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IMPROVING THE TECHNOLOGY FOR MANUFACTURING HOLLOW CYLINDRICAL PARTS FOR VEHICLES BY REFINING TECHNOLOGICAL ESTIMATION DEPENDENCES

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This paper shows that the technological preparation of production accounts for 20–70 % of the total labor intensity of technical preparation. An important role belongs to the applied programs of finite-element modeling. However, such software packages often cannot be purchased by small-scale industrial enterprises for various reasons. Therefore, special empirical and analytical calculation models are used, which have proved to be quite effective in typical metal processing processes. Drawing a cylindrical hollow part was used as an example of the improved analytical dependence to calculate meridional tensile stresses. Existing analytical models of the process accounted for the bending moment through additional stresses. However, this approach only roughly described the deformation process. It was possible to refine the existing analytical dependences by introducing a term into the differential equilibrium equations that takes into consideration the bending moment that acts in the meridional direction when a workpiece passes the rounding on the matrix edge. Analysis of the obtained expression revealed that the bending of a workpiece gives rise to the stretching meridional stresses, which depend on the ratio of the squares of the thickness of the workpiece and the radius of the matrix rounding. The results of the estimation data from the numerical and theoretical models coincided for small values of the radius of the matrix rounding of 1–2 mm, which confirms the adequacy of the analytical solution. In the numerical model, there is an extreme point where the tensile stresses have a minimum and, after it, begin to increase; this corresponds to the matrix rounding radius of 5 mm

Keywords: technological process, matrix, drawing, plastic deformation, flat workpiece, rounding radius, meridional stresses

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1. Introduction

The production of a modern car is a complex, knowledge-intensive, and costly task. Each stage of manufacturing is accompanied by the involvement of a large number of human and material resources, which leads to a high cost of the final product. Reducing the labor intensity and technological cost of the product is contributed to by integrated automation, as well as improving the technical preparation of production in general.

Technological preparation of production, in proportion to the total labor intensity of technical preparation, ranges from 20 to 70 % depending on the type of production and tends to increase when industrial production grows [1–3].

It is possible to reduce the labor intensity and duration of technological preparation of production by using the standardization and normalization of technological processes, unification of equipment, technological equipment, and documentation [4]. However, these techniques relate more to large-scale and mass types of production and to a lesser extent affect single and small-scale production, the share of engineering products of which increases year in year out [5]. This can be confirmed by the data from studies [6, 7], which claim that the share of enterprises belonging to single and small-scale production, according to expert estimates, is 75–80 % of the total share of machine-building plants. It should be noted especially that there are practically no recommendations for debugging technological processes. This applies to all types of production. Note that its implementation costs enterprises about 30–50 % of the time that is allotted for the technological preparation of production [8].

While the technological preparation of production is optimized by the introduction of an automated system of technological preparation of production, it is almost impossible to automate the debugging of the technological process on physical models [9]. It is possible to partially automate the debugging process with the involvement of applied programs for the finite-element modeling of technological processes [10, 11]. However, even here the accuracy of modeling depends on the skills and experience in the program of an engineering worker and does not exclude physical modeling, but only directs it in the right direction [12, 13]. In this regard, the issue of refining the existing universal computational technological dependences that could reduce the time of debugging equipment is acute. The results of such calculations are the basic data for the design of tooling and technological transitions [14]. It becomes obvious that the reduction of the labor intensity of technical preparation of production is inseparably related to the accuracy of mathematical models of technological processes and the possibility of their implementation by existing direct methods of integration. The obtained formulations of such decisions form the basis of technological calculations and lead to a reduction in the time of preparation of production, which indicates the relevance of this issue.

2. Literature review and problem statement

The technology of manufacturing cylindrical hollow parts by drawing from a flat sheet workpiece is a plastic deformation of its flange with its gradual retraction into the hole of the matrix with a punch. This process of metal processing by pressure is widely studied and covered in the literature. Thus, in work [15], the classical approach to the analysis of sheet stamping processes was reported, based on the solution to simplified differential equilibrium equations by direct integration. The simplicity and clarity of the solutions obtained made it possible to theoretically assess the role of the main factors of the drawing process in the field of emerging stresses. However, the issue of the loss of resistance by the flange part of a workpiece remained unresolved. In [16], an attempt was made to predict fold formation, based on the theory of energy conservation and the characteristics of longitudinal bending. However, in the cited work, the authors considered a rectangular plate, rather than a round workpiece, which does not fully reflect the features of drawing cylindrical parts. In [17], an attempt was made to take into consideration the influence of the anisotropy of workpiece material on the loss of flange stability. However, the issue of the influence of the radius of matrix rounding on folding remains unresolved. There are reference data for selecting the radius of the matrix and punch rounding, a gap, the coefficients of drawing for transitions, lubrication, clamping forces, etc. [18]. However, guided by the above data, technologists cannot avoid the complex process of adjusting the stamp for a new technological process of manufacturing hollow parts by drawing. Especially labor-consuming are operations associated with the stamping of thin sheets up to 0.5 mm, as well as large-sized complex-shape parts. Thus, in work [19], the design of the punch with the clamping of the flange to eliminate corrugation is proposed. However, the complexity of such a structure would increase the time of stamp adjustment. The authors of [20] use overhang ribs and thresholds when drawing complex-profile thin sheet parts. The most optimal installation zones on the stamp of these elements are indicated. However, their geometry is disregarded. For such parts, it is necessary to select a rational gap between the pressure ring and the flange of a workpiece to eliminate corrugation. As well as to prevent the destruction of a workpiece in the zone of transition of the wall to the bottom of the part. This entails the consumption of metal blanks for test runs and, accordingly, increases the duration of adjustment.

For complex parts, master models of the matrix, punch, and ring are first made of wood or plastic [21]. These activities require employing a highly qualified modeler in the staff. According to the master model, a gypsum model of the matrix for casting and a model for its processing on copying machines are made. The matrix model is used to manufacture a model according to which the punch casting mold is made and milling work is performed [22]. The foundry is not always available at enterprises with a small-scale type of production. After the manufacture of stamping equipment in metal, they proceed to the adjustment of the stamp [23]. There, the lack of means of mechanization of the installation of stamps reduces labor productivity on forging and stamping equipment since such equipment is idle over a significant part of the time. Adjustment implies fitting the mirror surfaces of the matrix and clamping, matrix and punch, the installation of stops, and test stamping [24]. These operations are quite time-consuming and the probability of making a mistake at each stage of preparation, even for an experienced technologist and adjuster, is quite high.

Particular attention is paid to the selection of the radius of matrix rounding as this is one of the most important design factors of the tool [25]. Its value affects the stability of the drawing process. Paper [26] considers the conditions of loss of stability but does not disclose the magnitude of the stresses at which this process occurs. The authors of [27] ignore the influence of the bending moment on the appearance of folds on the flange. In addition, the magnitude of the radius affects the distribution of dangerous tensile stresses [28], friction forces [29], lubrication conditions [30], the quality of the elongated part [31], as well as the total deformation force [32]. The incorrectly selected radius of matrix rounding leads to an increase in the duration of adjustment and setting works, especially for thin sheet metals and the region of small values of the radii of rounding the matrix [33]. There, in the process of adjustment, there is folding on the flange, a break in the bottom of the part, as well as chips and scratches on the finished part. The issue related to selecting the rational radius of matrix rounding originates from the early works of the theoretical analysis of the process [15]. There, researchers, while trying to simplify expressions as much as possible for better perception and analysis, consciously accepted a large number of assumptions in the mathematical model. That approach produced good results in the field of large thicknesses of blanks and, accordingly, sufficiently large radii of the rounding of the drawing edge of the matrix [34]. In [35], greater emphasis is put on the influence of the radius on the magnitude of the friction forces between a workpiece and the tool, which significantly reduces its role and influence on the distribution of stresses at the site of deformation. In turn, the desire to take into consideration as many components of differential equilibrium equations as possible led to the cumbersomeness of the resulting formulations and the inability to use them in engineering calculations. Thus, in [36], an attempt was made to linearize the differential equilibrium equations, which significantly improved the accuracy of the solution, but, due to its complexity, the analysis became ineffective. The author of [37] proposed a method of elastic solutions for differential equilibrium equations of round plates. The solutions make it possible to determine the critical stresses that cause a loss of stability of the workpiece but it is quite difficult to study the influence of the geometric parameters of the tool.

Our review suggests that the influence of the radius of matrix rounding on the conditions of drawing would have different intensities for different ratios of its size and thickness of workpieces. That is, probably, there is a set of intervals of the radius of matrix rounding and the thickness of the workpiece, where, for example, tensile stresses accept the greatest value. At the same time, there are intervals where the change in the radius of rounding has practically no effect on the distribution of meridional stresses. This premise is based on the complexity and duration of adjusting the technological process of drawing in the field of small thicknesses of blanks. To confirm it, it is necessary to consider the selected area of a workpiece at the site of plastic deformation, taking into consideration the greater number of terms included in the differential equilibrium equations, in comparison, for example, with solutions suggested in [38]. At the same time, it is necessary, nevertheless, to strive for expressions that would be explicitly integrated and subject to further analysis.

The above allows us to argue that the radius of the rounding of the drawing edge of the matrix is the most important technological factor in the drawing process. Existing analytical dependences do not fully disclose its effect on the field of emerging stresses and the formation of folds on the flange of a workpiece. Therefore, it is advisable to conduct research aimed at improving analytical solutions. This would make it possible to develop ideas about the laws of the stressed state and loss of stability when drawing hollow parts.

3. The aim and objectives of the study

The aim of this work is to refine existing estimation dependences for determining the stressed state during drawing. This would reveal the regularities of the influence of the basic technological factors of the process on the loss of flange stability and generally improve the technology of manufacturing hollow parts.

To accomplish the aim, the following tasks have been set:

– to estimate existing mathematical models of the process of drawing cylindrical parts;

– to solve the differential equilibrium equations by direct integration;

– to test the adequacy of the solution using the finite-element simulation of the process with different intervals of input parameters; to interpret and compare the results.

4. The study materials and methods

To be able to obtain a solution in quadratures, it is necessary to accept some assumptions and hypotheses that simplify the initial equilibrium equations. Since a part of the workpiece when entering the drawing edge of the matrix takes the shape of a rotation toroid, we accepted the assumptions from the general technical theory of shells, as well as the Kirchhoff-Love hypothesis [39, 40]. The stressed state was taken flat with axial symmetry. In the process of forming a workpiece, the wall thickness remains unchanged; the friction between the workpiece and tool is not taken into consideration. When a part of the workpiece enters the plastic state, the hypothesis of maximum tangent stresses was used without taking into consideration the hardening and influence of anisotropy.

To verify the adequacy of the analytical solution, a numerical experiment was conducted employing the finite-element package of 3D simulation Simullia Abaqus, student edition (Dassault Systèmes, France).

A workpiece was modeled with a flat round plate with a diameter of $D=100$ mm and a thickness of $s=1$ mm. The workpiece material had the following characteristics: Young's module, $E=210$ GPa; Poisson coefficient, $\mu=0.3$; density, $\rho=7{,}800$ kg/m^3, yield strength, $\sigma_s=230$ MPa; tensile strength, $\sigma_v=320$ MPa; relative elongation, $\delta=34$ %. The workpiece metal is isotropic, not hardening. This model of the material was adopted to minimize the effect of hardening on the distribution of stresses at the site of deformation.

The tooling was a matrix with an input diameter of $D_m=60$ mm and various radii of rounding the drawing edge, $r_m=1-10$ mm. The punch was a cylindrical body with a height of $h=35$ mm and the radius of rounding the transition of the wall to the bottom, unchanged for all experiments, $r_p=4$ mm. The tool was modeled by a non-deformable 3D solid body, by a «revolution» technique.

5. Results of studying the solution to a differential equation in partial derivatives for a round workpiece

5. 1. Evaluation of existing analytical solutions in stresses for drawing cylindrical parts

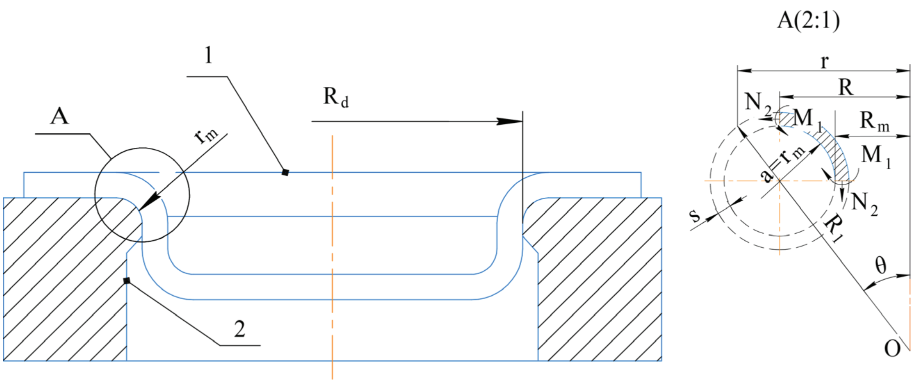
Existing mathematical models of the process of plastic deformation of a flat workpiece and its transformation into a hollow part are based on the provisions of the mechanics of a continuous medium and its branch – the theory of plasticity [15, 41, 42]. The equilibrium of the selected area of a workpiece in rectangular but, most often in polar coordinates, is considered. The system of differential equilibrium equations is simplified due to the fact that the size of the deformation site along the middle surface of a workpiece and the radii of curvature of the median surface are typically much greater than the thickness of a workpiece. Therefore, the stresses acting in the direction of the normal to the thickness of a workpiece are small compared to other stresses, so the stressed state scheme is assumed to be flat [15, 43]. To further simplify the system of equilibrium equations, a scheme of axisymmetric deformation is adopted; the stresses become a function of one coordinate. Solutions are obtained in quadratures using the plasticity equation according to the hypothesis of constancy of maximum tangent stresses or the energy hypothesis [44]. To find visual solutions, it is assumed that the thickness of a workpiece does not change during the plastic deformation.

Attempts to bring the schematized site of deformation as close as possible to the real conditions of shape change led researchers to devise reasonable techniques and ways to take into consideration various factors of the process for the distribution of stresses in it. For example, the authors of [45, 46] introduce anisotropy constants into the equilibrium equations, which complicates the calculations but brings the solution closer to the physical essence of the drawing process. Studies [15, 47] show how to take into consideration the change in the thickness of a workpiece and the friction between the tool and workpiece, which also leads to an increase in the accuracy of mathematical modeling. However, while the first authors introduce anisotropy constants into the equilibrium equations, the second authors do not take into consideration friction, changes in the thickness of a workpiece, and the action of the bending moment in the original differential dependences. Their influence is taken into consideration by summing the integral of the engineering solution to the differential equation with terms that depend on these parameters and are obtained on the basis of the logical conclusions of the authors.

An attempt is made here to refine the existing solutions for meridional stresses when drawing cylindrical parts by introducing a term into the equilibrium differential equations that takes into consideration the bending momentum acting in the meridional direction. It occurs when a workpiece is transferred to the drawing edge of the matrix.

5. 2. Solving a differential equilibrium equation by direct integration

A workpiece zone is considered that enters the drawing edge of the matrix, cut off from the rest of its part by planes perpendicular to the meridional direction. Internal efforts and momenta are applied to the selected area (Fig. 1).



a *b*

Fig. 1. Schematic drawing of a cylindrical workpiece (punch is not shown): *a* — selecting the investigated area: 1 — part; 2 — matrix; *b* — remote element of the workpiece section in the form of a part of the toroid surface

Taking the accepted assumptions into consideration, the equilibrium equation disregards the terms that depend on the circumferential coordinate, as well as the shifting forces and momenta acting in the tangential direction (axisymmetric problem) [38]:

$$\frac{\partial\left(\left(A_1\right)N_2\right)}{\partial\theta}-N_1\frac{\partial A_1}{\partial\theta}-M_1\frac{\partial A_1}{R_2\partial\theta}=0, \qquad (1)$$

where

$$A_1=R_1\sin\theta=\frac{r_m\left(1+k\sin\theta\right)}{k\sin\theta},$$

$R_2=a=r_m$, N_2 is the circumferential internal force.

The remaining components of formula (1) are intuitive from Fig. 1.

The drawing process is accompanied by the occurrence in the flange and walls of a workpiece of stretching meridional and compressing circumferential internal forces. The transition of the material into a plastic state, taking into consideration the action of the momentum, is formalized by the following dependence [15]:

$$\left.\begin{aligned}M_1=\frac{s^2}{4N_s}\left(N_s^2-N_2^2\right),\\ N_2-N_1=N_s,\end{aligned}\right\} \qquad (2)$$

where N_s is the intensity of internal efforts.

Thus, we have three unknown force parameters N_1, N_2, M_1, and three equations to determine them. A solution to equation (1) is to be found relative to the meridional effort since it is these forces that determine the stability of the deformation process and the necessary load on the press. We differentiate the equilibrium condition (1), exclude the momentum and the circumferential force, by substituting (2) into it, to obtain:

$$\frac{\partial N_2}{\partial\theta}+\frac{ks^2\cos\theta}{4N_s a^2\left(1+k\sin\theta\right)}N_2^2=-N_s\frac{k\left(1-\frac{s^2}{4a^2}\right)\cos\theta}{\left(1+k\sin\theta\right)}. \qquad (3)$$

The resulting linear inhomogeneous equation with respect to N_2 is an equation of the type $dy/dx+a(x)y^2+c(x)=0$, which is termed the generalized Riccati equation and, as Liouville showed, in a general case, it does not integrate with quadratures [48, 49]. It can, by substituting the variables, be transformed into Bernoulli's equation if one partial solution $y_1(x)$ to this equation is known [50]. Assuming $y=y_1(x)+z(x)$, $z(x)$ is a new unknown function of x, we find [51]:

$$\frac{dy_1}{dx}+\frac{dz}{dx}+ +a(x)\left(y_1^2+2y_1z+z^2\right)+ +b(x)\left(y_1+z\right)+c(x)=0. \qquad (4)$$

Hence, by virtue of the fact that $y_1(x)$ is the solution to equation (4), we obtain:

$$\frac{dz}{dx}+a(x)\left(2y_1z+z^2\right)+b(x)z=0, \tag{5}$$

or

$$\frac{dz}{dx}+a(x)z^2+\left(2a(x)y_1+b(x)\right)z=0. \tag{6}$$

A particular solution for meridional efforts was obtained earlier by the authors of [52, 53] under the assumption the deformation of a workpiece is moment-free. It takes the form:

$$N_2=-N_s\ln\left(1+k\sin\theta\right)+C. \tag{7}$$

Expand *ln*(1+*k*sinθ) into a series and, limiting ourselves to the first term of the series, substitute (7) in (6), then we obtain:

$$\frac{\partial N_2}{\partial\theta}+\frac{ks^2\cos\theta}{2a^2}N_2=-\frac{ks^2\cos\theta}{4N_s\,a^2\left(1+k\sin\theta\right)}N_2^2. \tag{8}$$

The derived equation is Bernoulli's equation of the form $dy/dx+p(x)y=q(x)y^n$ [51]. By replacing a variable $z=1/y^{n-1}$, Bernoulli's equation is reduced to a linear one and integrated as the linear one [49, 51, 54]. Then the formula for N_2 is converted to the following form:

$$N_2=N_s\left(\frac{2}{k}+\frac{2s^2}{a^2}\sin\theta\right)+C, \tag{9}$$

where *C* is the arbitrary integration constant of expression (8).

The authors of works [15–17] determined the stresses that act in the direction of the meridian in the flange of *s* workpiece during drawing $\sigma_\rho=\sigma_s\ln\left(R_z/\left(R_m+r_m\right)\right)$ at θ=0. In this case, the magnitude of these stresses can be taken as conditions at the border of the flange-radius of rounding, then:

$$\sigma_\rho=\sigma_s\left(\ln\left(\frac{R_z}{R_m+r_m}\right)+\frac{2s^2}{r_m^2}\right). \tag{10}$$

The transition from internal efforts to stresses is carried out according to the formulas $N_2=s\sigma_\rho$ and $N_s=s\sigma_s$.

5. 3. Finite-element simulation of a flat workpiece drawing operation

The finite-element modeling of the drawing operation was carried out without thinning the wall, as well as in the absence of friction between the workpiece and tool. This model of deformation did not meet the real conditions of drawing but contributed to reducing the influence of technological factors on the field of stresses caused directly by bending at the radius of matrix rounding. In the software, these conditions were realized by a gap between the workpiece and the wall of the matrix $z=0.3$ mm per side and by setting the friction mode in the program – «without friction».

It is quite difficult to assess the effect of bending on the value of meridional stresses at the site of deformation. Their distribution is mainly influenced, as mentioned above, by the hardening of the metal, the degree of deformation, the friction between the tool and workpiece. To a lesser extent, the anisotropy of mechanical properties, the geometry of the workpiece and tool, gaps, the pressure of pressing, etc. Techniques that limit the influence of some of those factors, as well as the absence of friction of the workpiece against the pressure ring, should have led to more accurate simulation results. Another technique that was superimposed on the process of deformation was the time limit of working stroke of the punch, equal to 10 s. The load on a workpiece was set by the directional movement of the punch at a speed of 2.5 mm/s. Then the punch, over a predefined time, was lowered by 25 mm, which, for each experiment, was a constant value and limited the degree of deformation of the workpiece.

Fig. 2 shows the construction of the problem with a finite-element grid.

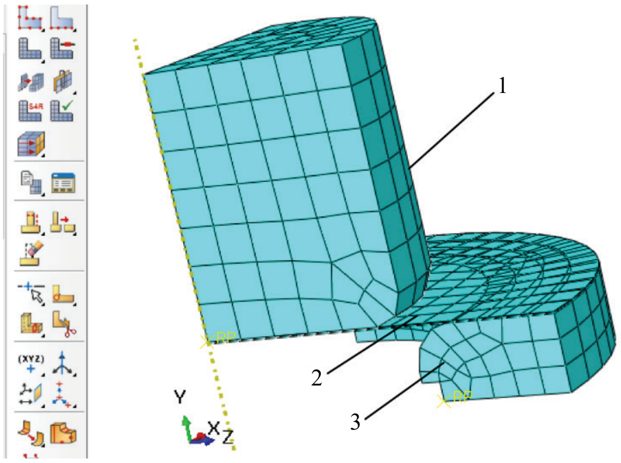
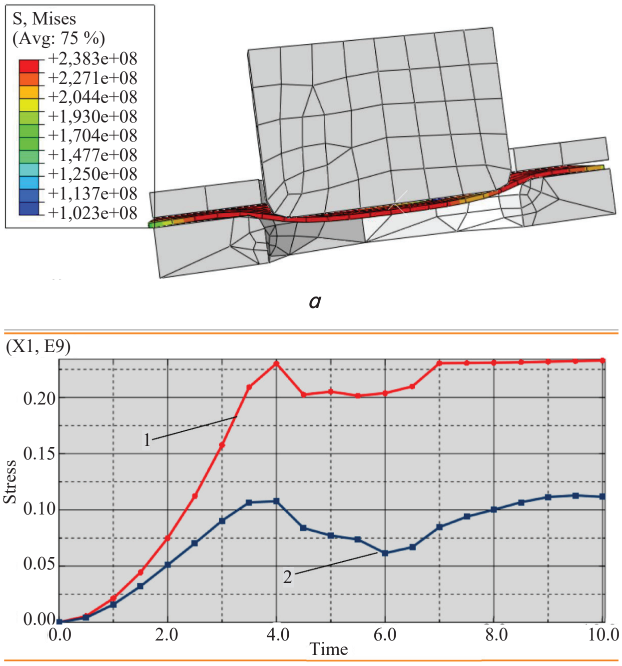


Fig. 2. The finite-element model of drawing a cylindrical part: 1 – punch; 2 – blank; 3 – matrix

Fig. 3 shows the deformed state of a workpiece at the end of the punch stroke, as well as typical plots of the drawing process.



a

b

Fig. 3. The process of drawing a cylindrical part: *a* – von Mises stress distribution diagram at the end of the punch stroke; *b* – typical plots of stress change at the end of the planned punch stroke: 1 – von Mises stresses; 2 – meridional stresses

Next, Fig. 4 shows graphical dependences that illustrate the change in meridional stresses over time for discrete values of the radii of the rounding of the drawing edge of the matrix and the constant thickness of a workpiece equal to s=1 mm, treated by the Excel tabular processor (Microsoft Corp., USA).

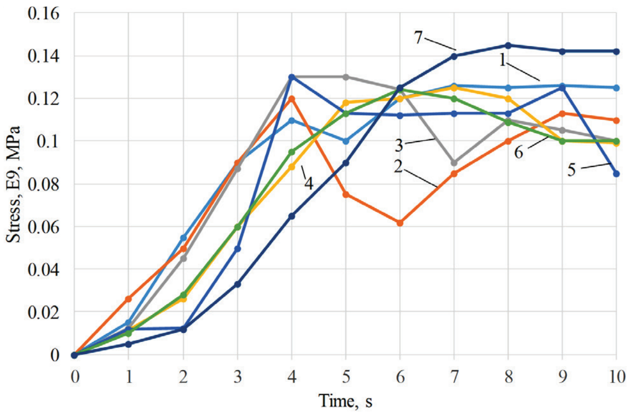


Fig. 4. Graphic dependences of the meridional stress-deformation time for workpiece thickness s=1 mm: 1 – r_m=2 mm; 2 – r_m=3 mm; 3 – r_m=4 mm; 4 – r_m=5 mm; 5 – r_m=6 mm; 6 – r_m=8 mm; 7 – r_m=10 mm

The plots (Fig. 4) demonstrate that the value of meridional tensile stresses does increase with a decrease in the radius of the rounding of the drawing edge of the matrix. However, not all values obey this statement. Moreover, the plots (Fig. 4) are not smooth but have extreme values.

Fig. 5 shows a combined plot demonstrating the values of meridional stresses at the end of the working stroke of the punch corresponding to 10 seconds for the model experiment, as well as the values calculated from dependence (10).

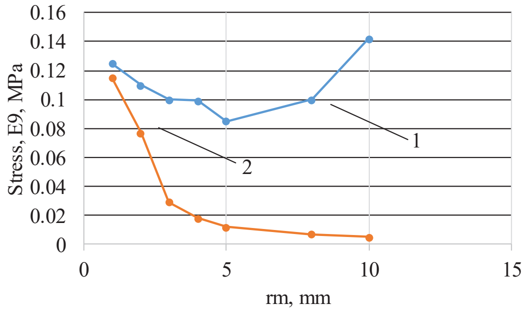


Fig. 5. Dependences of the meridional stresses – the radius of matrix rounding: 1 – model experiment; 2 – estimation dependence (10)

These curves (Fig. 5) have significant discrepancies in the magnitude of meridional stresses for the two procedures that determine them. However, the tendency to reduce them partially persists to the value of the radius of matrix rounding of r_m=5 mm.

6. Discussion of results of the study to refine the mathematical model of the drawing process

Comparing the results of calculations of the value of meridional stresses, we can conclude (Fig. 5) that the closest result was obtained for drawing with a radius of matrix rounding equal to 1 mm. The analytical calculation produces a value of σ_ρ=118 MPa, and the finite-element simulation – σ_ρ=122 MPa. Further, with an increase in the radius of matrix rounding, there is an increase in the discrepancy in the results of calculations for the two procedures being compared. The dependence (10) demonstrates a smooth decrease in tensile stresses with an increase in the value of the radius of the matrix rounding. This corresponds to the physical essence of the phenomena during drawing, due to a decrease in the resistance of a workpiece flange when it is drawn into the hole of the matrix with an increase in its rounding radius. The same nature of the change in stresses when changing the radius of matrix rounding was reported by researchers in [15, 16], but they did not take into consideration the influence of the bending momentum in the equilibrium equations. A particularly rapid drop is observed in the range r_m=1–3 mm from σ_ρ=118 MPa to σ_ρ=40 MPa. Then this decrease slows down and asymptotically approaches zero. Here, it is necessary to note the fact that with this geometry of a workpiece and its relative thickness, the recommended radius of matrix rounding for the process is 6 mm [18, 55]. This value of the radius of the matrix was derived experimentally under industrial conditions. For this radius size, the meridional stresses for the finite-element model are 90 MPa, the analytical solution produces the result of 10 MPa. This discrepancy of results indicates the imperfection of the analytical model and the ambiguity of approaches to the formation of boundary conditions. However, as shown in Fig. 4, the value of the stresses begins to increase with an increase in the radius of matrix rounding from 5 mm to 10 mm for numerical modeling. This fact gives reason to believe that the numerical model more accurately describes the process of deformation. The growth of meridional stresses is caused by the beginning of the process of folding on the flange, which is facilitated by an excessive increase in the radius of matrix rounding, as indicated in the recommendations from [18, 56].

The resulting analytical dependence (10) does not take into consideration the formation of folds on the flange of the semi-finished product, since, to this end, it is necessary to solve an extreme problem, which is beyond the scope of this study. However, it makes it possible to estimate the value of meridional stresses depending on the value of the radius of matrix rounding and the geometry of a workpiece. Based on (10), it is also possible to predict the onset of folding. It likely begins at the radius of matrix rounding where the decrease in meridional stresses practically stops (section r_m=6–10 mm, Fig. 5).

Our results are explained by the presence of assumptions in the mathematical model about the axisymmetric loading of a workpiece and the absence of momentum in the tangential direction. The numerical model is formed in a three-dimensional statement and is devoid of these shortcomings. Therefore, it takes into consideration folding.

A feature of the resulting analytical dependence (10) is the expansion of ideas about the influence of the radius of matrix rounding on the distribution of meridional stresses. In a given formula, it is included in the second power, which explains the sensitivity of the process on the slightest change in the radius of matrix rounding. This is confirmed by production operations for adjusting equipment for stamping thin sheet blanks. In existing analytical models [15–17], the radius of matrix rounding is included in the first power and its role in the drawing process is somewhat underestimated.

The numerical verification of the adequacy of the proposed formula (10) revealed the imposed limitations for its use.

The closest results correspond to small radii of rounding in the range of 1–4 mm and small thicknesses of workpieces. Therefore, we can recommend the use of the resulting dependence for workpiece thicknesses up to 1 mm.

The disadvantage of the proposed expression is the lack of a possibility to determine the moment of the beginning of folding. However, the same shortcomings are characteristic of existing analytical models [15–17]. The appearance of folds corresponds to certain intervals in the geometric and technological parameters of drawing (the relative thickness of a workpiece, degree of deformation, geometry of the matrix) and is the solution to the problem of stability of the deformation process. Therefore, in this study, no solving such a problem was considered. However, determining those critical stresses that cause a loss of flange stability is a promising task that can advance this field. Then, it would be necessary to solve an extreme problem regarding meridional and tangential stresses, taking into consideration the hardening of the material, the movement of a workpiece flange in time, which could cause certain difficulties of a mathematical nature.

7. Conclusions

1. We have performed a critical review of the techniques for the mathematical description of the process of drawing cylindrical parts, which revealed that existing analytical models demonstrate the accuracy sufficient for engineering calculations. The models containing anisotropy constants are more cumbersome; their application in practice is difficult due to the high complexity and duration of calculations. However, their use is justified in the case where it is not possible to adjust the technological process using calculations that do not take anisotropy into consideration. The mathematical models based on the solution to the simplified equilibrium equations, together with the plasticity equation, show simplicity and clarity in terms of analysis. They reveal the basic regularities of the drawing process, make it possible to determine the factors that have a decisive impact on the stability of the shape change. However, the influence of the bending momentum is taken into consideration in such a solution approximately as the increment of stresses from the bending of the band of unit length. However, a workpiece for drawing is a round plate; taking into consideration the momentum when bending the strip does not fully reveal the physical aspect of the issue.

2. The solution to the reduced differential equilibrium equation has been derived for a workpiece of a constant thickness, supplemented by a term taking into consideration the bending momentum in the meridional direction. The second-order partial differential equation, by the co-solution to the plasticity equation, was reduced, by replacing the variable, to a Bernoulli's linear differential equation. The integral of this equation is the dependence of meridional stresses on the ratio of squares of the thickness of a workpiece and the radius of matrix rounding. That largely distinguishes the resulting solution from the existing ones in which the thickness of a workpiece and the radius of matrix rounding are included in the first power. This circumstance indicates that a slight change in the radius of matrix rounding with a constant thickness of a workpiece leads to a sharp increase or decrease in meridional stresses, especially in the region of small thicknesses of workpieces. Hence the complexity of selecting the geometry of the matrix for drawing under industrial conditions, since the slightest deviation of the radius of matrix rounding from the optimal one leads to a loss of stability of the flange part of the workpiece.

3. The finite-element simulation of drawing a workpiece with a thickness of 1 mm on matrices with rounding radii from 1 mm to 10 mm was carried out. The results of the calculated data matched for small values of the radius of the rounding of the matrix, 1–2 mm, which confirms the adequacy of the analytical solution. In the numerical model, there is an extreme point where the tensile stresses have a minimum, and, after it, they begin to increase, this corresponds to $r_m=5$ mm. This result indicates the beginning of the formation of folds on the flange of a workpiece during drawing. The analytical solution is deprived of the possibility to determine the moment of formation of folds but it has universality and contributes to the understanding of the drawing process, the identification of the most significant factors affecting the stability of the shape change.

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