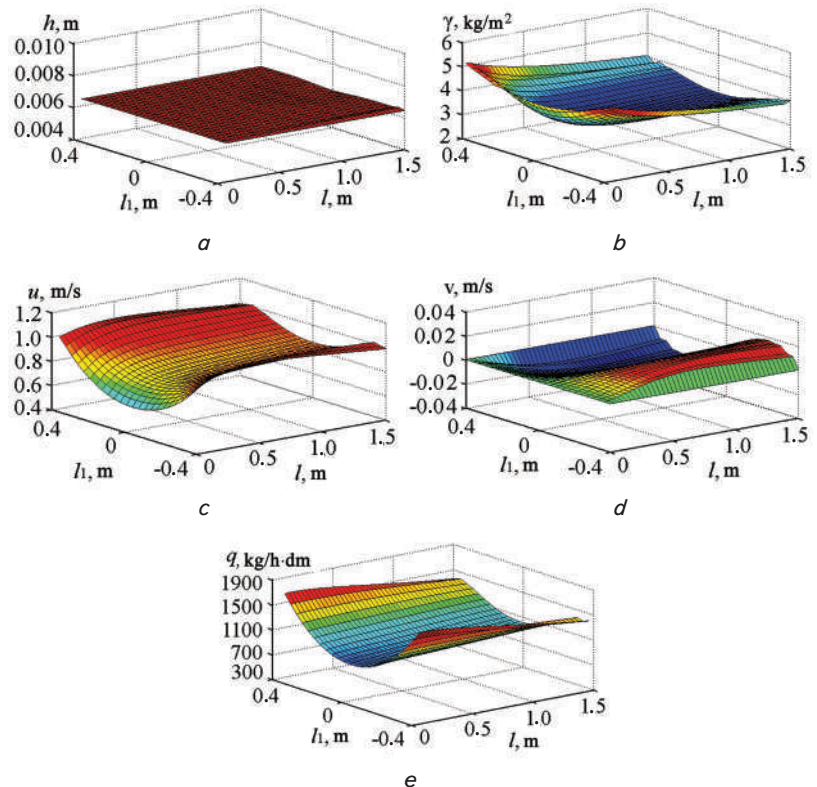


Fig. 4. Characteristics of the flow of a loose mixture with the concave, in terms of width, profile of the initial velocity: *a* – thickness of the layer; *b* – surface density of the mixture; *c, d* – longitudinal and transverse velocity components; *e* – specific load

Fig. 5 shows characteristics of the flow of a mixture on the sieve with the triangular for width profile of the initial velocity:

- a) – thickness of the layer;
- b) – surface density of the mixture;
- c), d) – longitudinal and transverse components of the velocity;
- e) – specific loading.

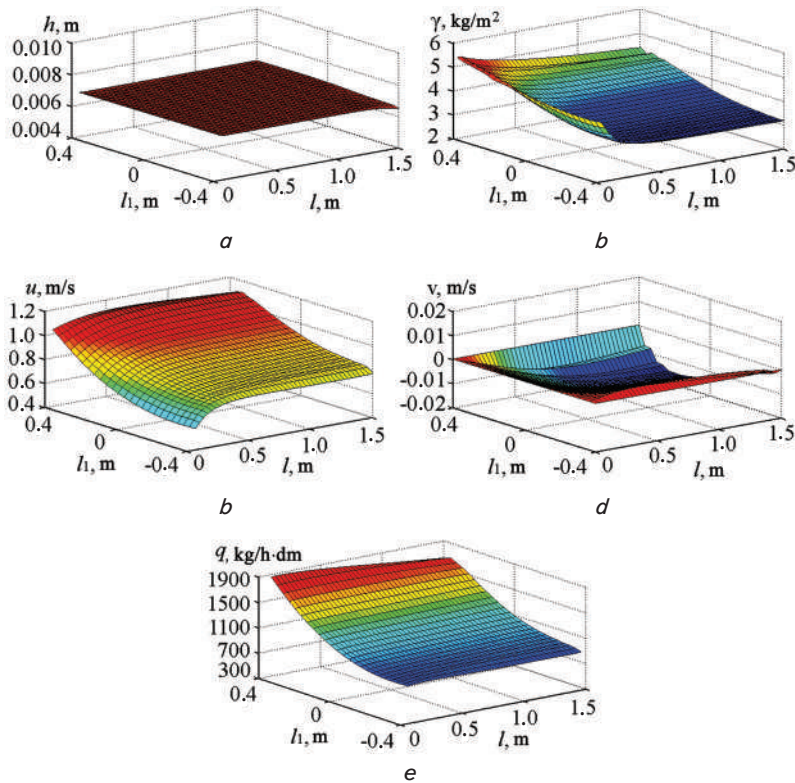


Fig. 5. Characteristics of the flow of a loose mixture with the triangular, in terms of width, profile of the initial velocity: *a* – thickness of the layer; *b* – surface density of the mixture; *c*, *d* – longitudinal and transverse velocity components; *e* – specific load

We express the profile of the initial velocity of the mixture for width of the sieve by the equation  $u(y) = 0,8 + 0,6y + 0,4y^2$ .

The defined regularities of distribution of layer thickness, surface density, longitudinal and transverse components of the velocity of the mixture and specific loading determine the influence of uneven feeding on the process of loading over the entire area of a vibrating sieve. The obtained dependences determine areas of overload or underload of a sieve, their location, magnitudes of deviations from the initial loading and determine causes of their occurrence. The results of the study are necessary for the design of feeders and distributors of loose mixtures and calculation of separation modes.

## 6. Discussion of results of studying the influence of an uneven, in terms of width, vibrating sieve feeding of a loose mixture on the process of its loading

The thickness of the layer remains unchanged over the entire surface of the sieve (Fig. 3, *a* – 5, *a*) for the considered

initial velocity profiles. An appropriate change in density of the layer and the velocity of the mixture ensure its constancy.

Kinematic characteristics of the flow of the mixture vary as follows if the profile of the initial velocity is convex. Surface density of the mixture decreases sharply at the inlet section of the flow (Fig. 3, *b*), and then, it decreases slightly as the mixture progresses along the sieve. The surface density aligns for width closer to the initial cross section of the flow. We can observe an increase in density of the layer near the longitudinal axis of the sieve.

The longitudinal velocity component of the mixture increases at the inlet cross section of the flow (Fig. 3, *c*), and then, as the mixture progresses along the sieve, it does not change much. We observe an increase in velocity near the longitudinal axis of the sieve, due to the convex profile of the initial velocity at the sieve inlet. As the mixture progresses along the sieve, the profile of the longitudinal component of the velocity aligns for width gradually.

The transverse component of the velocity of the mixture (Fig. 3, *d*) is very small in comparison with the longitudinal one. The nature of the change in the transverse velocity component leads to the compression of the flow closer to the longitudinal axis of the sieve. Because of the change in the longitudinal velocity component along the width of the sieve. Particles of the mixture move faster and capture particles from edges near the longitudinal axis of the sieve. Particles from edges move with lower velocity, which leads to the increase in density of the layer along the axis of the sieve.

The specific load of the sieve varies across its surface and has a convex profile (Fig. 3, *e*). Because of the change in the longitudinal velocity and density of the mixture, which take the largest values along the axis of the sieve.

The largest deviations in specific loading from the average value occur at the inlet cross section of the flow, which we explain by the increase of density of the mixture in this area. The deviation is greater than the average value along the axis of the sieve. This indicates that the area is overloaded. And it is less along the edges. This indicates that the area is underloaded. As the mixture progresses along the sieve, the deviation of the specific loading from the average value decreases, which we explain by the stabilization of density and velocity with length.

Kinematic characteristics of the mixture flow vary as follows with the concave profile of the initial velocity. Surface density of the mixture decreases smoothly near the side walls as it progresses along the sieve. It decreases more sharply in the central part. The density function of the mixture has a concave profile, that is, there is consolidation of the layer at the side walls of the sieve. The surface density aligns for width gradually closer to the initial cross section of the flow.

The nature of the longitudinal velocity changes to the opposite – it is smaller in the central part of the sieve than at the side walls and it has the concave profile. At the input of the sieve, there is a growth of the longitudinal velocity in

the central part of it, and then it does not change much as the mixture progresses along the sieve. The dependence of the longitudinal velocity on the length of the sieve is smoother on the side walls.

The transverse component of the speed of the mixture (Fig. 4, *d*) is much smaller than the longitudinal, and is directed from the lateral walls to the axis of the sieve, which leads to the sealing of the mixture in this direction.

The nature of the specific loading of the surface of the sieve has changed in accordance with the change in the longitudinal velocity and density of the mixture. There is material underloading of the surface in the central part of the sieve, and overloading near the sides. This occurs due to the concave profile of the initial velocity and density of the mixture at the sieve input, which determines the amount of mixture fed to the working surface. Deviations of the specific loading from the average value decrease gradually with the length of the sieve.

The kinematic characteristics of the flow of the mixture vary as follows at the triangular for width profile of the initial velocity. Surface density of the mixture is the largest near one side wall and it decreases in the opposite direction. The surface density decreases gradually in the direction of the output cross section of the flow along the length of the sieve.

The longitudinal velocity component of the mixture has the profile of the initial velocity at the sieve inlet, that is the largest values appear at one side wall and they decrease in the opposite direction. The nature of the change in velocity along the length is more pronounced near the wall with the smallest values.

The transverse component of the speed of the mixture (Fig. 5, *d*) is much less than longitudinal one, and it is directed from one wall to the other, which leads to the increase in the density of the mixture in this direction.

The specific load of the working surface changes according to the width of the sieve, in accordance with the triangular profile of the longitudinal velocity and density of the mixture on the sieve. There is overload of the working surface at one side wall and underload – near the opposite one. The specific loading changes insignificantly along the length of the sieve.

The initial velocity profiles at the sieve inlet change characteristics of the flow along the length almost equally, and for width – in accordance with the profile. We can determine the specific load of the sieve by the surface density and the velocity of the mixture. Regularities of their changes cause areas of overload and underload of the working surface.

Thus, the uneven for width feeding of a loose mixture determined by the profile of the initial velocity at the sieve inlet has a significant effect on kinematic characteristics of the flow and loading of the working surface. The nature of the initial velocity profile is decisive for distribution of areas of overload or underload of the working surface of the sieve.

The results of the study are useful for sowing and sorting vibrating sieves, which sift a small part of the mixture – 5–10 %, and 2–3 % in some cases. Application of the results for the unloading and grain sieve, which sift 40 % and 90 % of the mixture, respectively, is limited by the boundary conditions on the surface of the sieve, which should take into account passage of material through a sieve.

The prospect of the development of the study is to develop methods and devices for control of loading of a vibrating sieve according to the known distribution of kinematic characteristics of a mixture flow and areas of overload and underload of a working surface. It is quite complicated to consider possible problems of any nature at the stage of determination of the prospect due to a large variety of possible designs of devices, methods for their implementation and mathematical modeling of processes.

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## 7. Conclusions

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1. We set the unevenness of feeding of a loose mixture by a convex, concave, and triangular profile of the initial velocity, in terms of width, of the sieve inlet. Flow characteristics vary equally over the length of a sieve in the considered profiles. The thickness of the layer remains constant, the surface density decreases, the longitudinal component of the velocity increases, the transverse component is very small in comparison with the longitudinal one and it almost does not change.

2. Flow characteristics change according to the initial velocity profile, in terms of width, of the sieve. For a convex profile, the thickness is constant, the surface density and the longitudinal velocity component are the largest along the longitudinal axis of the sieve and they are the smallest at the side walls, the transverse component of the velocity is directed from the longitudinal axis. For a concave profile, the thickness is constant, the surface density and the longitudinal velocity component are the largest along the side walls, and they are the smallest along the longitudinal axis; the transverse component of velocity is directed from the side walls to the axis. For a triangular profile, the thickness is constant, the surface density and the longitudinal velocity component are the largest along one side wall and they are the smallest at the opposite side, the transverse component of the speed is directed to the first mentioned side wall.

3. If the profile of the initial velocity is convex, there are overload of the surface along the longitudinal axis of the sieve and underload near the side walls. If the profile is concave – there are overload of the surface is near the side walls and underload along the longitudinal axis of the sieve. If the profile is triangular, there are overload of the surface along one side wall and underload along the opposite wall.

The largest deviations occur at the inlet cross section of the flow, the smallest ones – at the outlet cross section.

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