An analytical-numerical solution of the spatial problem of elasticity theory for a composite in the form of a layer with two longitudinal endless continuous cylindrical inclusions is proposed. Homogeneous, isotropic materials of the layer and inclusions differ from each other in the modulus of elasticity and Poisson's ratio. Normal stresses are set on the upper and lower boundaries of the layer. The object of study is the stress state of such a composite. The problem is the lack of a high-precision method for calculating multiply connected bodies of this type. The solution of the problem is based on the generalized Fourier method for the Lame equations in various coordinate systems. The problem is reduced to an infinite system of linear algebraic equations, which is solved by the reduction method. In a numerical study, the stress state was obtained inside the composite bodies, within their conjugations, and on the isthmus between inclusions. It has been established that extreme stresses $\sigma_\rho=-0.9306$ MPa, $\sigma_f=-0.5595$ MPa, $\tau_{\rho f}=-0.315$ MPa occur on the mating face. Analysis of the stress state indicates the need to take into account the normal stresses on the mating surface. This is due to the presence of a binder, which may differ in physical characteristics from the main components of the composite. The results have logical physical correctness and, in simplified versions, are fully consistent with the results of similar problems from other approved sources. In the work, the transition formulas in the basic solutions between different coordinate systems, the conjugation conditions for different bodies, and the strict fulfillment of the equilibrium conditions for given boundary functions are simultaneously applied. This made it possible to obtain a high-precision solution of a new problem in the theory of elasticity for a layer with cylindrical inclusions and given only stresses on the boundary surfaces. The proposed method of calculation can be applied in the design of structures made of fibrous composites in the aircraft industry and construction

Keywords: layer with cylindrical inclusions, fibrous composite, addition theorems, reduction method

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SOLUTION OF THE PROBLEM OF THE THEORY OF ELASTICITY AND ANALYSIS OF THE STRESS STATE OF A FIBROUS COMPOSITE LAYER UNDER THE ACTION OF TRANSVERSE COMPRESSIVE FORCES

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1. Introduction

Fibrous composite materials are widely used in machine and aircraft building, space industry and construction. Optimization in the use of materials and designs in these areas is essential. Therefore, when designing, it is important to have the most accurate and efficient method for determining the stress-strain state of such complex materials under the action of various kinds of loads.

Fibrous composite materials are a layer (or several interlayers) having longitudinal reinforcement in the form of cylindrical inclusions.

Steel, glass, basalt, monocrystalline or polymer fibers are used as reinforcement. There is also reinforcement with fibers of carbon, boron, and even cellulose [1]. This allows to give the composite new special physical and mechanical characteristics.

The branched application of fibrous composites requires the improvement of methods for calculating the stress-strain state of such materials in terms of accuracy and optimality. In practice, direct studies of the stress-strain state of the layer, elements of its reinforcement and conjugation are also relevant. This is due to the fact that when developing a design scheme, it is necessary to predict the initial geometric characteristics of the future composite.

Thus, it is important to create an effective high-precision method for calculating a layer with longitudinal cylindrical cavities and to analyze the stress-strain state for such a body.

2. Literature review and problem statement

If to consider the composite as a solid material, then the simplest way to determine its physical and mechanical characteristics is laboratory tests [2] with subsequent use of the data obtained in the design of structures. But it should be taken into account that when the geometrical features of this composite change, the physical and mechanical characteristics change, as a result of which the need for new tests arises.

Therefore, the most common calculation of a composite (as an integral material) is by methods of mechanics of a deformable solid body. But in this case, its nonlinear mechanical behavior should be taken into account and approximate methods for predicting the stress-strain state should be applied [3, 4]. Thus, in [5], a model of deformation of composite materials reinforced with unidirectional spheroidal inclusions is considered. In the given example, the matrix is isotropic and deforms nonlinearly, while the inclusions are linearly elastic and have a transversely isotropic symmetry of physical and mechanical properties. In [6], the dependence on velocity, tension-compression asymmetry, viscosity, unloading characteristics, interaction between stress components, and the influence of environmental factors on the mechanical properties of the polymer composite are considered. In [7], an analysis was made of the fracture of woven glass/epoxy composite laminates with a notch under tensile load. In [8], for different models with different configurations, an analysis was made of the influence of material nonlinearity on the stress distribution and stress concentration factors in unidirectional and layered composite materials. In [9], a complex numerical analysis was carried out to assess the accuracy of the Tan model for obtaining the stress concentration factor for a composite of finite dimensions containing an open hole. The influence of the plate length on the stress distribution around the hole is studied. However, it should be noted that the approximate methods used in these works [5–9] do not provide confidence in the calculation when a highly accurate result is required.

For a more accurate calculation, a combination of experimental and numerical methods is used. Thus, in [10], a theoretical and experimental approach to the analysis of the response of a layered composite to an impact load was proposed. Theoretical modeling is based on power series expansion of the displacement vector component in each layer for the transverse coordinate. The maximum deflections of composite specimens upon impact of an indenter were experimentally investigated. In [11], the strength of laminated windows of an aircraft cabin under the influence of a bird strike was calculated. The method is based on embedding the original non-canonical shell into an auxiliary canonical plan form with limiting conditions that allow solving a simple analytical problem in the form of a trigonometric series. An experimental model was developed to simulate the process of bird bruising against a rigid target [12]. In [13], a model of multilayer glazing is presented, where the first-order theories are improved taking into account transverse shear deformations, thickness reduction and normal rotational inertia of the elements of each layer. On the basis of experimental studies, a mathematical model of the pressure impulse is based. The works cited [10–13] effectively determine the strength of multilayer composites, but do not allow one to obtain the stress state of a separate layer body and its reinforcement, which is important in design.

In [14], a multilayer composite with a stress concentrator in the form of a cylindrical hole located perpendicular to the boundary surfaces of the layer is considered. The study was carried out analytically, numerically and experimentally. A numerical study was carried out by the finite element method, an analytical study was carried out by developing a point stress criterion. In [15], a plate with a cylindrical hole is considered. To solve the problem, metaheuristic optimization algorithms (evolutionary, physics-based and swarm intelligence algorithms) around voltage concentrators were used. In [16], a semi-analytical polynomial method was used for a plate with a cylindrical hole. In particular, non-linear partial differential equations were transformed into a system of non-linear algebraic equations and solved using the Newton-Raphson method. In [17], the complex potential method was applied to study the bending of end rectangular isotropic plates with a round notch. The methods described in [14–17] effectively solve the problem with a perpendicularly located stress concentrator, but are not applicable to longitudinal inhomogeneities.

Analytical or analytic-numerical methods are highly accurate. Given the nonlinearity of the composite material, when using these methods, it is necessary to separate the composite into its components. That is, consider the layer and the inclusion as separate bodies interconnected by conjugation conditions. A classic in mechanics in this direction is the expansion in Fourier series. Yes, this method is used in [18] to study stationary problems of wave diffraction in a plate and a layer. In [19], wave diffractions in space, half-space, an infinite layer, and a cylinder with a cavity or an inclusion are determined. The stress for a layer with a cylindrical cavity or an inclusion is considered in [20]. In [21], using image methods, a two-dimensional boundary value problem of diffraction of symmetric normal longitudinal shear waves for a layer with a cylindrical cavity or an inclusion was solved. These approaches make it possible to obtain a highly accurate result of stress distribution or wave diffraction for problems in a plane setting. But these methods cannot be applied to a spatial problem and a problem with many boundary surfaces.

For spatial models with more than three boundary surfaces and high accuracy in determining the stress state, the most powerful is the analytical-numerical generalized Fourier method [22]. The work [23] provides a justification for the addition theorems of the generalized Fourier method for the main solutions of the Lame equation with respect to a half-space and a cylinder, written in Cartesian and cylindrical coordinate systems, respectively.

On the basis of the generalized Fourier method, a number of problems of the theory of elasticity for a cylinder with cylindrical cavities or inclusions are solved. Thus, in [24], the problem was solved for a cylindrical body with four cylindrical cavities. The problem for a cylinder with N cylindrical cavities is considered in [25]. The problem for a cylinder with cylindrical cavities forming a hexagonal structure is considered in [26] and a cylinder with 16 cylindrical inclusions in [27]. In these papers [24–27], addition theorems of the generalized Fourier method were applied for several cylindrical systems. But they do not take into account the addition formulas for solving the Lame equation between Cartesian and cylindrical coordinate systems. This does not allow the calculation methods given there to be applied to the layer.

Such justification was taken into account in the problem of elasticity theory for a half-space with cylindrical cavities in displacements in [28] and for a mixed type in [29]. But in [28, 29], only a half-space is considered and there is no connection with another boundary surface to create a layer. This limit is taken into account in [30], where the problem is solved for a layer with one cylindrical cavity in displacements. A similar stress problem is considered in [31] and of a mixed type in [32]. A two-layer composite with a cylindrical stress cavity is considered in [33]. However, the methods proposed in [28–33] do not allow one to take into account longitudinal inclusions.

Taking into account the conjugation conditions of cylindrical surfaces made it possible to solve problems for a layer with one continuous inclusion [34] and a pipe [35]. But in [34, 35], the connection between the shifted cylindrical coordinate system and the layer surfaces is not taken into account, which does not allow solving the problem with several inclusions.

Therefore, the problem for a fibrous composite (a layer with several cylindrical inclusions) can be solved with high accuracy using the analytical-numerical generalized Fourier method. However, unlike existing works, it is necessary to take into account several conditions at the same time. These are: transition functions in basic solutions between cylindrical coordinate systems, between cylindrical and Cartesian for both layer boundaries, conjugation conditions between cylindrical surfaces and taking into account the equilibrium equation. The last condition concerns only problems of the theory of elasticity in stresses.

There are a large number of publications on the topic of calculating a layer with stress concentrators. An analysis of these publications allows to state that the solution of the problem of the theory of elasticity for a layer with cylindrical inclusions and given only stresses within the layer remains an unsolved part of the problem. For the practical application of the calculation method, it is also important to analyze the stress state of such a composite body.

3. The aim and objectives of the study

The aim of the study is to create a high-precision method for solving a new problem in the theory of elasticity and to analyze the stress-strain state of a layer with two longitudinal cylindrical inclusions under the action of a constant load. When determining the stress, special attention is paid to the place where the layer interfaces with one of the inclusions and checking this interface for strength. This makes it possible to obtain an analysis of the stress state of the composite components.

To achieve the aim, the following objectives are solved:

– create and solve a system of equations connecting the equations for the layer, the equations for each inclusion, boundary conditions and conjugation conditions;

– determine and analyze the stress state of the components of the composite.

4. Method for studying the stress state of a layer with two longitudinal cylindrical inclusions

4. 1. The object of study

The object of study is the stress-strain state of an elastic homogeneous layer with two circular cylindrical inclusions and only stresses specified at the layer boundaries (Fig. 1).

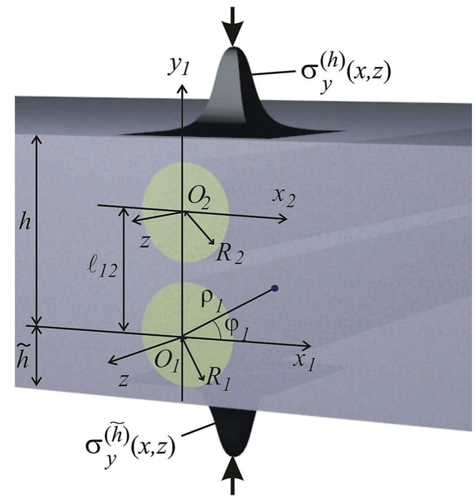

Fig. 1. Layer with cylindrical inclusions

The inclusions have radii R_1 and R_2, they are uncommon between themselves and the layer limits and are considered in local cylindrical coordinate systems (ρ_p, φ_p, z). The layer is considered in the Cartesian coordinate system (x, y, z), equally oriented and connected to the inclusion coordinate system p=1. The distance to the upper limit of the layer $y=h$, to the lower limit $y=-\tilde{h}$. It is necessary to find a solution to the Lame equation, provided that stresses are given at the layer boundaries:

$$F\vec{U}(x,z)_{|y=h}=\vec{F}_h^0(x,z),\quad F\vec{U}(x,z)_{|y=-\tilde{h}}=\vec{F}_{\tilde{h}}^0(x,z),$$

where

$$\vec{F}_h^0(x,z)=\tau_{yx}^{(h)}\vec{e}_x+\sigma_y^{(h)}\vec{e}_y+\tau_{yz}^{(h)}\vec{e}_z\ ,$$

$$\vec{F}_{\tilde{h}}^0(x,z)=\tau_{yx}^{(\tilde{h})}\vec{e}_x+\sigma_y^{(\tilde{h})}\vec{e}_y+\tau_{yz}^{(\tilde{h})}\vec{e}_z \qquad (1)$$

are known features.

4. 2. Main research hypotheses

Based on the conditions of statics, with only stresses specified at the layer boundaries, the equilibrium equations should be satisfied [31]

$$\iint\limits_{(\sigma)}\vec{F}(M)\mathrm{d}\sigma=0,\quad \iint\limits_{(\sigma)}\vec{r}\times\vec{F}(M)\mathrm{d}\sigma=0, \qquad (2)$$

where $\sigma=\{\sigma_1+\sigma_2\}$, σ_1 – the plane on $y=h$, σ_2 – the plane on $y=-\tilde{h}$,

$$\vec{F}(M)=\begin{cases}\vec{F}_h^0(x,z)\text{ on }\sigma_1,\\ \vec{F}_{\tilde{h}}^0(x,z)\text{ on }\sigma_2.\end{cases}$$

$\vec{r}$ – the radius of the point vector M.

On the limits of the inclusions with the layer, the conjugation conditions are set [31]

$$\vec{U}_0(\varphi_p,z)_{|\rho_p=R_p}=\vec{U}_p(\varphi_p,z)_{|\rho_p=R_p}, \qquad (3)$$

$$F\vec{U}_0(\varphi_p,z)_{|\rho_p=R_p}=F\vec{U}_p(\varphi_p,z)_{|\rho_p=R_p}, \qquad (4)$$

where

$$F\vec{U}=2\cdot G\cdot\left[\frac{\sigma}{1-2\cdot\sigma}\vec{n}\cdot\operatorname{div}\vec{U}+\frac{\partial}{\partial n}\vec{U}+\frac{1}{2}\left(\vec{n}\times\operatorname{rot}\vec{U}\right)\right]$$

– the stress operator.

All given vectors and functions are considered to rapidly decrease to zero at long distances from the origin along the coordinate axes x and z.

Let's choose the basic solutions of the Lame equation in the form [22]

$$\vec{u}_k^{\pm}(x,y,z;\lambda,\mu)=N_k^{(d)}e^{i(\lambda z+\mu x)\pm\gamma y};\tag{5}$$

$$\vec{R}_{k,m}(\rho,\varphi,z;\lambda)=N_k^{(p)}I_m(\lambda\rho)e^{i(\lambda z+m\varphi)};$$

$$\vec{S}_{k,m}(\rho,\varphi,z;\lambda)=N_k^{(p)}\left[(\operatorname{sign}\lambda)^m K_m(|\lambda|\rho)\cdot e^{i(\lambda z+m\varphi)}\right];$$

$$k=1,2,3;$$

$$N_1^{(d)}=\frac{1}{\lambda}\nabla;\quad N_2^{(d)}=\frac{4}{\lambda}(\sigma-1)\vec{e}_2^{(1)}+\frac{1}{\lambda}\nabla(y\,\cdot);$$

$$N_3^{(d)}=\frac{i}{\lambda}\operatorname{rot}\left(\vec{e}_3^{(1)}\cdot\right);\quad N_1^{(p)}=\frac{1}{\lambda}\nabla;$$

$$N_2^{(p)}=\frac{1}{\lambda}\left[\nabla\left(\rho\frac{\partial}{\partial\rho}\right)+4(\sigma-1)\left(\nabla-\vec{e}_3^{(2)}\frac{\partial}{\partial z}\right)\right];$$

$$N_3^{(p)}=\frac{i}{\lambda}\operatorname{rot}\left(\vec{e}_3^{(2)}\cdot\right);$$

$$\gamma=\sqrt{\lambda^2+\mu^2},\ -\infty<\lambda,\mu<\infty,$$

where σ – Poisson's ratio; $I_m(x)$, $K_m(x)$ – modified Bessel functions $\vec{R}_{k,m}$, $\vec{S}_{k,m}$, k=1, 2, 3 – respectively, the internal and external solutions of the Lame equation for the cylinder; $\vec{u}_k^{(-)}$, $\vec{u}_k^{(+)}$ – solutions of the Lame equation for the layer.

For the transition of basic solutions between coordinate systems, formulas [22] are used:

– for transition from solutions $\vec{S}_{k,m}$ of the cylindrical coordinate system to solutions of the layer $\vec{u}_k^{(-)}$ (for y>0) and $\vec{u}_k^{(+)}$ (for y<0)

$$\vec{S}_{k,m}(\rho_p,\varphi_p,z;\lambda)=$$
$$=\frac{(-i)^m}{2}\int_{-\infty}^{\infty}\omega_{\mp}^m\cdot e^{-i\mu\bar{x}_p\pm\gamma\bar{y}_p}\cdot\vec{u}_k^{(\mp)}\cdot\frac{d\mu}{\gamma},\quad k=1,3;\tag{6}$$

$$\vec{S}_{2,m}(\rho_p,\varphi_p,z;\lambda)=\frac{(-i)^m}{2}\times$$
$$\times\int_{-\infty}^{\infty}\omega_{\mp}^m\cdot\left(\begin{matrix}\left(\pm m\cdot\mu-\dfrac{\lambda^2}{\gamma}\pm\lambda^2\bar{y}_p\right)\vec{u}_1^{(\mp)}\mp\\ \mp\lambda^2\vec{u}_2^{(\mp)}\pm4\mu(1-\sigma)\vec{u}_3^{(\mp)}\end{matrix}\right)\cdot\frac{e^{-i\mu\bar{x}_p\pm\gamma\bar{y}_p}d\mu}{\gamma^2},$$

where

$$\gamma=\sqrt{\lambda^2+\mu^2},\quad\omega_{\mp}(\lambda,\mu)=\frac{\mu\mp\gamma}{\lambda},\quad m=0,\pm1,\pm2,\ldots;$$

– for transition from interchanges $\vec{u}_k^{(+)}$ and $\vec{u}_k^{(-)}$ a layer to solutions $\vec{R}_{k,m}$ of a cylindrical coordinate system

$$\vec{u}_k^{(\pm)}(x,y,z)=e^{i\mu\bar{x}_p\pm\gamma\bar{y}_p}\cdot\sum_{m=-\infty}^{\infty}(i\cdot\omega_{\mp})^m\vec{R}_{k,m},\ (k=1,\ 3);\tag{7}$$

$$\vec{u}_2^{(\pm)}(x,y,z)=e^{i\mu\bar{x}_p\pm\gamma\bar{y}_p}\times$$
$$\times\sum_{m=-\infty}^{\infty}\left[\begin{matrix}(i\cdot\omega_{\mp})^m\cdot\lambda^{-2}\left(\left(m\cdot\mu+\bar{y}_p\cdot\lambda^2\right)\cdot\vec{R}_{1,m}\pm\right.\\ \left.\pm\gamma\cdot\vec{R}_{2,m}+4\mu(1-\sigma)\vec{R}_{3,m}\right)\end{matrix}\right],$$

where $\vec{R}_{k,m}=\vec{\tilde{b}}_{k,m}(\rho_p,\lambda)\cdot e^{i(m\varphi_p+\lambda z)}$;

$$\vec{\tilde{b}}_{1,n}(\rho,\lambda)=\vec{e}_\rho\cdot I_n'(\lambda\rho)+i\cdot I_n(\lambda\rho)\cdot\left(\vec{e}_\varphi\frac{n}{\lambda\rho}+\vec{e}_z\right);$$

$$\vec{\tilde{b}}_{2,n}(\rho,\lambda)=\vec{e}_\rho\cdot\left[(4\sigma-3)\cdot I_n'(\lambda\rho)+\lambda\rho I_n''(\lambda\rho)\right]+$$
$$+\vec{e}_\varphi i\cdot m\left(I_n'(\lambda\rho)+\frac{4(\sigma-1)}{\lambda\rho}I_n(\lambda\rho)\right)+$$
$$+\vec{e}_z i\lambda\rho I_n'(\lambda\rho);$$

$$\vec{\tilde{b}}_{3,n}(\rho,\lambda)=-\left[\vec{e}_\rho\cdot I_n(\lambda\rho)\frac{n}{\lambda\rho}+\vec{e}_\varphi\cdot i\cdot I_n'(\lambda\rho)\right];$$

$\vec{e}_\rho$, $\vec{e}_\varphi$, $\vec{e}_z$ are the unit vectors of the cylindrical coordinate system;

– to move from the interchanges of the cylinder with number p to the interchanges of the cylinder with number q

$$\vec{S}_{k,m}(\rho_p,\varphi_p,z;\lambda)=\sum_{n=-\infty}^{\infty}\vec{b}_{k,pq}^{mn}(\rho_q)\cdot e^{i(n\varphi_q+\lambda z)},\quad k=1,2,3,\tag{8}$$

$$\vec{b}_{1,pq}^{mn}(\rho_q)=(-1)^n\tilde{K}_{m-n}(\lambda\ell_{pq})\cdot e^{i(m-n)\alpha_{pq}}\cdot\vec{\tilde{b}}_{1,n}(\rho_q,\lambda);$$

$$\vec{b}_{3,pq}^{mn}(\rho_q)=(-1)^n\tilde{K}_{m-n}(\lambda\ell_{pq})\cdot e^{i(m-n)\alpha_{pq}}\cdot\vec{\tilde{b}}_{3,n}(\rho_q,\lambda);$$

$$\vec{b}_{2,pq}^{mn}(\rho_q)=$$
$$=(-1)^n\left\{\begin{matrix}\tilde{K}_{m-n}(\lambda\ell_{pq})\cdot\vec{\tilde{b}}_{2,n}(\rho_q,\lambda)-\dfrac{\lambda}{2}\ell_{pq}\times\\ \times\left[\begin{matrix}\tilde{K}_{m-n+1}(\lambda\ell_{pq})+\\+\tilde{K}_{m-n-1}(\lambda\ell_{pq})\end{matrix}\right]\cdot\vec{\tilde{b}}_{1,n}(\rho_q,\lambda)\end{matrix}\right\}\cdot e^{i(m-n)\alpha_{pq}},$$

where α_{pq} – the angle between the x_p axis and the segment ℓ_{pq}; $\tilde{K}_m(x)=(\operatorname{sign}(x))^m\cdot K_m(|x|)$.

5. Results of the study of the stress state of the constituent elements of the composite

5. 1. Creation and solution of a system of equations

In the elastic homogeneous isotropic layer, parallel to its surfaces, there are two continuous inclusions (Fig. 1).

Layer material – plastic, Poisson's ratio σ=0.38, modulus of elasticity E=1.7·10[3] MPa. The material of both inclusions is steel σ_2=0.25, E_2=2·10[5] MPa.

Geometric parameters:

h=27 mm, R_1=R_2=7 mm, ℓ_{12}=17 mm.

Stresses are set on the upper and lower surfaces of the layer (Fig. 1)

$$\sigma_y^{(h)}(x,z)=\sigma_y^{(\tilde{h})}(x,z)=-10^8\cdot\left(z^2+10^2\right)^{-2}\cdot\left(x^2+10^2\right)^{-2},$$

$$\tau_{yx}^{(h)}=\tau_{yz}^{(h)}=\tau_{yx}^{(\tilde{h})}=\tau_{yz}^{(\tilde{h})}=0.$$

The solution of the problem is presented in the form

$$\vec{U}_0=\sum_{k=1}^{3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\begin{pmatrix}H_k(\lambda,\mu)\cdot\vec{u}_k^{(+)}(x,y,z;\lambda,\mu)+\\+\tilde{H}_k(\lambda,\mu)\cdot\vec{u}_k^{(-)}(x,y,z;\lambda,\mu)\end{pmatrix}\mathrm{d}\mu\mathrm{d}\lambda+$$
$$+\sum_{p=1}^{N}\sum_{k=1}^{3}\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty}B_{k,m}^{(p)}(\lambda)\cdot\vec{S}_{k,m}(\rho_p,\varphi_p,z;\lambda)\mathrm{d}\lambda, \quad (9)$$

$$\vec{U}_p=\sum_{k=1}^{3}\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty}A_{k,m}^{(p)}(\lambda)\cdot\vec{R}_{k,m}(\rho_p,\varphi_p,z;\lambda)\mathrm{d}\lambda, \quad (10)$$

where $\vec{U}_0$ – displacement in the body of the layer; $\vec{U}_p$ – displacement in the inclusion body p;

$$\vec{S}_{k,m}(\rho_p,\varphi_p,z;\lambda),\ \vec{R}_{k,m}(\rho_p,\varphi_p,z;\lambda),$$

$\vec{u}_k^{(+)}(x,y,z;\lambda,\mu)$ and $\vec{u}_k^{(-)}(x,y,z;\lambda,\mu)$ – the basic solutions given by formulas (5), while the unknown functions $H_k(\lambda,\mu)$, $\tilde{H}_k(\lambda,\mu)$, $A_{k,m}^{(p)}(\lambda)$ and $B_{k,m}^{(p)}(\lambda)$ must be found from the boundary conditions (1) and conjugation conditions (3), (4).

To fulfill the boundary conditions at the layer boundaries, let's rewrite the vectors $\vec{S}_{k,m}$ in (9), using the transition formulas (6), in the Cartesian coordinate system through the basic solutions $\vec{u}_k^{(-)}$ at $y=h$, and $\vec{u}_k^{(+)}$ at given $y=-\tilde{h}$ in terms of the double Fourier integral.

The system of 6 equations has a determinant coinciding with [31].

From these equations let's find the functions $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$ through $B_{k,m}^{(p)}(\lambda)$.

The next step is the fulfillment of conjugation conditions (3) of each inclusion p with the layer. To do this, the right side of (9), using the transition formulas (7) and (8), let's rewrite in the local cylindrical coordinate system of this inclusion through the basic solutions $\vec{R}_{k,m}$, $\vec{S}_{k,m}$.

If to apply the stress operator to the obtained equations, let's obtain three more equations for the conjugation conditions (4).

The system determinant for each cylinder p coincides with [29].

These systems can be solved by the cutting method and convergence of approximate solutions to the exact one.

From the resulting system of equations, let's eliminate the previously found functions $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$ through $B_{k,m}^{(p)}(\lambda)$.

Freed from series in m and integrals in λ, let's obtain a set of 12 infinite systems of linear algebraic equations for determining the unknowns $B_{k,m}^{(p)}(\lambda)$ and $A_{k,m}^{(p)}(\lambda)$.

Let's substitute the functions $B_{k,m}^{(p)}(\lambda)$ found from the infinite system of equations into the expressions for $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$. This will determine all unknown problems.

5. 2. Analysis of the stress-strain state of the body

To obtain numerical results, the mathematical software package Maple 16 was used. The calculation of integrals for functions without oscillations was performed using Simson's quadrature formulas, for functions with oscillations, using Philo's quadrature formulas.

The infinite system has been truncated to $m=8$. The accuracy of fulfilling the boundary conditions for the indicated values of m and geometric parameters was 10^{-4} for stress values from 0 to 1.

Fig. 2 shows the stresses at the upper boundary of the layer along the x axis, at $z=0$.

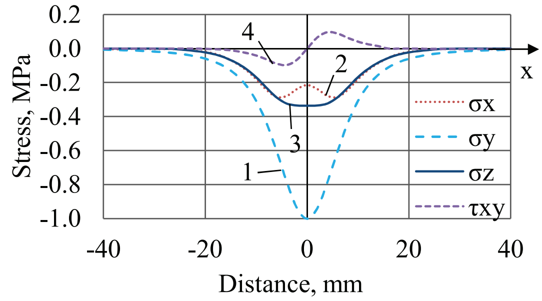

Fig. 2. Stress along the x-axis at z=0

In accordance with the given stresses σ_y (Fig. 2, line 1), maximum stresses $\sigma_z=-0.3359$ MPa arise on the surface of the layer at $x=0$ (Fig. 2, line 3). Maximum stresses $\sigma_x=-0.2846$ MPa at $x=6$ mm (Fig. 2, line 2) and maximum stresses $\tau_{xy}=0.0966$ MPa at $x=4$ mm (Fig. 2, line 4).

Fig. 3 shows the stresses along the z-axis, at $x=0$.

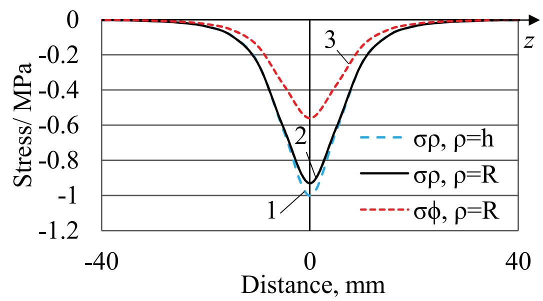

Fig. 3. Stress along the z-axis at φ=π/2

At given unit compressive stresses (Fig. 3, line 1) along the z axis, maximum stresses $\sigma_\rho=-0.9306$ MPa appear on the interface between the layer and the inclusion. (Fig. 3, line 2). Such an insignificant decrease in stress (relative to the given ones) arises due to the proximity of the limiting surfaces $R_1/\tilde{h}=0.7$. The stress σ_z almost coincides with the stresses σ_φ (Fig. 3, line 3).

Fig. 4 shows the stressed state along the limit of the interface between the layer and the lower inclusion ($p=1$) at $z=0$.

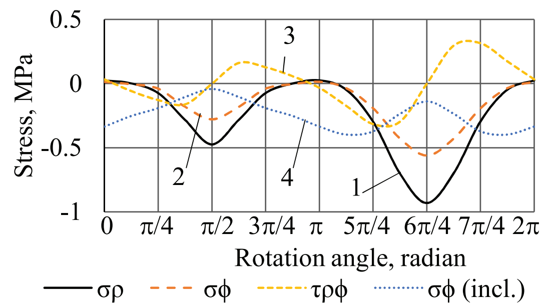

Fig. 4. Stress on the limit of the interface between the layer and the inclusion at $z=0$

The stress σ_z on the mating surface in the body of the layer almost coincides with the stresses σ_φ, therefore they are shown in Fig. 4 only along line 2. The same applies to the stresses on the reinforcement surface σ_z (incl.), which almost coincide with the stresses σ_φ (incl.) (Fig. 4, line 4). The stress σ_ρ (incl.) and $\tau_{\rho\varphi}$ (incl.) are equal to σ_ρ and $\tau_{\rho\varphi}$ on the layer surface, respectively (Fig. 4), which corresponds to conjugation conditions (3) and (4). Maximum values σ_ϕ (incl.)=–0.39672 MPa, $\tau_{\rho\phi}$ (incl.)=±0.31501 MPa.

On the isthmus (Fig. 5), with appropriate geometric characteristics ((R_1+R_2)/ℓ_{12}=0.82), some stresses increase, but not significantly.

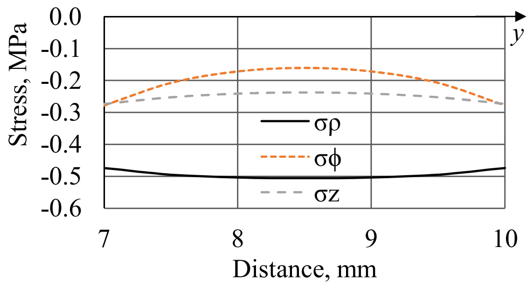

Fig. 5. Stress state on the isthmus between inclusions at z=0

At the isthmus, the stresses σ_ρ increase, σ_ϕ and σ_z decrease. Maximum values σ_ρ=–0.5059 MPa.

Thus, the extreme stresses arising in the conjugation are normal σ_ρ=–0.9306 MPa and adjoining $\tau_{\rho\phi}$=–0.315 MPa.

6. Discussion of the results of studying the stress-strain state of a layer with two cylindrical inclusions

Using the generalized Fourier method, a connection was created between the basic solutions of the Lame equation in different coordinate systems. Taking into account the conjugation conditions for cylindrical fields (3), (4), it was possible to write the entire system of equations (9), (10) in one local coordinate system. After representing the limiting conditions in terms of the double Fourier integral, an infinite system of linear algebraic equations was obtained to determine the unknowns.

Solutions (9), (10) were used to obtain the stress state. Thanks to the transition formulas (6)–(8), it became possible to write these solutions in one coordinate system and obtain a numerical result.

The high accuracy of the fulfillment of the boundary conditions, indicated in Section 5. 2, indicates the high accuracy of the obtained stress-strain state (the error is always higher on the boundary surfaces) and the efficiency of the applied method as a whole. This indicator can be improved by increasing the parameter m.

As a result, a solution to the problem was obtained, in contrast to [14–17], which makes it possible to analyze the stress-strain state for bodies with longitudinal inhomogeneities.

The proposed analytic-numerical solution method uses the generalized Fourier method. This allows

– to obtain high-precision results, which prefers the accuracy of calculations over works [5–9];

– to analyze the stress state of the layer body separately and its reinforcement separately, which cannot be done by some other methods [10–13];

– to solve the problem in a spatial formulation and with a larger number of boundary surfaces than the traditional expansion into Fourier series used in [18–21].

In contrast to the works [24–35], which also use the generalized Fourier method with separate addition theorems, in this work a number of tools were applied simultaneously. These are transition functions in basic interchanges between cylindrical coordinate systems, transition functions between cylindrical (including shifted) and Cartesian coordinate systems for both layer boundaries. The conditions of conjugation between cylindrical surfaces are also applied, the equilibrium equations are taken into account. This made it possible to obtain a solution to this problem.

The proposed solution method makes it possible to carry out a high-precision calculation of the strength of fibrous composite materials, which are used in machine and aircraft building, the space industry and construction. The above analysis of the stress state allows predicting the geometric parameters of structures or their composite elements. The effect of applying the proposed method can be to optimize the use of materials at the design stage.

In the numerical determination of the stress state, limitations should be taken into account: the method does not allow solving problems when the boundaries of the bodies intersect. Also, some functions cannot be represented due to the double Fourier integral.

Among the disadvantages, it should be noted that with a decrease in the distance between the limits of the bodies, the accuracy of the calculation decreases. To restore accuracy, it is necessary to increase the order of the system of equations m, which entails a significant increase in the time for calculating the integrals of matrix elements.

Further development of this direction is possible for a similar problem under other boundary conditions, including the contact type between the inclusions and the layer. Such conditions in a real structure can arise when the conditions of rigid coupling are lost.

7. Conclusions

1. A new problem in the theory of elasticity for a layer with two longitudinal circular cylindrical inclusions is solved for stresses specified on the boundary surfaces. The solution of the problem is based on the analytic-numerical generalized Fourier method, the conditions of conjugation between cylindrical surfaces are additionally applied and the equilibrium equations are taken into account. The problem is reduced to an infinite system of linear algebraic equations, to which the reduction method is applied. This allows to get a result with a given accuracy.

2. The performed analysis of the stress state indicates the need to take into account the normal stresses on the mating face when assessing the strength of the composite material, in particular, on the mating surface. It has been established that all maximum stresses are concentrated in the lower part of the interface (closer to the load source) and have negative values (except for $\tau_{\rho\phi}$). If the given load instead of a single one is equal to the ultimate strength of the plastic ([σ]=26 MPa), the stresses in the interface will get the value σ_ρ=–2424.2 MPa. Such stress values may exceed the tensile strength of this interface (for example, for VK-5 glue, [σ]=22 MPa). As for shear stresses, there is still a significant margin of safety.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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