UDC 621.771.004.15
DOI: 10.15587/1729-4061.2022.257013

ESTIMATING THE RESIDUAL RESOURCE OF BASIC STRUCTURES USING A MODEL OF FATIGUE DURABILITY UNDER COMPLEX LOADING

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This paper reports the construction of a durability model of basic structures, which takes into consideration the complex stressed state under the cyclic action of the complex load. The models that take into consideration this factor are categorized on the basis of equivalent for a certain indicator of the stressed-strained state. The equivalence models based on the tangent stresses and strains have been recognized as the most effective ones. However, they hold when the ratio of the limits of fatigue under tangent and normal stresses exceeds 0.5. In addition, determining the latter requires specific testing equipment. The concept of basic bearing structures for industrial equipment has been formulated. The issue related to the multi-axis fatigue of basic structures was considered from the standpoint of combining the reliability indicators of systems. The durability model has been derived from the rule of combining resource safety indices. The load is represented as a combination of individual subprocesses of simple types of deformation with their amplitudes and asymmetries. A model of durability with multi-axis fatigue has been built, which takes into consideration the parameters of the form of the strain cycle, and the type of process (synphase, disproportionate, unchanging static stress). The possibility of obtaining parameters for the multi-axis fatigue model during tests for three-point bending under conditions of varying the multiplicity of the span has been confirmed. According to this scheme, fatigue tests of prismatic samples of the steels 09G2 and 40H were carried out. For them, the parameters of fatigue resistance were found; additionally, the ratio of the fatigue limit for tangent stresses of displacement and fatigue limits for normal bending stresses, which is equal to 0.385, was established. A test procedure has been devised to determine the initial data for the multi-axle fatigue model, which is suitable for conventional test machines and simple-shape samples. The latter advantage is important precisely for basic structures, from fragments of which it is difficult to fabricate a sample of a complex shape

Keywords: basic load-bearing structures, multi-axis fatigue, safety index, shear stresses

Received date 13.04.2022
Accepted date 10.06.2022
Published date 30.06.2022

***How to Cite:** Belodedenko, S., Hrechanyi, O., Hanush, V., Vlasov, A. (2022). Estimating the residual resource of basic structures using a model of fatigue durability under complex loading. Eastern-European Journal of Enterprise Technologies, 3 (1 (117)), 33–41. doi: https://doi.org/10.15587/1729-4061.2022.257013*

1. Introduction

According to the canons of functional and cost analysis, the structure of the mechanical system of technological equipment for industrial production can be represented as composed of such units as the engine (E), transmission (Tr), executive mechanism (EM), and tool (T) (Fig. 1).

All these elements are installed on basic bearing structures (BS), which include frames, beds, bodies, casings, etc. If the equipment is stationary, the basic elements are fixed on foundations (F).

These elements perform the main function of the equipment and determine its functionality.

In addition to the main structures, there are subsidiary devices (SD), which define the quality of functions. In industrial units, elements E, Tr, EM are absent while subsidiary elements acquire greater importance.

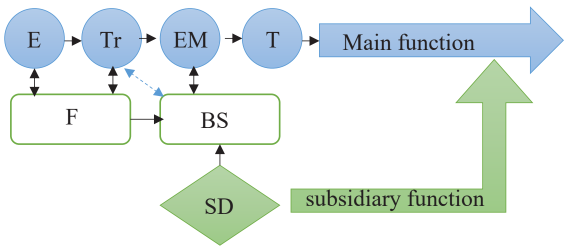

Fig. 1. Structural diagram of the mechanical system of technological equipment

All those structural elements differ in the approach to the purpose of the resource. The resource of the equipment, in general, must at least correspond to the depreciation period of the group to which the object under consideration can be attributed. Over the period, most nodes of structural units can be replaced several (2 to 6) times. This does not apply, firstly, to the tools, which are mostly treated as consumables. Secondly, the exception is the basic structures with foundations, the replacement of which is not implied while their repair is possible. In fact, the BS resource defines the object's service life.

It should be noted that now there is a global tendency to prolong the intended service life of the main technological equipment in all industries. According to data from the Ukrainian company Energoatom, the annual effect of the extension of one NPP unit for 10–15 years is approximately USD 1.5 billion. Among railway transport specialists, it is believed that the economic effect of extending the service life is about UAH 100 thousand (for 2020, that corresponded to USD 3,500–4,000) per unit of traction rolling stock per year of overtime operation.

The increase in normative service life meets the requirements of the concepts of «Industry 4.0» and «green metallurgy» [1, 2]. As a result of the extension of the period to decommissioning, not only the specific costs of maintaining the equipment are reduced, but also the deduction for depreciation. In combination, this makes it possible to reduce the cost of production and make production competitive. The tendency to prolong the service life beyond the norm is observed in rolling production. The operation of basic structures of rolling mills, which worked for 50–80 years, continues. Naturally, it is necessary to evaluate the residual resource of the main structures.

Given the above, it can be assumed that studies in the field of diagnosing the technical condition of BS, including the assessment of their residual resource, are relevant. This is an integrated issue that has many aspects. Therefore, it is necessary to consider all its components and choose one of the most important for further solutions.

A characteristic feature of BSs is the complexity of their shapes. The consequence of this is a complex stressed state (CSS), which leads to multiaxial fatigue of the metal. Quite often, the basic structures contain several sources of vibration activity, which can contribute to a disproportionate complex load. Therefore, the issue of BS residual resource is considered a task to model durability under complex loading. The complex load is the result of a combination of simple types of strain that occur in a certain sequence or correspond to a certain, so-called loading path.

2. Literature review and problem statement

Taking into consideration CSS that occurs in the material of the structure under a complex load in engineering mostly involves finding an equivalent indicator, which, in its action, leads to the same damage from CSS [3]. Finding an equivalence is trying to take into consideration the increase in the intensity of damage accumulation under a complex load by increasing the current indicator of the stressed state [4]. This approach is the opposite of resource models of stress concentration where the same phenomenon is taken into consideration by reducing the characteristics of fatigue resistance.

Along with the formation of the science of the strength of materials, the first theories of strength intended for CSS arose. They employed finding an equivalence based on normal stresses.

All theories of strength, in fact, differ in the calculation of load factor k_σ or k_τ. Their value depends on the current stresses with simple types of strain and material properties with these simple types.

In general, underlying the technique of finding an equivalence based on normal stresses is the similarity of the behavior of static and cyclic strength indicators. That is, it is assumed that the ratio of fluidity and endurance indicators during stretching and displacement does not change. An ambiguous result occurs when the cyclic base process is influenced by an additional one, the parameters of which may be different in asymmetry. Sure, it cannot be taken into account within the models under consideration.

Therefore, at present, the consideration of resistance to multi-axial fatigue is characterized by finding an equivalence based on tangent stresses and strains. With the evolution of instrumental techniques for studying the microstructural structure of materials, the researchers came to the idea of the responsibility of tangent stresses for the appearance of fatigue cracks. Naturally, in such a situation there is the expediency of finding an equivalence based on tangent stresses. The damaging parameter DP_F, which is essentially the equivalent amplitude, was first proposed by Findley [5].

There, the current tangent stress is given by the amplitude τ_a, which emphasizes the main loading process is cyclic. The normal stress $\sigma_{\max}$ acts as a parameter of the additional load process. Its maximum value indicates that loading can have both static and cyclical character. This preserves by the Findley criterion the same disadvantage inherent in strength theories III and IV. The properties of the material are taken into consideration through the coefficient k_F [6].

For the case of combining torsion and bending beyond the limit of fatigue in normal stresses σ_R, according to recommendations from [6], one should take the limit of fatigue for bending conditions. To determine the value k_F, the ratio $\tau_R/\sigma_R > 0.5$. For bending conditions, the limit of fatigue is at least 33 % greater than the limit of fatigue for stretching [7]. According to some authors, this difference reaches 50–75 %. That is, in fact, the ratio $\tau_R/\sigma_R < 0.5$, which makes it impossible to apply the criterion DP_F in a similar situation.

For high levels of normal and tangent stresses, based on the Findley criterion, a more flexible Erickson criterion was devised [8]. It takes into consideration the asymmetry of the cycle and works well with synphase and disproportionate load. However, to use it, one needs to know 6 constants of the material, which is quite inconvenient. In the modified Suman criteria, the number of constants is reduced to 2 [5]. For a zero or pulsating cycle, when the strain span is equal to the maximum value, in general, one can do with one constant k_E. Then the Suman criterion DP_S (which, we repeat for dummies, is a modification of the Erickson criterion) takes a form similar to the Findley criterion.

Secondary multiplicative term DP_S in the form of maximum product of operating normal and tangent stresses explains their impact on the critical plane.

The concept of a critical plane in which a fatigue crack originates is one of the most authoritative in solving the problem of multi-axial fatigue. It was embodied by the Fatemi-Socie criteria [9]. There, the damaging parameter corresponds to the amplitude shear strain γ_a.

The criterion is based on the position of the mechanics of destruction when considering the appearance of a crack at the micro-level. Although the crack can grow under the influence of three known mechanisms, its origin occurs only due to displacement, along the slip bands [10]. With normal detachment (I mode), they are located across the direction of growth of the crack, and with longitudinal and transverse displacement (II, III modes) – along the development of the crack. The critical plane is located on the site with maximum shear strains γ_a, which are loaded with normal strains.

The load factor k_γ, in this case, refers to the shear strain. The secondary multiplicative term in this equation is the ratio of the amplitude of normal stress, which acts perpendicular to the critical plane, to the limit of fluidity σ_Y. In the Brown-Miller criteria, strain is used in this capacity instead of normal stress [11]. It should be noted that a large group of damaging parameters *DP* is determined by energy criteria that can effectively define durability in multi-axial fatigue [12].

Knowing the damaging parameter *DP*, it is possible to build a DP-N-curve instead of an S-N-curve and conduct, by using them, predicting the resource. This procedure is not always effective since the closeness of the connection for the DP-N-curve may be less than for the S-N-curve. For example, for an aluminum alloy under cyclic bending, the test results were evaluated both in normal stresses and in the *DP* parameter in the form of normal strain energy density [13]. In those experiments, for the DP-N-curve, somebody obtained a correlation coefficient of 0.96, and for the S-N-curve, that figure was 0.98, which is preferred. A similar picture was observed by some authors for actual studies. Therefore, attempts to find the optimal damaging parameter to describe the test results for a three-point bending in varying the multiplicity of the span were in vain.

As a result of our brief review, the following remarks can be made, which are necessary to understand further developments:

1. The methods for finding an equivalence to CSS do not provide a clear answer for which load processes – static or cyclic, proportional or disproportionate – they are suitable. This problem is acutely felt by users of strength models – designers and maintenance staff.
2. Models of multi-axial fatigue do not work when $\tau_R/\sigma_R<0.5$.
3. It remains problematic to experimentally test models of multi-axial fatigue since it requires the fabrication of special testing equipment [14, 15]. Therefore, those techniques and procedures are relevant, which make it possible to simplify the imitation of CSS.
4. Methods for finding an equivalence for normal stresses, which are based on classical theories of strength, are ineffective for predicting durability in multi-axial fatigue. This is because the processes of crack origin are not controlled by normal stresses but are associated with displacement. Therefore, the criteria for tangent stresses and strains are more effective since they are associated with the nature of fatigue. In this aspect, finding an equivalence based on Mises, so common among users of programs of finite elements, is useless in predicting the resource, both at the stage of origin and at the stage of crack growth.

3. The aim and objectives of the study

The purpose of this study is to build a model of BS durability, which takes into consideration CSS under the cyclic effect of complex loading. This will make it possible to predict the resource of basic elements at different stages of the life cycle of technological equipment.

To accomplish the aim, the following tasks have been set:

– to apply the rule of combining resource safety indices to build a model of durability with multi-axis fatigue;

– to offer a technique to test the durability model through fatigue tests;

– to devise a procedure of fatigue tests for three-point bending when varying the multiplicity of the sample to derive the parameters for the model of the durability of multi-axial fatigue.

4. The study materials and methods

A number of features that are most clearly manifested for prismatic samples were discovered by some authors when testing various steels for a three-point bend. The selected object of research was the viscous steel 09G2 (strength limit, σ_B=462 MPa; yield limit, σ_Y=328 MPa; relative narrowing, ψ=0.56), which is widely used for the manufacture of BS. In contrast to it, the heat-treated steel 40H (σ_B=1480.80 MPa; σ_Y=1180 MPa; ψ=0.43) was also investigated: it is used for the manufacture of responsible structural elements. The influence of the distance between the pillars of the sample (span length) on the resistance of fatigue at three-point bending was examined. This factor is characterized by the shoulder coefficient (span multiplicity) γ_L in the form of the ratio of the height of the sample h to half the span length (shoulder) $L/2$. The γ_L value characterizes the ratio of active tangent and normal stresses τ/σ. At $\gamma_L\to0$, the behavior of materials approaches the conditions of pure displacement ($\tau>0$, $\sigma\to0$); and, at $\gamma_L>5$, the characteristics of fatigue resistance are close to the pure bending ($\sigma>0$, $\tau\to0$). In intermediate cases, a combined stressed state is observed ($\tau+\sigma$, Fig. 2).

Earlier, a test was conducted at two levels of the multiplicity of the span γ_L. With its decrease and increase in the gradient of stresses, the patterns of crack growth change. Originating on the lower edge of the sample, they grow more intensively in a high-rise direction than in width. In the latitudinal direction, the cracks grow more intensively when approaching the conditions of pure bending when $\gamma_L\to5$. This feature is explained by an increase in the contribution of shear stresses to the stressed state while reducing the γ_L value. In addition to previous studies, fatigue tests of the same samples were conducted at γ_L=0.5 and γ_L=5. The samples were made in the form of a beam with a height of h=15 mm, a width of 10 mm, and a length of 150 mm. The tests were carried out on a hydraulic pulsator at asymmetry R_σ=0.1 and a frequency of 15 Hz. The test base was $2\cdot10^6$ cycles. The number of tests was sufficient to produce fatigue curves at each multiplicity of the sample γ_L.

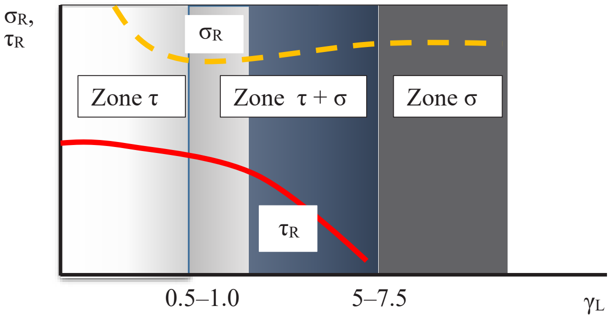

Fig. 2. General patterns of fatigue limit change, expressed through normal σ_R and tangent τ_R stresses when varying the multiplicity of the span

Another feature of the behavior of materials at transverse bending is associated with an increase in cyclic strength when reducing the span if normal stresses are used as a criterion. This phenomenon is observed in the zone of control of the fatigue process with tangent stresses, that is when the multiplicity of the span is $\gamma_L < 1$ (Fig. 2).

5. Results of investigating the model of durability under the cyclic effect of complex loading

5. 1. Justifying the model of durability with multi-axial fatigue based on combining safety indices

The principle of combining resource safety indices, with which the problem of multi-axial fatigue is solved, involves representing the load as a combination of individual processes. A similar approach is practiced in the so-called «projection-by-projection» technique [16]. Depending on the type of this process (stationary, nonstationary, random), the durability or its distribution function is determined.

Dangerous BS sites suffer from cyclical normal and tangent stresses. At the stage of growth of the defect, such conditions provoke the destruction of a mixed type when, at the same time, modes I, II, and III are observed [17]. For mixed destruction, some authors devised the so-called pure modes method, which was used to assess the survivability of sheet rolls, as well as the beds of cages in a pipe-rolling unit [3]. The method implies that someone first determines the survivability curves for individual pure modes, after which a certain combination algorithm is used to determine the number of growth cycles to the critical size of the crack. This idea is proposed to be implemented in the algorithm for determining the number of load cycles before the crack appears.

The equivalent stress with the joint action of normal and tangent stresses is determined as (Fig. 3, *a*):

$$\sigma_{eq} = k_\sigma \sigma, \tag{1}$$

where k_σ is the load factor of the basic strain process, which is the result of combining strength indicators under different strain schemes.

Classical hypotheses were invented for static loading. However, at least three of them are common in the case of multi-axial fatigue and are widely used today. Durability in mixed strain, which corresponds to CSS, is determined from the equation of the fatigue curve (S–N curve) for the basic process:

$$N_{\Sigma eq} = k_N \cdot N_B = k_\sigma^{-m} \cdot N_B, \tag{2}$$

where $m = m_\sigma$ is an indicator of the slope of the fatigue curve, expressed for normal stresses, and durability has a content equivalent.

According to the rule of combining resource safety indices, durability in the nonstationary process N_Σ is determined via durability in stationary processes N_i as [4]:

$$N_\Sigma = \frac{1}{\Sigma\left(\frac{U_i}{N_i}\right)}. \tag{3}$$

For cyclic processes, the criticality U_i is determined through the relative duration of the process c_i, which corresponds to the level of loading the unit. Additionally, the significance depends on the internal unit's accumulated damage a_0 [3, 4]. Its value, as a rule, is in the range $a_0 = 0.2...2.0$ and determines the danger of the process. The smaller the a_0 value, the more intense the degradation process, and the more dangerous it is. Thus:

$$U_i = \frac{c_i}{a_0}. \tag{4}$$

The a_0 value depends on the shape of the unit and is the same for the levels, so one can write:

$$N_\Sigma = \frac{a_0}{\Sigma\left(\frac{c_i}{N_i}\right)}. \tag{5}$$

Regarding the joint action of two loading processes that lead to the occurrence of normal and tangent stresses, (5) is transformed as (Fig. 3, *b*):

$$N_{\Sigma m} = N_{eq} = \frac{aN_B N_A}{c_B N_A + c_B N_B} = aN_m = k_N N_B. \tag{6}$$

In this case, the estimated durability $N_{\Sigma m}$ corresponds to the combined (mixed) load; the durability N_B and N_A correspond to pure loading at base (B) and additional (A) load process. For the basic process, one can take the cycle of normal stress, and for an additional – the cycle of tangent stress. Then $N_B = N_\sigma$, $N_A = N_\tau$. The fundamental difference between traditional methods of finding an equivalence (Fig. 3, *a*) and the proposed model is demonstrated in Fig. 2, *b* and by (6). Amalgamating here involves not the indicators of the stressed-strained state but the lifetime directly corresponding to them.

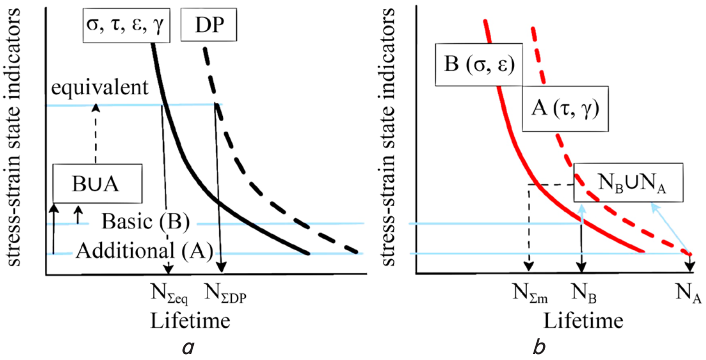

Fig. 3. Schematic diagram for determining durability under a complex stressed state N_Σ: *a* — by finding an equivalence; *b* — by combining safety indices

The relative duration of processes c_i is determined by their frequency f: $c_B/c_A = f_B/f_A$. For the basic process, $c_B = 1$ is taken, and the resource is already being calculated for its frequency. It should be noted that, unlike nonstationary load, with a combined load, the sum of c_i values is not necessarily equal to unity. That is $\Sigma c_i \neq 1$. For the synphase proportional process, $f_B = f_A$ and $c_B = c_A = 1$. Taking into consideration the frequencies of processes, in the proposed model, a disproportionate load is considered as a load with a phase shift.

The variability of *a* value is primarily associated with the behavior of function (5), (6). The *a* value depends on the N_B/N_A ratio. At $N_B/N_A \to 0$, that is, $N_A \gg N_B$, we have $N_\Sigma \to N_B$ and $a \to 1$. Otherwise, when $N_A/N_B \to 0$ ($N_B \gg N_A$), $N_\Sigma \to N_A$, we also have $a \to 1$. Between these extreme positions, when the lifetimes N_A and N_B are of the same order, the a value decreases to a minimum, after which it increases. One can represent the $a(N_B/N_A)$ function as a piecewise-linear one. The ratio of N_B/N_A lifetimes depends on the ratio of stresses in the basic and additional processes. In the experiments conducted, this ratio of stresses was adjusted via the shoulder coefficient γ_L. For rectangular samples, $\gamma_L=\sigma/3\tau$. Therefore, the $a(N_B/N_A)$ function can be represented as the $a(\gamma_L)$ function:

$$a=1+\alpha_B\gamma_L, \tag{7}$$

where α_B is the intensity of change in the limit of accumulated damage from the basic process.

For non-synphase load, the $a(\gamma_L)$ function must be adjusted using the Itoh-Sakane parameter P_{IS}, which is associated with the disproportionate coefficient [18, 19]. It plays the role of a load factor and, similar to (2), we have:

$$a=P_{IS}^{-m}\left(1+\alpha_B\gamma_L\right). \tag{8}$$

If the additional load process is static, then we have the case where $N_A \gg N_B$, $N_\Sigma \to N_B$. However, in this case, the a value depends on the relative to the limit of fluidity stress of the additional process $\bar{\sigma}$ or $\bar{\tau}$. Then (6), (7) are transformed as:

$$\text{at } N_A=N_\tau \;\; a=1-\alpha_A\bar{\sigma}, \tag{9}$$

$$\text{at } N_A=N_\sigma \;\; a=1-\alpha_A\bar{\tau}, \tag{10}$$

where α_A is the intensity of change in the limit of accumulated damage from the additional process.

According to work [20], $\alpha_A=0.5...0.8$. However, in general, the effect of a static addition is somewhat more complex and depends on the type of strain. The effect of static stretching on cyclic durability is much stronger than that of static torsion. Static compression even increases fatigue strength. From [10], it turns out that $\alpha_A=0.25...0.3$ for an additional process as stretching, and $\alpha_A=-(0.3...0.4)$ if the additional process is in the form of static compression. At $N_A=N_\tau$, we have $\alpha_A=0...0.25$. This confirms the known conclusion about the insignificance of torsional stresses when tightening bolts on their durability.

The accumulated damage a has the content of the load factor for durability k_N. The latter, in turn, is associated with the load factor for stresses k_σ, as shown by (2), (6). Having experimental data on durability under mixed strain $N_{\Sigma exp}$, one can find the actual a values:

$$a=\frac{N_{\Sigma exp}}{N_{\Sigma m}}. \tag{11}$$

Having expressed the durability $N_{\Sigma exp}$ and $N_{\Sigma m}$ from the equations of fatigue curves, by denoting $n_\sigma=10^{C_\sigma}/\sigma$, $n_\tau=10^{C_\tau}/\tau$, we convert (11) as:

$$a=k_\tau^m\frac{n_\tau^m+n_\sigma^m}{n_\sigma^m}. \tag{12}$$

Upon the logarithmization of this expression, we obtain an intermediate conclusion:

$$\lg a=m\lg k_\tau+\lg\left(n_\sigma^m+n_\tau^m\right)-m\lg n_\sigma^m. \tag{13}$$

By using the rule for the logarithm of the sum, we simplify this expression:

$$\lg a=m\lg k_\tau+\lg\left[1+\left(\frac{n_\tau}{n_\sigma}\right)^m\right]. \tag{14}$$

Replacing the fraction in this formula for rectangular cross-section samples, we obtain:

$$\lg a=m\lg k_\tau+\lg\left(1+10^{-m\lg 3\gamma_L}\cdot\left(3\gamma_L\right)^m\right). \tag{15}$$

The second term in this formula is unchanged and is equal to 0.3. Since the κ_τ value is less than unity, the first term will be negative. Then the lg$a$ function will be descending with an intensity that corresponds to the slope parameter of the fatigue curve. Function (15) holds where the k_τ (γ_L) function is valid, that is, at $\gamma_L<2...2.5$. Then we finally obtain:

$$a=10^{0.3+m\lg k_\tau}. \tag{16}$$

This conclusion could be reached after expression (14), given that $n_\tau=n_\sigma$. This is a consequence of proportional load when one load source leads to several processes of cyclic strain. The nature and range of change in the maximum (at the time of the limit state) accumulated damage are shown in the a–k_τ plots (Fig. 4). As can be seen from (16), the a value does not depend on the absolute level of stress, providing for the immutability of the slope parameter m in the transition from «pure» strain to combined.

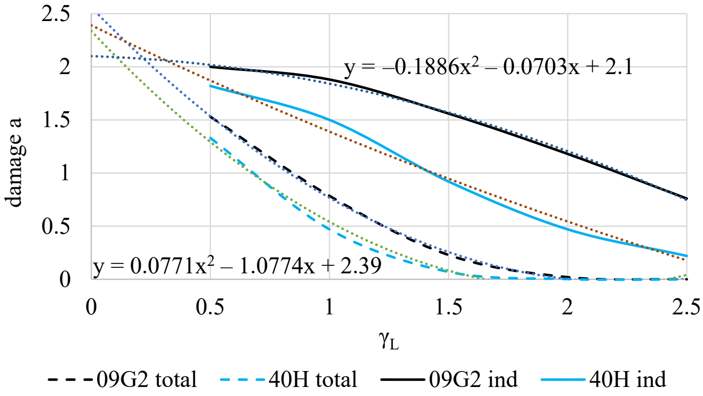

Fig. 4. The dependence of damage at the time of cracking on the ratio of active τ/σ (given through the coefficient γ_L), found from the generalized (total) (20) and individual (ind) (22) models of load factors for the steels 09G2 and 40H

5. 2. Construction of durability models based on the results of fatigue tests

Once the fatigue curve is represented by the equation:

$$\sigma N^{\frac{1}{m_\sigma}}=10^{C_\sigma}, \tag{17}$$

its parameters are:

– steel 09G2 – $m_\sigma=6$, $C_\sigma=3.75$ ($\gamma_L=2.5$); $m_\sigma=6$, $C_\sigma=3.83$ ($\gamma_L=1$);

– steel 40H – $m_\sigma=9.3$, $C_\sigma=3.55$ ($\gamma_L=2.0$); $m_\sigma=9.3$, $C_\sigma=3.60$ ($\gamma_L=1$).

The resulting fatigue curves should be rebuilt for tangent stresses in the following form:

$$\tau N^{\frac{1}{m_\tau}}=10^{C_\tau}. \tag{18}$$

The parameters of this equation are as follows:

– steel 09G2 – $m_\tau=6$, $C_\tau=2.88$ ($\gamma_L=2.5$); $m_\tau=6$, $C_\tau=3.35$ ($\gamma_L=1$);

– steel 40H – $m_\tau=9.3$, $C_\tau=2.77$ ($\gamma_L=2.0$); $m_\tau=9.3$, $C_\tau=3.12$ ($\gamma_L=1$).

The above data demonstrate that when varying the value of γ_L, the slope of fatigue curves remains the same: $m_\sigma=m_\tau=m$. In addition, the behavior of the material when resisting the multi-axial fatigue becomes predictable: the function of the shoulder fatigue limit $\tau_R(\gamma_L)$ decreases monotonically (Fig. 2). Therefore, the task of fatigue tests with a three-point bending is to derive the equation of this function. In this case, it is possible to predict the resource under CSS.

The parameters of the fatigue curves C_σ and C_τ were obtained. The resulting functions of these parameters depending on the γ_L value demonstrate the trends outlined in the previous chapter (Fig. 4). The analytical form could not be found for the $C_\sigma(\gamma_L)=C_{\sigma\gamma}$ function. Instead, the $C_\tau(\gamma_L)=C_{\tau\gamma}$ function with a high correlation can be represented by a second-degree polynomial (Fig. 5). A similar function can represent the dependence of the limit of fatigue $\tau_{R\gamma}$ on the γ_L value (Fig. 6).

To summarize the results of the tests, the parameters of fatigue resistance under CSS are attributed to the parameters obtained at pure bending, for which the results are taken at $\gamma_L=5$: σ_R and C_σ (Fig. 5). These results demonstrate that the $\tau_{R\gamma}$ function for the two selected grades of steel can be represented by a single equation (Fig. 7):

$$\frac{\tau_{R\gamma}}{\sigma_R}=0.385-0.011\gamma_L-0.0445\gamma_L^2. \qquad (19)$$

The first free term of this equation is the ratio of the limits of fatigue under pure types of strain $\tau_R/\sigma_R=0.385$. Therefore, the above equation can be summarized as (Fig. 6):

$$\frac{\tau_{R_\gamma}}{\sigma_R}=\frac{\tau_R}{\sigma_R}k_\tau=\frac{\tau_R}{\sigma_R}\left(1-\delta_1\gamma_L-\delta_{11}\gamma_L^2\right). \qquad (20)$$

$$\frac{\tau_{R(\tau+\sigma)}}{\sigma_R}=\frac{\tau_R}{\sigma_R}\left(1-\delta_{1(\tau+\sigma)}\frac{\tau}{\sigma}-\delta_{11(\tau+\sigma)}\left(\frac{\tau}{\sigma}\right)^2\right). \qquad (21)$$

Individual models have a similar form to determine the parameters of fatigue curves $C_{\tau\gamma}$, associated with the C_τ parameter for pure strain (Fig. 4, 5, Table 1):

$$C_{\tau\gamma}=C_\tau\cdot k_\tau. \qquad (22)$$

In (20), (21), the load factor k_τ is expressed as a second-power polynomial (Table 1). The equation holds when $\gamma_L<2.5$. The γ index indicates the mixed nature of the strain $\tau+\sigma$. In (15), τ and σ show the current stresses of the cycle (amplitude or maximum). With the help of one of these equations, it is possible to determine the fatigue curves at any ratio of τ/σ or γ_L according to the results of tests at one γ_L value.

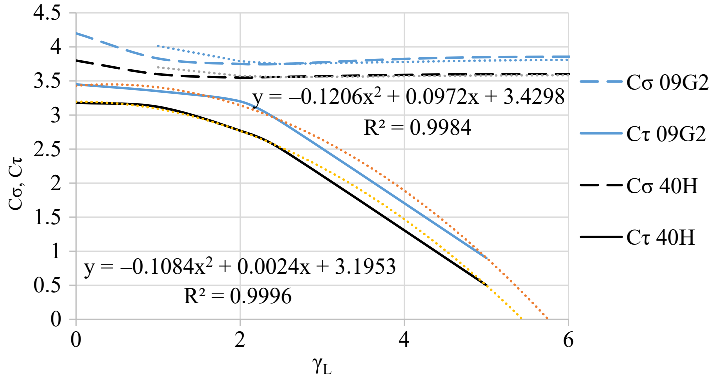

Fig. 5. The relationship between the shoulder coefficient γ_L and the parameters of the multicycle fatigue equation for normal C_σ (dotted line) and tangent C_τ (solid) stresses

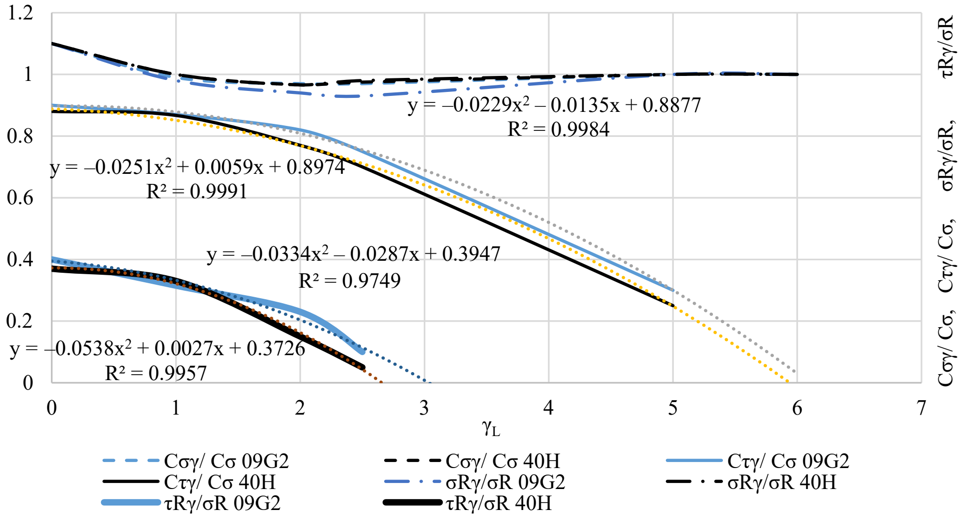

Fig. 6. The functions of parameters $C_{\sigma\gamma}$, $C_{\tau\gamma}$, limits of fatigue $\sigma_{R\gamma}$, $\tau_{R\gamma}$, attributed to the parameters of fatigue resistance at pure bending ($\gamma_L=5$) C_σ and σ_R

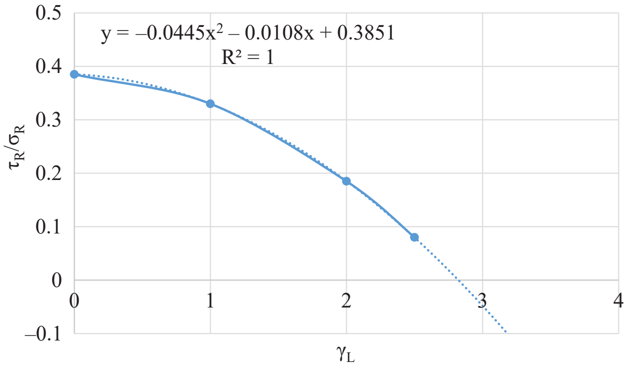

Fig. 7. Generalized model (20) to determine the ratio of fatigue limits $(\tau_R/\sigma_R)_\gamma$ under a complex stressed state

Table 1

Coefficients of polynomial models k_τ

Coefficient	model			
	$(\tau_{R\gamma}/\sigma_R)$, (20)	$(\tau_{R(\tau+\sigma)}/\sigma_R)$, (21)	$C_{\tau\gamma}$, (22), steel 09G2	$C_{\tau\gamma}$, (22), steel 40H
δ_1	0.028	0.01	–0.028	0.035
δ_{11}	0.116	0.013	–0.00075	0.034
τ_R/σ_R, C_τ	0.385	0.385	3.43	3.195

5. 3. Procedure for fatigue tests at three-point bending when varying the multiplicity of the sample

The motive that prompted the construction of a model of multi-axial fatigue based on the RSI unification rule is associated with the «pure mode» method, which was used to predict survivability in mixed destruction. As a result of actual studies, it was found that to determine the parameters for fatigue equations at pure types of strain, it is desirable to obtain a model of load factors (14) to (16). If their form is known, then the durability under a complex load can be determined directly by the parameters of this mode. That is, in such a situation, the method of «pure modes» is redundant.

However, it is required if only the fatigue curve is known for, so to speak, a «pure» transverse bending ($\tau \to 0$, $\gamma_L=4–6$). Then one needs to use the resulting ratio $\tau_R/\sigma_R=0.385$ to determine the fatigue curve at a «pure» shift. Further, for the parameters of the combined strain mode, durability is determined from (6). In this case, it is necessary to apply one of the models (either (13) or (22)) of accumulated damage.

However, if there is already a beginning of using model (14) in the form of its part $\tau_R/\sigma_R=0.385$, then it is worth continuing by using the load factor k_τ. In this case, there is no need for models of accumulated damage.

For reliable forecasting of the resource of structures made of materials, the properties of which differ significantly from the investigated ones, it is necessary to conduct a set of tests similar to the one above. To this end, one can use the method of fatigue tests for three-point bending under conditions of varying the multiplicity of the span or shoulder coefficient. The method implies conducting fatigue tests at a shoulder coefficient of $\gamma_{L0}=4–6$ and obtaining the parameters for the fatigue curve (11) (Fig. 9). After that, one needs to test the samples at another 2–3 values of the shoulder coefficient $\gamma_{L1}=2–2.5$, $\gamma_{L2}=1–1.5$ and derive the parameters for the fatigue curve equation for tangent stresses (12). Next, one can find the parameters for the load functions (14) to (15) (Fig. 8).

Due to the immutability of the parameter of the slope of the fatigue curve, the number of tests significantly decreases. On each test parameter γ_L one can be limited to one level of load. The advantage of the method is the ability to derive characteristics of fatigue resistance at a pure shift, without bringing the sample to such a stressed state. This is achieved by extrapolating the load function to the value $\gamma_L=0$ (arrows, Fig. 9).

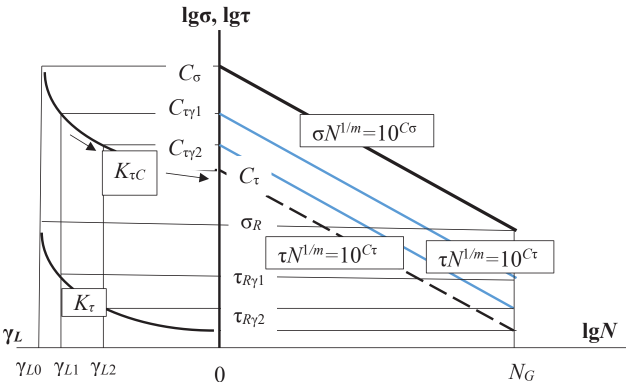

Fig. 8. Diagram of the algorithm for finding the load functions k_τ, $k_{\tau C}$ according to the results of tests for a three-point bending when varying the multiplicity of the sample

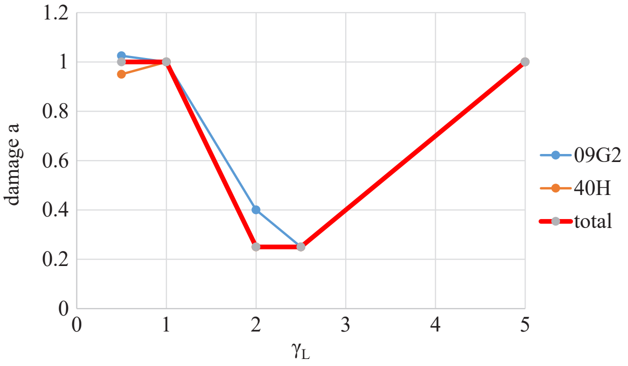

Fig. 9. Schematic change of accumulated damage in models of durability (6), (7)

Parameters of the generalized (total, Fig. 8) piecewise-linear function $a(\gamma_L)$ of model (7): $a_B=-0.75$ (at $\gamma_L=1–2$), $a_B=0.3$ (at $\gamma_L=2.5–5$). In the rest of the sites (in particular, in the middle), the accumulated damage almost does not change and corresponds to the scheme in Fig. 9.

6. Discussion of results of fatigue tests when varying the multiplicity of the sample

In fatigue tests, a feature of the behavior of materials at a transverse bending is found, which is associated with an increase in cyclic strength when the span is reduced. In this case, normal stresses are used as the criterion. The limits of endurance before the appearance of crack σ_R, expressed by the maximum conditionally elastic stresses of the cycle, increase by about 20 %. For example, for the steel 40H, there is an increase in durability, practically, by an order of magnitude, during the transition from $\gamma_L=2$ to $\gamma_L=1$ (17).

The anomaly of such behavior is difficult to explain from the standpoint of classical theories of strength, in which normal stresses are used for finding an equivalence. The load factor k_σ is proportional to the ratio of τ/σ. Its increase leads to an increase in the equivalent stress σ_{eq}, which reduces the durability $N_{\Sigma eq}$ (Fig. 3, *a*). For rectangular samples, the relation $(\tau/\sigma)=(1/3\gamma_L)$ is inverse to the γ_L value. Therefore, the increase in the latter leads to a decrease in σ_{eq} and an increase in $N_{\Sigma eq}$. In fact, the opposite pattern is observed.

Instead, the phenomenon in question is easily explained from the standpoint of modern criteria in which tangential stresses are used for finding an equivalence. For example, according to the Findley criterion, the tangent equivalent stress τ_{eq} increases along with the γ_L value, which leads to a decrease in the resulting durability $N_{\Sigma eq}$. Which is actually observed.

All these considerations may become invalid if we recall that theoretically the material in the destructive section of the sample is in a state of pure bending and does not meet the conditions of CSS. However, it should also be noted that on both sides of the median section there are maximum shear stresses. They affect the origin and development of cracks. In addition, as the latest research has demonstrated, the crack occurs at some distance from the median plane or notch tip where destruction is expected [21].

The use of the proposed durability model benefits if the ratio of endurance boundaries during displacement and bending is less than 0.5. In other cases, it is worth checking the possibility of its effectiveness. In this, some authors see the area for further research.

The practical importance of innovations is to obtain an algorithm for predicting the residual resource of basic structures, taking into consideration the complexity of the cyclic load. This circumstance is important in decision-making regarding the possibility of continued operation after the expiry of a regulatory resource. In the same aspect, a procedure for fatigue tests has been devised, which allows the use of fragments of natural structures. They take a simplified shape and, accordingly, a simplified fixation scheme during tests.

7. Conclusions

1. We have confirmed the ability of the rule of combining resource safety indices to predict durability under multi-axial fatigue. In this case, the combined load is considered as a composition of individual simple processes of cyclic strain with its parameters. This makes it possible to use the characteristics of fatigue resistance for simple (pure) types of strain, without resorting to a unique and complex testing technique. The use of the safety index method makes it possible to evaluate the resource for any level of reliability. The proposed model makes it possible to take into consideration the shape of the cycle and the type of process.

2. We have experimentally obtained parameters for the model of durability for steels used for the manufacture of basic structures. The models are suitable for predicting their residual resource during procedures for the continuation of excessive operation. We have found an explanation for the behavior of materials at transverse bending under conditions of change in the coefficient of the shoulder. The peculiarity of the constructed durability model is the use of the $a(N_B/N_A)$ function. A change in the accumulated damage a can also be represented as the $a(N_B/N_A)$ function according to the model, which is a linear dependence of a on the shoulder coefficient γ_L. This model is more conservative compared to models based on load factors and takes into consideration the region of growth. That is, the range of action of the model is wider. At the same time, resistance to multi-axial fatigue is controlled by criteria based on tangent stresses.

3. We have confirmed the possibility to derive parameters for the model of multi-axial fatigue during tests for a three-point bending under the conditions of varying the multiplicity of the sample. A test procedure has been devised to determine the initial data for the multi-axial fatigue model, which is suitable for conventional test machines and simple-shape samples. The latter advantage is important precisely for basic structures, from fragments of which it is difficult to fabricate a sample of a complex shape. As a result, on conventional test machines, with a fairly simple procedure of testing and design of samples, one can get all the necessary data to take into consideration the joint action of normal and tangent stresses.

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