

*This paper proposes a method that makes it possible to study the patterns of changes in the concrete strength of reinforced concrete crossbars depending on the heating temperature under fire conditions by interpreting the results of their standard fire tests. For the implementation of this method, it is proposed to use similar data obtained using mathematical modeling based on the finite element method and given material properties, including the curve of concrete strength reduction recommended by the guidelines, as data included in the set of measurement results during fire tests depending on temperature. Such data are time dependences of temperature indicators at individual cross-section points and time dependence of the maximum deflection of the crossbar. The article proposes an interpolation method that makes it possible to set the temperature at any point of the section based on the approximation of the isotherms by parabolas with a variable indicator of their power. A method based on the mathematical interpretation of temperature indicators obtained using the proposed interpolation method and the curve of the dependence of the maximum deflection on time using a deformation model for describing the stress-strain state is proposed to identify the dependence of the concrete strength of reinforced concrete crossbars.*

*The work also shows that the results obtained using the proposed method of identifying concrete strength reduction coefficients are adequate as their relative error is on average no more than 7%. Based on the results, the possibility of its application to study the regularities of the decrease in the strength of reinforced concrete crossbars under fire conditions has been proven*

**Keywords:** reinforced concrete crossbar, concrete strength reduction factor, fire resistance, concrete strength

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# METHOD FOR IDENTIFYING THE STRENGTH CHARACTERISTICS OF CONCRETE OF A REINFORCED CONCRETE CROSSBAR DURING HEATING UNDER CONDITIONS OF FIRE

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## 1. Introduction

The main drawback in predicting the effective functioning of reinforced concrete building structures is the assumption of an insufficient reserve of their strength under fire conditions. The greatest danger of collapse under such conditions is represented by bent elements of structures. Such elements include reinforced concrete crossbars and beams because they have the largest dimensions and loads.

To increase the reliability of buildings under the conditions of the thermal impact of fire, one of the most effective

ways is to ensure a standardized limit of fire resistance based on methods of assessing the fire resistance of building structures of a given facility, in particular, reinforced concrete crossbars. Existing methods are sufficiently developed and form a hierarchical structure from the simplest ones in its basis to complex, carried out with the involvement of computer software and hardware. However, for the implementation of these methods, a set of initial data is required, including the thermomechanical characteristics of materials. At the current stage, the perfect information about the concrete strength of reinforced concrete crossbars under the

conditions of the thermal effect of fire needs to be clarified in order to increase the reliability of the data obtained by the methods of calculating the fire resistance of reinforced concrete crossbars, since this makes it possible to significantly increase the accuracy of forecast data when analyzing the compliance of reinforced concrete crossbars with the requirements of fire resistance standards.

Therefore, studies aimed at determining changes in temperature distributions and concrete strength in cross-sections of reinforced concrete crossbars depending on the duration of the thermal effect of the standard fire temperature regime are relevant.

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## 2. Literature review and problem statement

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Experimental and computational methods of assessing fire resistance are used to predict the behavior of reinforced concrete crossbars under fire conditions [1]. These methods are currently employed to assess the fire resistance of reinforced concrete structures, in particular reinforced concrete crossbars, but they have advantages and disadvantages that impose certain specific conditions for their use and actualize efforts aimed at their improvement.

A first method is based on conducting fire tests. High-temperature tests of reinforced concrete crossbars and beams take place in accordance with standards [2]. According to these standards, the reinforced concrete crossbar must be subjected to thermal action under the conditions of mechanical load, which fully corresponds to the actual load in them according to the design scheme of the building. The strength of concrete changes differently at elevated temperatures depending on the load and heating conditions. In works [3, 4], for a more accurate description of the strength characteristics of concrete, a range of values of the material properties, which were determined due to variations in the experimental situation, is used. The method of fire tests is universal for all building structures when determining all types of limit states. However, the implementation of this method has a number of disadvantages, namely: conducting research requires significant material and labor costs, and there is a possibility of accidents during tests.

Also, the experimental method of predicting the functioning of reinforced concrete crossbars under conditions of high temperatures based on high-temperature tests cannot always be correctly applied. This is mainly due to the impossibility of reproducing in the experiment the full compliance with the conditions of fastening and loading of the element during its operation as a component part of this structure.

Calculation methods for predicting the functioning of reinforced concrete structures under fire conditions do not have such limitations and are widely used at the stage of designing buildings and structures that would meet the requirements of building regulations regarding their fire resistance [5]. These methods, with the help of computer simulation, make it possible to reproduce the operating conditions of the building structure in the event of a fire in full [6]. Calculations are implemented with the help of special software, directly intended for simulating the development, spread, and extinction of high-temperature processes in rooms. The algorithms of these programs incorporate modern numerical methods [7]. These include methods such as the method of finite or boundary elements, non-connection methods, the Galerkin method, optimization methods,

etc. That is, methods for solving systems of differential equations of continuous media such as Navier-Stokes equations, heat conduction equations, which take into account all possible factors, the description of which is impossible when applying analytical solution methods. Such factors include any complexity of the boundary conditions, as well as various non-linearities in these equations [8, 9]. The advantages of calculation methods for evaluating the fire resistance of reinforced concrete crossbars are universality, flexibility, the ability to take into account any set of boundary conditions and structural features, as well as regimes of thermal effects of fire. As disadvantages, one should point out the laboriousness of preparing calculation models and conducting calculations, the need for highly qualified specialists who conduct these calculations, increased requirements for computational equipment, the need to verify the results of calculations, as well as the sufficient limitation of initial data on the phenomenology of the behavior of materials under fire conditions.

The strength characteristics of concrete given in the European standards are used as initial data for calculation methods for assessing the fire resistance of building structures. In work [10], the given coefficient of reduction of the characteristic strength of concrete depending on the temperature is determined for a concrete sample – a cube. It is universal and is used in the assessment of fire resistance by calculation method for all types of reinforced concrete structures. However, the study of samples whose dimensions are close to the real dimensions of building elements gives more accurate results.

To obtain reliable results in predicting the fire resistance of reinforced concrete structures, the method of fire tests and mathematical processing of the results is considered, which provides flexibility when considering the boundary conditions for reinforced concrete elements [11]. This variant of conducting research involves the use of various geometric parameters of structures, the application of any level of load, using various theories of material strength, which significantly reveals the possibilities of computational experiments [12]. This approach has prospects for clarifying the characteristics of materials that are components of reinforced concrete directly through the results of measurements during fire tests of crossbars as an alternative to obtaining similar data through testing laboratory samples of the corresponding materials.

The procedures proposed in [8–12] are based on the assumption that the strength of the concrete of the inner layers of the crossbars corresponds to the strength determined by testing laboratory samples of concrete in compression, cylindrical in shape. However, the analysis of the results reported in [10–12] allows us to assume that the nature of the deformation of the inner layers of the concrete of the crossbars when they are heated under fire conditions may differ significantly, which in turn may affect the accuracy of the calculation results. To increase the accuracy of calculation methods for assessing fire resistance, it is promising to identify the inner layers of concrete of reinforced concrete crossbars based on the results of fire tests of their full-size samples in installations for their experimental study of fire resistance.

According to work [11], this approach to studying the strength of concrete based on the results of fire tests can be applied by establishing temperature indicators in the inner layers of reinforced concrete crossbars in each minute of the tests based on the results of point temperature measure-

ments. Knowing the temperature indicators, it is possible to identify the strength characteristics of concrete based on the solution of the equations describing the stressed-strained state in the inverted statement. To this end, it is necessary to devise a method of temperature interpolation and a method of drawing up equations of the stressed-strained state to implement the proposed approach. However, in order to implement this approach, it is necessary to investigate the adequacy of the methods based on this approach since there are significant sources of errors at the stages of its implementation. Such stages are the restoration of the temperature field in every minute of the test and the assumptions introduced during the construction of a mathematical model that describes the stressed-deformed state of a reinforced concrete crossbar under the conditions of its heating during a fire. Determining such errors will justify the application of the experimental and computational research method as an effective tool for researching the patterns of concrete strength reduction under high-temperature heating conditions based on fire tests of reinforced concrete crossbars that directly reproduce the effects of fire.

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### 3. The aim and objectives of the study

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The purpose of this study is to devise an experimental and computational method for identifying patterns of changes in concrete strength depending on the temperature of its heating based on the results of measurements during fire tests of reinforced concrete crossbars.

To accomplish the aim, the following tasks have been set:

- to obtain data that reproduces a set of measurement results during fire tests of reinforced concrete crossbars by performing calculations based on the finite element method;
- to substantiate a complex of mathematical models for temperature interpolation along the cross-section of a reinforced concrete crossbar based on the approximation of isotherm curves by parabolas based on temperature indicators at individual points of the cross-section of a reinforced concrete crossbar under the conditions of its heating under the influence of a standard fire temperature regime;
- to substantiate a mathematical model for identifying patterns of changes in concrete strength depending on the temperature of its heating, based on temperature indicators in the cross-section of a reinforced concrete crossbar, obtained as a result of interpolation, and the curve of maximum deflection depending on time;
- to investigate the adequacy of the results of identifying the change in concrete strength depending on the temperature of the reinforced concrete crossbar by comparing them with the data provided when determining the temperature indicators and the maximum deflection curve using the finite element method.

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## 4. The study materials and methods

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### 4. 1. The object and hypothesis of the study

The object of our study is the method of identifying the compressive strength of concrete based on the results of measurements made during fire tests of reinforced concrete crossbars for fire resistance. This method was studied from the point of view of the adequacy of the obtained results, in order to justify its use as a tool for the study of new, more ac-

curate data on the properties of concrete, which can be used as part of the initial data for the calculated evaluation of the fire resistance of reinforced concrete crossbars according to the refined methods.

To implement the experimental and calculation method, a set of hypotheses and assumptions was accepted. The main hypothesis, which is the basis for conducting research, assumes that the temperature-force response in the inner layers of a reinforced concrete crossbar during its fire tests for fire resistance can be obtained by calculation using the approximation of differential equations of mathematical models of thermal conductivity and the stress-deformed state using the finite element method. It was also assumed that the behavior of materials that are components of reinforced concrete is determined by mathematical models of materials recommended by guidelines [10]. The mechanical characteristics of concrete and reinforcing steel for models of their behavior at different temperatures are precisely known as they are part of the initial data for the calculation. The identification of strength characteristics is carried out only for concrete since the chemical composition and structure of one or another reinforcing steel is controlled much more strictly. Given this, concrete strength indicators that can be identified can be compared with concrete strength indicators that have been embedded in a mathematical model. So, the idea of the work is to compare these indicators to establish the adequacy of the results obtained by the proposed method.

### 4. 2. Research methodology

When conducting research, the following main stages are performed. At the first stage, a mathematical model of a reinforced concrete crossbar with a section of 150×300 mm and a length of 1.7 m is built, similar to the series of experiments described in [13]. This model is based on the discretization of the calculation area using the finite element method, taking into account the physical and geometric nonlinearity of the behavior of reinforced concrete under heating conditions.

Using the model built, a numerical experiment is conducted, during which the temperature distributions in the cross-section of the reinforced concrete crossbar and its movement during the thermal effect of the standard fire temperature regime are determined.

At the second stage, an appropriate interpolation procedure is devised to determine the temperatures at any point of the sections of reinforced concrete cross-sections based on point measurements of the temperature in their inner layers during fire tests; the adequacy of the results obtained during its application is established.

At the third stage, using the obtained temperature distributions and the curve of dependence of the maximum deflection on time, a method of identifying the mechanical characteristics of concrete is devised based on the equilibrium equations constructed using the deformation model. At the last stage, the adequacy of the identified concrete strength indicators is investigated by comparing them with the concrete strength indicators that were included in the mathematical model.

### 4. 2. Procedure for determining the initial data for the identification of strength properties of concrete

The set of data required to identify the strength characteristics of concrete under the conditions of thermal exposure of the standard fire temperature regime is deter-

mined using mathematical modeling based on the finite element method.

At the initial stage, the temperature indicators and parameters of the stressed-strained state of the reinforced concrete crossbar under the conditions of thermal influence of the standard fire temperature regime should be calculated. The mechanical properties of the concrete of the reinforced concrete crossbar are part of the initial data and are clearly established according to the recommendations of the second part of Eurocode 2 EN 1992-1-2: 2004 [10]. After that, the given mechanical characteristics of concrete are identified according to the developed method and their adequacy is established by comparing them with the previously specified ones.

To study the behavior of a reinforced concrete crossbar under the conditions of thermal influence of the standard fire temperature regime, the crossbar is considered, the structural scheme of which is shown in Fig. 1.

To carry out a thermal calculation of a reinforced concrete crossbar under the conditions of thermal influence of the standard fire temperature regime, it is enough to consider the heat exchange in its cross-section. The equation of heat conduction in quasi-linear notation, taking into account convection and radiation heat exchange at the boundaries of the calculation area with the fire environment under boundary conditions (BC) of II kind, is written in a two-dimensional statement. The temperature regime in the environment of a room with a fire is described by a standard fire temperature curve. Since the thermal conductivity of concrete and reinforcing steel is significantly different, the process of heat transfer is considered only in concrete. Thermophysical characteristics of concrete correspond to temperature dependences in accordance with the recommendations of Eurocode 2 EN 1992-1-2: 2004 [10]. The heat conduction equation is approximated using the finite element method with the use of the ANSYS APDL computer system.

Modeling of the behavior of a reinforced concrete crossbar takes place during the sequential solution of the thermal engineering problem and the problem of strength. The results of solving the thermal engineering problem are used to solve the strength problem when using the calculated nodal temperatures as a load.

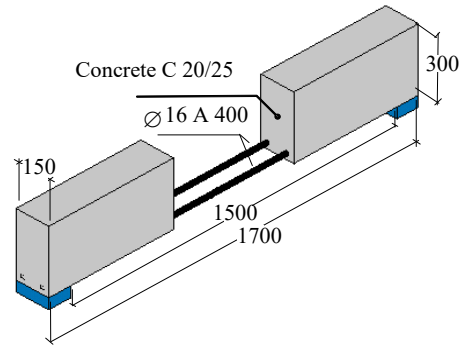


Fig. 1. Structural diagram of a reinforced concrete crossbar for studies of its behavior under the influence of a standard fire temperature regime

Mathematical models were used for the calculation; their parameters are given in Table 1.

To solve the thermal problem using the finite element method, a hexahedral finite element of the Lagrangian type was used [14], the geometric form of which is shown in Fig. 2.

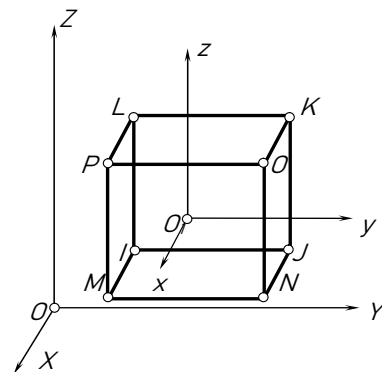


Fig. 2. The geometric form of the finite element used for the calculation

The finite-element diagram of a reinforced concrete crossbar in accordance with Fig. 1 and the scheme of application of boundary conditions (BC) when setting the boundary value problem is shown in Fig. 3.

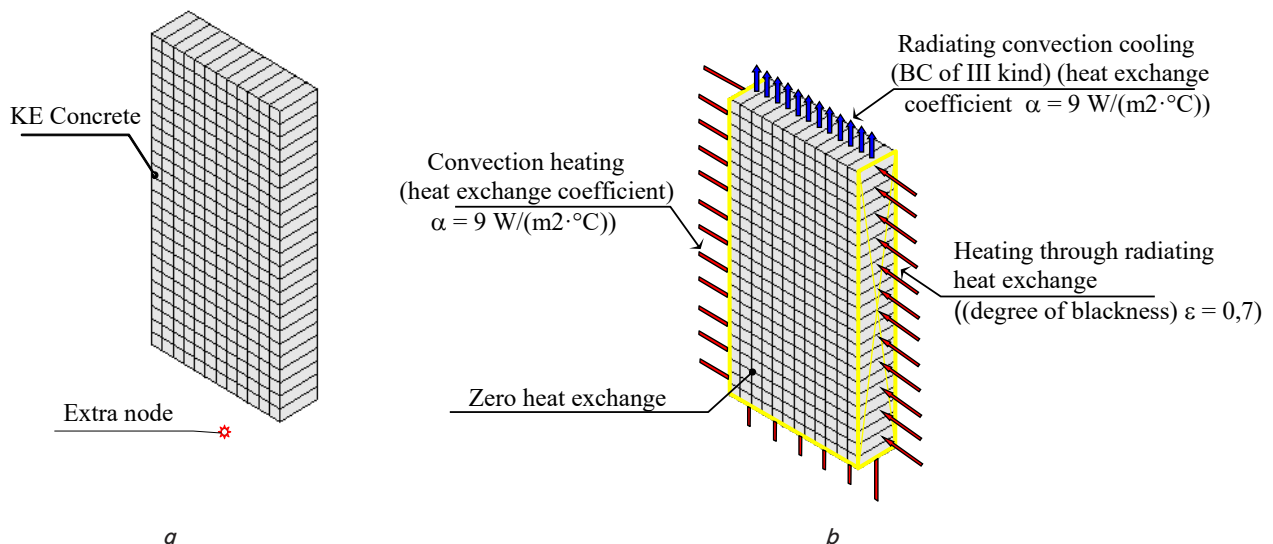


Fig. 3. Finite-element diagram of a reinforced concrete crossbar for thermal calculation (a) and diagram of application of boundary conditions (b)

After the calculations, temperature data was obtained in the inner layers of the reinforced concrete crossbar. These data are used as initial data for mathematical modeling of the behavior of the studied reinforced concrete crossbar under the conditions of thermal influence of the standard fire temperature regime.

Table 1

Basic methods of numerical research of reinforced concrete crossbars under conditions of fire tests

Mathematical model component	Calculation methods used
Thermal problem	
Thermal conductivity	Differential nonstationary equation of thermal conductivity approximated by finite element method [3, 8]
Boundary conditions	Boundary conditions of the third kind, taking into account convection and radiant heat transfer [7]
Physical nonlinearity	Newton-Raphson method [8]
Thermophysical characteristics	Recommended by EN 1992-1-2 [10]
Characteristics of boundary conditions	Recommended by EN 1992-1-2 [10]
Structural problem	
Stressed and deformed state	Finite element method in nonlinear implementation [3, 9, 11]
Plastic deformation	Associative theory of plasticity [5]
Crack formation criterion	Willem-Warnke concrete strength criterion [13]
Physical and geometric nonlinearity	Newton-Raphson method [13]
Mechanical and thermomechanical characteristics	Deformation diagrams recommended by EN 1992-1-2 [10]

For mathematical modeling, a system of differential equations of the stressed-strained state is used, which is solved numerically using the finite element method with the implementation of nonlinear calculation according to the Newton-Raphson method. The solution implies the step-by-step addition of the active mechanical load at the beginning of the calculation and the step-by-step addition of temperature loads at the final stage with a time step of 1 min.

When stating a nonlinear strength problem, the recommendations of the second part of Eurocode 2 EN 1992-1-2:2004 [10] are used to determine the mechanical properties of concrete and reinforcing steel. The mechanical characteristics of concrete and reinforcing steel in accordance with these guidelines can be illustrated by the deformation diagrams shown in Fig. 4.

Taking into account temperature deformations, the coefficient of thermal expansion is specified in the calculation. This coefficient is defined as the first derivative of temperature deformation with respect to temperature. Fig. 5 shows the temperature curves of the coefficient of thermal expansion of concrete and reinforcing steel.

In order to calculate the strength of the reinforced concrete crossbar under the conditions of thermal exposure of the standard temperature regime of the fire, the scheme of the action of the mechanical load and fastening was considered. Using this scheme, a finite-element scheme was built, shown in Fig. 5. This diagram shows the applied BCs.

The results of our calculations using this mathematical model are the temperature values at certain points of the

cross-section of the reinforced concrete crossbar and the dependence curve of the maximum deflection of the reinforced concrete crossbar on the test time.

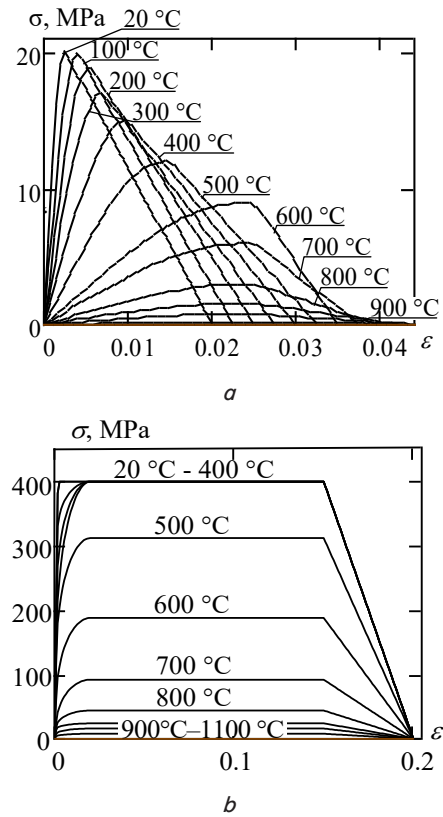


Fig. 4. Reinforced concrete deformation diagrams: a – concrete deformation diagram; b – deformation diagram of reinforcing steel

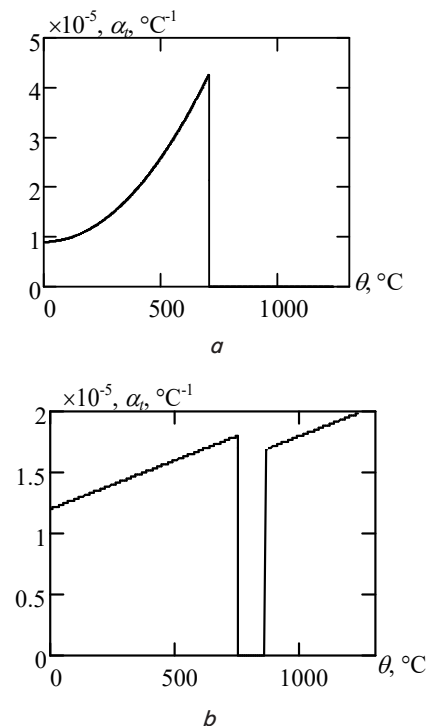


Fig. 5. Dependences of the thermal expansion coefficient on the heating temperature: a – concrete; b – reinforcing steel

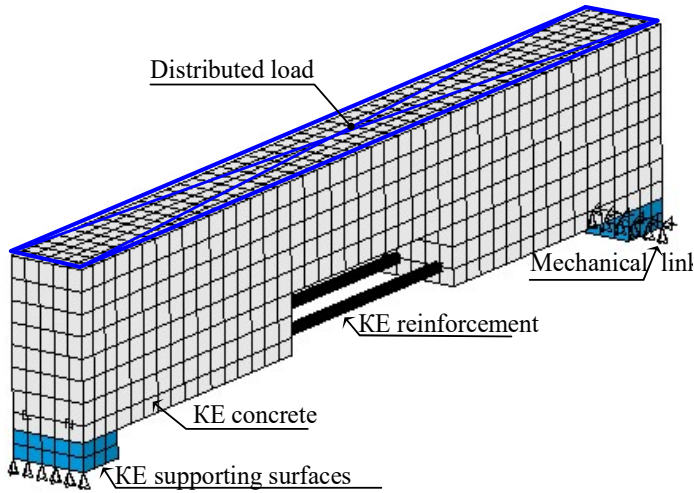


Fig. 6. Design scheme of a reinforced concrete crossbar for solving the strength problem

**5. Results of identification of the strength characteristics of the reinforced concrete crossbar and their adequacy**

**5.1. Results of data calculation regarding the behavior of a reinforced concrete crossbar under the conditions of fire tests using the finite element method**

After the calculations, temperature data was obtained in the inner layers of the reinforced concrete crossbar. Fig. 7 shows temperature distributions across the cross-section of the crossbar at different times.

Fig. 8 shows the distribution of the total deformation along the middle cross-section of a reinforced concrete cross-section at the time before the start of the test when only mechanical loads are applied, and at the time before the crossbar collapses (conditionally on min 20 of the test).

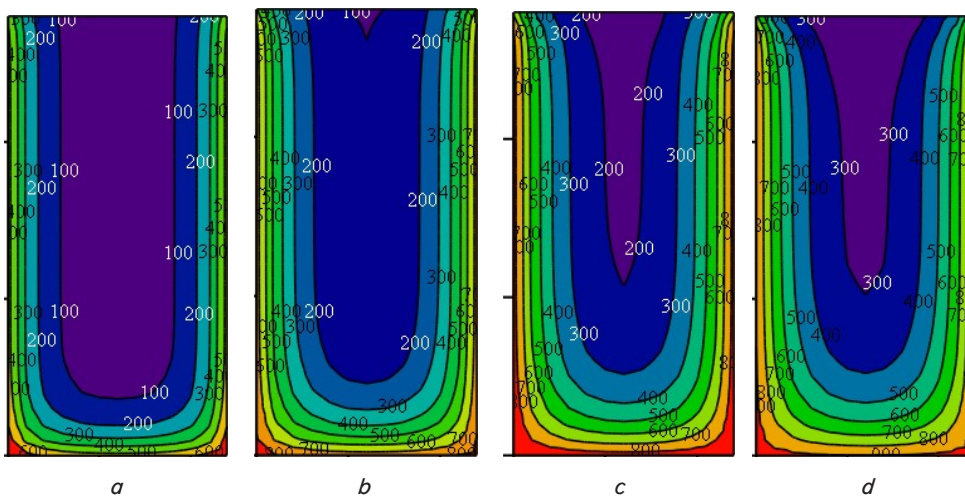


Fig. 7. Temperature distributions across the cross-section of a crossbar at different moments of exposure to the standard fire temperature regime (°C): a – 15 min; b – 30 min; c – 45 min; d – 60 min

According to Fig. 8, it can be concluded that the deformations in the upper layers of compressed concrete and the concrete layers in the locations of the reinforcing bars take approximately the same values. Such a pattern, in turn, allows us to assume that the stressed-strained state in the cross-section of a reinforced concrete crossbar can be de-

scribed using the hypothesis of flat sections and the deformation model based on the concept of pure bending.

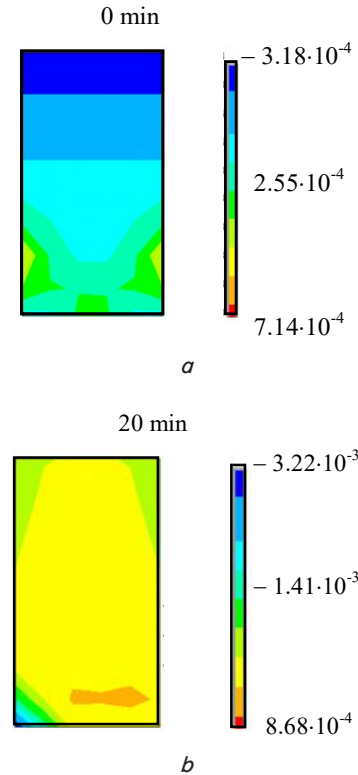


Fig. 8. Distribution of the total relative deformation along the middle section of the reinforced concrete crossbar: a – at the beginning of the test under the action of only mechanical load; b – on minute 20 of the test before destruction

The necessary data for identifying the strength characteristics of concrete, which were obtained as a result of the calculation, are the dependence curve of the maximum deflection of a reinforced concrete crossbar depending on the time of thermal exposure of the standard fire temperature regime, represented in the form of the corresponding curve in Fig. 9.

Fig. 9 demonstrates the initial stage of loading by the active mechanical load, the gradual increase of the deflection as a result of the temperature increase in the layers of concrete and reinforcing steel of the reinforced concrete crossbar. At

the last stage, one can see a jump-like increase in the maximum deflection, which can be associated with the destruction of the reinforced concrete crossbar as a result of the degradation of concrete during its high-temperature heating.

The experimental results obtained in the course of the calculation can be used to justify the method of researching

the patterns of changes in concrete strength depending on the temperature based on measurements obtained during fire tests. Fire tests are carried out according to a special procedure. This makes it possible to refine the deformation diagrams for concrete recommended in [10]. To achieve this goal, the approach described in works [13, 15, 16] was used.

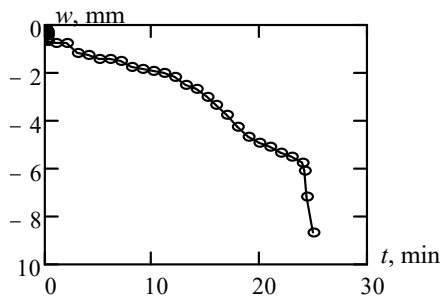


Fig. 9. Dependence plot of the critical deflection of a reinforced concrete crossbar depending on the time of the test

**5. 2. A set of mathematical models for temperature interpolation in the cross-section of reinforced concrete crossbars based on point temperature indicators**

In order to identify the strength of concrete under heating conditions, an algorithm was developed for determining the temperature at nodal points of the cross-section by interpolating temperatures based on temperature indicators at control points of the cross-section, the block diagram of which is shown in Fig. 10.

Here:

- $\theta_{dk}, \theta_{gk}, \theta_{vk}$  are the temperatures determined on the heating surfaces along the diagonal and average vertical and horizontal sections, respectively;
- $\theta_0$  is the temperature of the least heated cross-section point;
- $Q_d, Q_g, Q_v$  - indicators that are determined, respectively, for the diagonal and average horizontal and vertical sections;
- $d_0$  is the length of the segment diagonally from the origin of the coordinates to the given nodal point;
- $d$  is the length of the radial segment from the origin of the coordinates to the point with the current coordinates;
- $h, a$  are the height and width of the section, respectively.

The scheme of interpolation between approximated isotherms is shown in Fig. 11.

Using the proposed algorithm, appropriate calculations were performed for a reinforced concrete cross-section, the diagram of which is shown in Fig. 1. As an imitation of the temperature indicators measured during fire tests, the calculated data described above were used.

For a qualitative assessment of the adequacy of the results, temperature fields in the cross-section in the form of isolines were constructed. Fig. 12 shows the constructed temperature fields.

The analysis of the results of the interpolation of temperature distributions showed that the control points should be located along the main vertical, horizontal, and diagonal sections.

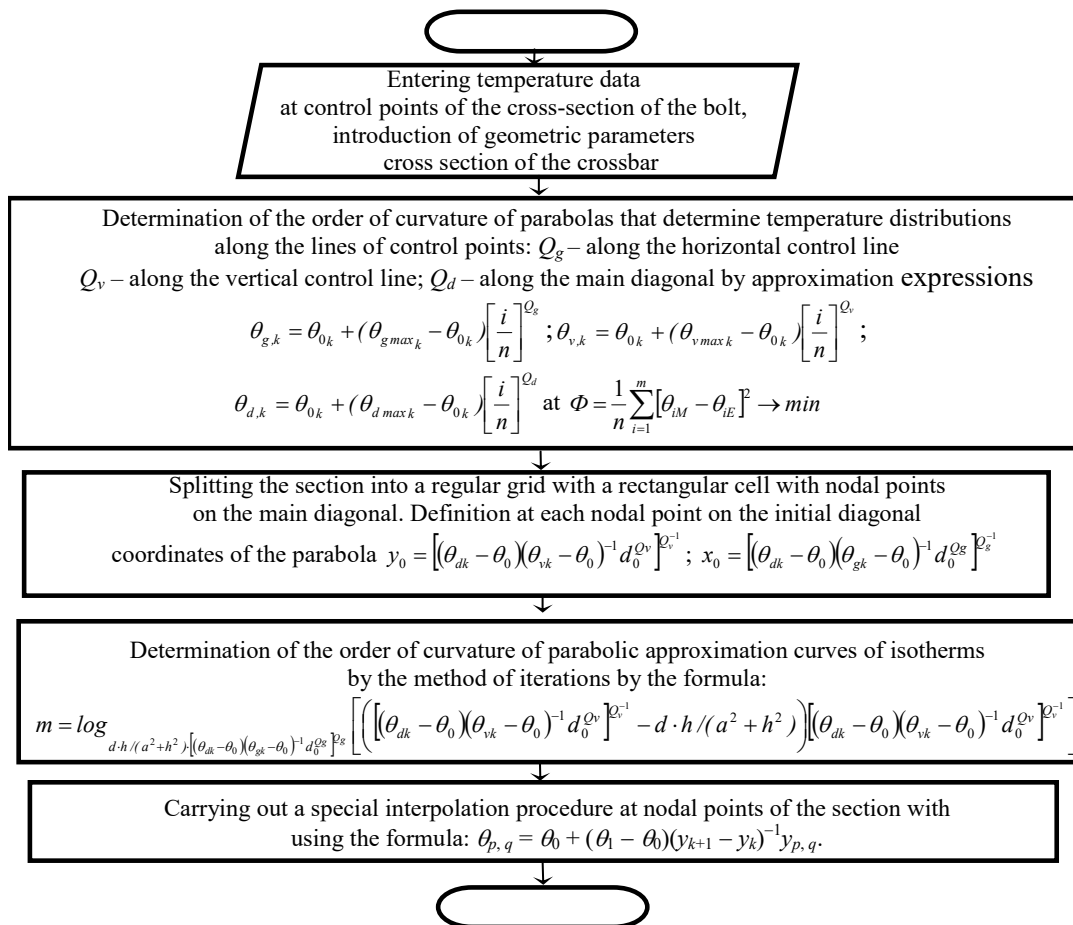


Fig. 10. Block diagram of the algorithm for determining the temperature at the nodal points of the cross-section by interpolating the temperatures according to the temperature indicators at the control points of the cross-section

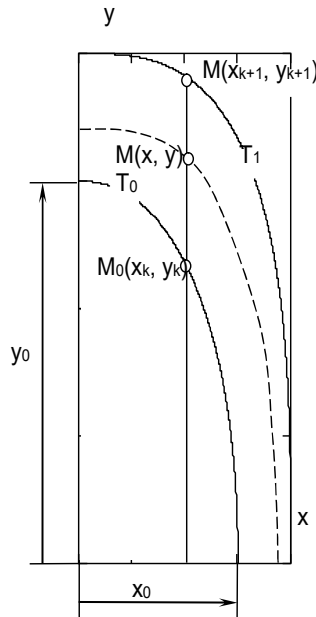


Fig. 11. Scheme of temperature interpolation in cross-section of a fragment of a reinforced concrete crossbar

approximation, but the surfaces of the temperature distributions have discontinuities in the first and second derivatives.

For a more accurate analysis, the initial data, that is, the data obtained by the theoretical approach, and the corresponding interpolation results obtained by the proposed algorithm were compared. The results of the comparative analysis are given in Table 2.

Table 2

Data on comparative analysis of results obtained by interpolation

Maximum deviation, °C	Mean relative deviation, %	Standard deviation, °C
76	8.4	18.9

The data in Table 2 indicate the high accuracy of the interpolation algorithm, and the obtained results are adequate. Therefore, it can be used to approximate temperature distributions based on discrete temperature measurements at control points where thermocouples are installed.

**5. 3. Mathematical model for identifying the concrete strength of reinforced concrete crossbars under the conditions of thermal exposure of the standard fire temperature regime**

The calculation to identify the coefficient of reduction in concrete strength was carried out according to the scheme shown in Fig. 13. This scheme makes it possible to write down the system of equilibrium equations for the layers into which the section is divided, in a convenient form for determining the specified values of the coefficient of reduction in the concrete strength of the reinforced concrete crossbar.

To identify concrete strength characteristics, a system of linear algebraic equations (SLAE) is used, which takes the form:

$$[F]\{k\} + \{S\} = M_{Ed}, \tag{1}$$

where  $\{k\} = (k_{c1} \ k_{c2} \ \dots \ k_{cm})^T$  is a vector of values of coefficients of reduction of concrete strength of a reinforced concrete crossbar corresponding to tabular values of temperatures  $\{\theta_m\} = (0 \ 100 \ 200 \ \dots \ \theta_m)^T$ , which are unknowns in the recorded SLAE;

$[F]$  is the matrix of SLAE coefficients, which takes the following form:

$$[F] = \begin{pmatrix} Z_{11} & \dots & Z_{1j} & \dots & Z_{1m} \\ \vdots & & \vdots & & \vdots \\ Z_{i1} & & Z_{ij} & & Z_{im} \\ \vdots & & \vdots & & \vdots \\ Z_{m1} & \dots & Z_{mj} & \dots & Z_{mm} \end{pmatrix}. \tag{2}$$

The parameters of the coefficient matrix are calculated by the expressions:

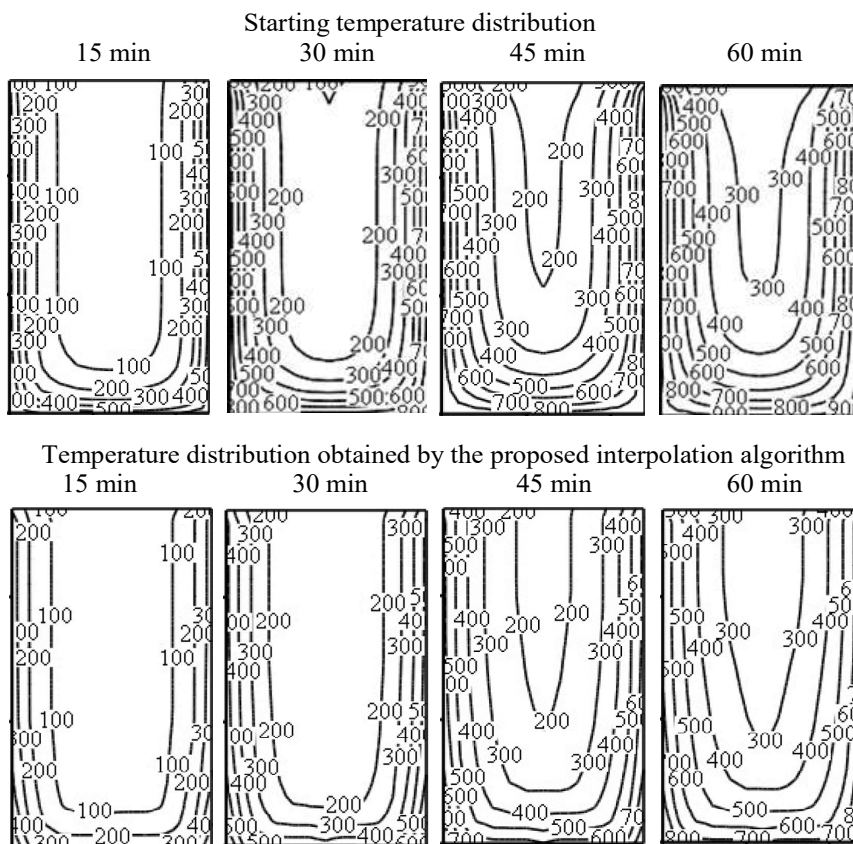


Fig. 12. Temperature distributions in the cross-section of a 300×150 mm reinforced concrete crossbar made of heavy concrete at different points in time during the action of a “standard” fire, with the arrangement of thermocouples according to the standard scheme

From the surfaces of the temperature distributions, it can be seen that the interpolation performed according to the proposed algorithm gives adequate results. It is possible to note a noticeable difference in the shapes of the obtained surfaces. The proposed interpolation procedure gives a sufficiently accurate



$$\begin{aligned}
 Z_{i1} &= \sum_n [1 - 0.01(\theta_{ni} - \theta_1)] F_n(\epsilon, \theta_{ni}) A_c y_n Z_{im}, \\
 Z_{im} &= \sum_n 0.01(\theta_{ni} - \theta_{m-1}) F_n(\epsilon, \theta_{ni}) A_c y_n, \\
 Z_{ij} &= \sum_n [1 - 0.01(\theta_{ni} - \theta_j)] F_n(\epsilon, \theta_{ni}) A_c y_n + \\
 &+ \sum_n 0.01(\theta_{ni} - \theta_{j-1}) F_n(\epsilon, \theta_{ni}) A_c y_n.
 \end{aligned} \tag{3}$$

The value  $m = \lceil T_{\max} \cdot 0.01 \rceil$  is the number of equations and variables in SLAE (1), which is calculated based on the maximum heating temperature of a reinforced concrete crossbar during fire tests.

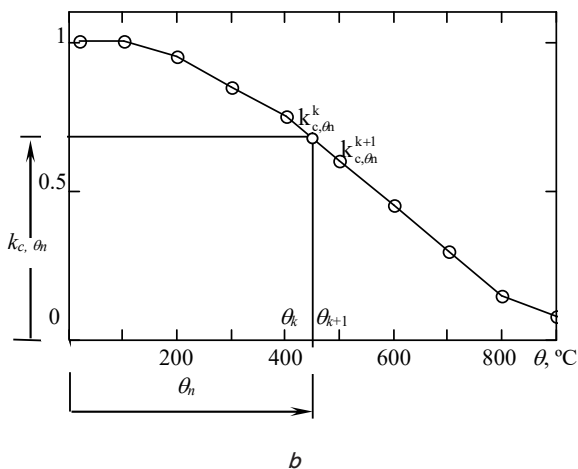
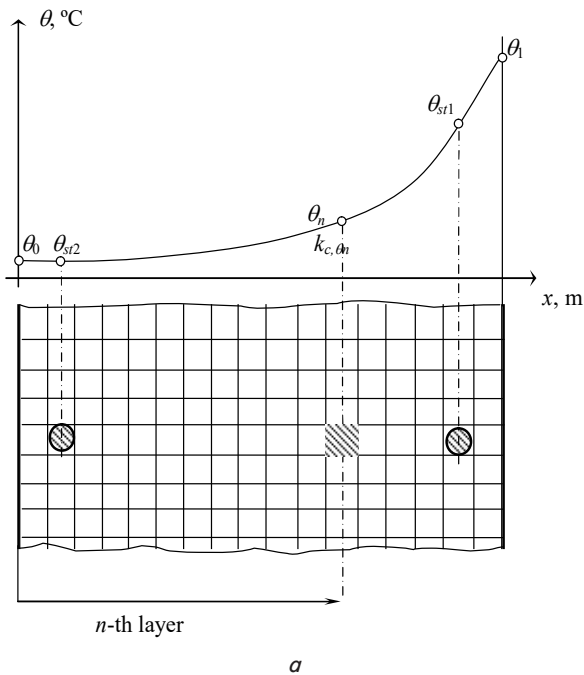


Fig. 13. Scheme for calculating the coefficient of reduction of the concrete strength of a reinforced concrete crossbar according to the temperature in the inner layer: *a* – diagram of division into layers of the cross-section of a reinforced concrete crossbar; *b* – scheme of linear interpolation of the coefficient of reduction of concrete strength of a reinforced concrete crossbar according to tabular data

When writing the equations for specific test moments, the value of the maximum deflection is set evenly between the first test moment and the time when the correspondence of the deformation and stress diagram to the linear dependence is maintained.

In SLAE (1),  $\{S\}$  is the force vector in reinforcing bars, which is calculated using the following formula:

$$\{S\} = (S_1 \dots S_i \dots S_m)^T, \tag{4}$$

where  $S_i = \sum_p F_{st,p}(\epsilon, \theta_{st,p})$ . The forces in the *p*-th reinforcing bar at the *i*-th moment of time are calculated according to the expressions recommended by the guidelines for the estimated fire resistance of reinforced concrete structures [10].

Stresses in the inner layers of concrete and reinforcing steel for the calculation of matrices that are components of SLAE (1) are calculated using deformation diagrams of reinforced concrete crossbars according to the expressions:

$$\begin{aligned}
 \sigma_{ci} &= f_1(\epsilon_{ci}), \quad \sigma_{sj} = f_2(\epsilon_{sj}), \\
 \epsilon_{ci} &= \epsilon_0 + y_{ci} \chi_t, \quad \epsilon_{sj} = \epsilon_0 + y_{sj} \chi_t,
 \end{aligned} \tag{5}$$

where  $\epsilon_0$  is the relative deformation of the upper cross-section point of the reinforced concrete crossbar in the transverse direction;

$\chi_t$  is the curvature of the reinforced concrete crossbar at a certain moment in time.

The relative deformation of the upper point and the curvature of the section of the reinforced concrete crossbar is calculated using the hypothesis of flat sections according to the formulas:

$$\epsilon_0 = \frac{h^2 \chi_t}{2h - z_0}, \quad \chi_t = \frac{48w_t}{5L^2}, \tag{6}$$

where *L* is the span length of the reinforced concrete crossbar;  $w_t$  – the maximum deflection of a reinforced concrete crossbar at a certain moment in time;

$z_0$  is the axial distance from the outermost reinforcing bar to the lower edge of the reinforced concrete cross-section.

#### 5. 4. Adequacy of the results of identification of the dependence of the concrete strength of the reinforced concrete crossbar on temperature

The obtained results made it possible to carry out the necessary calculations and obtain the dependences of the concrete strength of the reinforced concrete crossbar on the temperature, which are shown in Fig. 14.

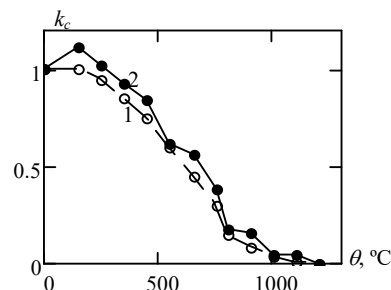


Fig. 14. Dependences of the concrete strength reduction factor: 1 – standard dependence, based on which the initial data for identification are determined; 2 – refined dependence

Analysis of the curves of the concrete strength reduction factor shown in Fig. 14 showed that when comparing them, the average relative error is no more than 7 %. Thus, using the results of temperature measurements and the maximum deflection of a reinforced concrete crossbar obtained during fire tests, the concrete strength reduction coefficient was experimentally refined.

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### 6. Substantiation of the adequacy of the results of identification of strength characteristics of concrete of reinforced concrete crossbars

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Based on the results of our research, it is shown that the obtained data on the temperature and maximum deflection of a reinforced concrete crossbar under heating conditions during fire tests of full-size samples allow us to refine the value of the coefficient of reduction of concrete strength for reinforced concrete crossbars. An algorithm based on the approximation of isotherms by parabolas with a variable exponent is proposed to restore the temperature field based on individual temperature indicators. The proposed algorithm for temperature interpolation based on indicators at control points showed their sufficient accuracy and adequacy since the relative error of temperature indicators does not exceed 8.4 %, and the root mean square deviation of temperature does not exceed 18.9 °C. This makes it possible to use it to identify the dependence of concrete strength on the heating temperature, using at the same time the data of fire tests of reinforced concrete crossbars. Calculations carried out to identify the coefficient of reduction in concrete strength were performed according to the scheme shown in Fig. 13. This scheme makes it possible to write a system of equilibrium equations for the layers into which the section is divided. To describe the stressed-strained state in the cross-section of a reinforced concrete crossbar under heating conditions, a deformation model was used, on the basis of which SLAE was compiled for the implementation of the method for identifying concrete strength depending on the heating temperature (1).

Using the data obtained by interpolation of temperatures in cross-sections of reinforced concrete cross-sections and mathematical apparatus (1) to (6), the curve of the dependence of the coefficient of reduction of concrete strength on the heating temperature was identified. Analysis of the curves of the concrete strength reduction factor shown in Fig. 14 reveals that the average relative error is no more than 7 %. This appropriately confirms the effectiveness of the devised method for identifying the mechanical characteristics of concrete.

A comparative analysis of our results (Fig. 14) with the results that were included in the model to obtain a set of initial data for identification showed that the use of the proposed method of identification of the concrete strength reduction factor will make it possible to increase the accuracy of the forecast data when assessing the fire resistance of reinforced concrete crossbars.

Our studies of this method of identification are limited only to reinforced concrete cross-sections and crossbars of rectangular cross-section. Also, this method uses the hypothesis of plane sections and the deformation model used to analyze the stressed-strained state. All this imposes restrictions on the use of this identification method.

The next step, which will make it possible to determine the real regularities of the decrease in the concrete strength

of reinforced concrete crossbars, is the application of the proposed method of identification using measurement data during real fire tests.

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### 7. Conclusions

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1. On the basis of calculations based on the finite element method, the temperature indicators at the control points of the cross-section and the curve of dependence of the maximum deflection on the time of the tests, which are necessary for the implementation of the method of identifying the strength of concrete depending on its heating temperature, were obtained. It is shown that isotherms in the cross-section of a reinforced concrete crossbar can be approximated by parabolas with a variable exponent, and the maximum deflection curve reproduces the corresponding data obtained during fire tests.

2. A set of mathematical models of interpolation of temperatures in the inner layers of a reinforced concrete crossbar based on the approximation of isotherms by parabolas with a variable exponent has been substantiated, and the adequacy of the results obtained by these methods has been shown. It is demonstrated that the obtained results are adequate since the relative error of the temperature indicators does not exceed 8.4 %, and the root mean square deviation of the temperature does not exceed 18.9 °C.

3. A mathematical model for identifying the coefficients of reduction in concrete strength of reinforced concrete crossbars and crossbars of rectangular cross-section has been substantiated. This mathematical model can be applied to identify the strength of concrete depending on the temperature using the obtained data of fire tests of reinforced concrete crossbars by calculation.

4. The adequacy of the concrete strength indicators obtained as a result of the identification was analyzed by comparing them with the concrete strength indicators included in the mathematical model. As a result, it was confirmed that the obtained data are adequate as their relative error does not exceed 7 % on average. This makes it possible to assert that the experimental-calculation method for identifying patterns of changes in concrete strength depending on the temperature of its heating based on the results of measurements during fire tests of reinforced concrete crossbars allows obtaining new data, which is the basis for improving the methods of calculating the fire resistance of reinforced concrete structures.

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### Conflicts of interest

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The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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### Data availability

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All data are available in the main text of the manuscript.

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