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DEVELOPMENT OF THE METHOD OF GENERAL INTERPOLATION FOR Z-NUMBER-VALUED IF-THEN RULES

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Rule interpolation-based methods are used when the rule base is sparse. This frequently being the case, as information relevant to real-world problems is not usually comprehensive. At the same time, relevant information is often characterized by both fuzziness and partial reliability. To deal with such kind of information, the concept of Z-number was introduced by Zadeh. This paper is devoted to an extension of the general interpolation method for fuzzy rules to the case of if-then rules with Z-number-valued antecedents and consequents. The proposed approach relies on the determination of the distance between the current observation vector and vectors of rules antecedents. By determining the distance between the current vector and the antecedents of the rules, decisions can be made based on the nearest antecedents. In this context, rule antecedents are vectors that represent certain conditions. The resulting output is computed as a weighted sum of rules consequents. Weighting factors are used to account for the importance of each rule in the interpolation. Weights of interpolations are found on the basis of mentioned distance values. The results of this study are aimed at developing an approach to decision-making in terms of Z-valued information. The method is characterized by relatively low computational complexith. Regarding the application of the proposed approach, the job satisfaction evaluation problem is considered. Consequently, the obtained results confirm the efficiency of the proposed approach. The proposed method can be a useful tool for decision-making in various applications, especially where high computational complexity is unacceptable or impractical

Keywords: Z-number, fuzzy number, partial reliability, if-then rules, interpolation, distance, weights

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1. Introduction

In recent years, fuzzy systems and fuzzy data analysis methods have become increasingly popular in various fields of science and technology. One of the key components of such systems is the fuzzy rule base itself, which describes the dependencies between input and output parameters. However, in real applications, a situation may arise when the rule base contains an insufficient number of rules to fully describe all possible variants of input parameters. Such a base is called sparse, that is, not all possible variants of input parameters are provided for in the rules. In this case, it becomes necessary to use reasoning methods based on fuzzy rule interpolation (FRI). Fuzzy rule interpolation reasoning methods allow to use several different rules that can interact with each other to determine the result. Thus, this method allows more efficient use of the rule base, which can contain only a small number of rules, while still ensuring the accuracy and reliability of the result.

In situations with high levels of uncertainty, information is often characterized by both fuzziness and partial reliability. The concept of Z-number was introduced by Zadeh to deal with this issue [1]. A Z-number is an ordered pair $Z=(A, B)$ of fuzzy numbers A and B. A is used as a fuzzy restriction on a value of a variable of interest, whereas B plays a role of reliability described as a probability measure of A [2].

At the same time, it is not always necessary to consider all combinations of linguistic terms to form an adequate model of a considered phenomenon. Zadeh's Z-numbers are a key element in the interpolation of fuzzy rules, which is widely used in fuzzy control systems for decision-making based on fuzzy logic.

In the interpolation of fuzzy rules, Zadeh's Z-numbers are used to determine the degree of membership of an element to a fuzzy set that describes the state of the system. Based on these values and a rule base containing fuzzy statements about how to control the system depending on its state, an output value is formed.

The values of Zadeh's Z-numbers can be used to aggregate statements from different rules and determine the output value of the system. Interpolation of fuzzy rules can use various methods, such as minimum or maximum operators, or the weighted average of Zadeh's Z-numbers to calculate the final result.

In general, Zadeh Z-numbers and fuzzy rule interpolation are key components of fuzzy control systems and allow working with uncertainty and fuzziness in decision-making. They allow to make decisions based on fuzzy logic and achieve optimal results in the face of uncertainty.

Thus, Zadeh's Z-numbers play an important role in the interpolation of fuzzy rules and are a key element in fuzzy control systems. By using Z-numbers to model the uncertainty and imprecision of real-world problems, fuzzy logic, and fuzzy control have proven to be useful in a wide range of applications in various fields such as control engineering, decision-making, pattern recognition, and machine learning.

2. Literature review and problem statement

A systematic analysis of state-of-the-art FRI is proposed in [3]. According to the analysis, existing approaches are divided into two categories: FRI with non-weighted rules and with weighted rules. In the first category-based approach, it is assumed that all rule antecedents are of equal importance for computing the conclusion. The second category-based approaches rely on the assumption that antecedents are of different importance levels. This allows to improve the interpolation performance. The authors conduct intra-category and inter-category comparative analyses of the approaches and uncover their advantages and limitations for different problems. Determining the levels of importance of antecedents can be subjective and depend on the views and expertise of researchers, which can lead to ambiguous results. The use of categories can increase the complexity of calculations, especially if it is necessary to process a large amount of data or a large number of categories.

In [4] an updated and extended version of the original FRI toolkit is described, as well as the analysis of various fuzzy rule interpolation (FRI) methods (KH, KH Stabilized, MACI, IMUL, CRF, VKK, GM, FRIPOC, LESFRI, and SCALEMOVE) based on various features, using unified numerical reference examples, and their classification in accordance with the conditions of anomaly and linearity. At the same time, it should be noted that the classification of data by linearity can ignore non-linear relationships between variables, which can be important for understanding complex relationships in data. In addition, the unified numerical reference examples may only cover certain types of data and conditions, which may limit their applicability to various tasks and situations.

As described in [5] the D-FRI system is an efficient method for selecting, combining, and generalizing informative interpolated rules to combine with an existing rule base and perform interpolation reasoning. Numerous experiments by the author show that D-FRI is superior to traditional FRI methods in accuracy and reliability. As an example, the article explores the use of D-FRI for network security analysis and the development of an intrusion detection system (IDS) that integrates with Snort software, one of the most common open-source IDS. In this article, it should be noted that fuzzy logic rules can be ambiguous in interpretation, which can make it difficult to determine precise and unambiguous actions when threats or attacks are detected. Network security analysis using fuzzy rules can be computationally intensive, especially when processing large amounts of data, which can slow down the process of detecting and responding to threats.

A novel approach to FRI based on fuzzy geometry is proposed in [6]. The motivation is to derive a closed mathematical form of interpolation. The proposed approach includes two different stages. Firstly, all the fuzzy rules are transformed into higher-dimensional fuzzy points and joined with a class of fuzzy line segments. It is then considered as the problem of the identification of the interpolated piecewise linear fuzzy polynomial which allows mapping a given observation to the desired conclusion. The approach provides a strong mathematical basis and geometrical visualization of FRI. Interpolation may be a good strategy for exact matching data, but may not be able to generalize to unknown data outside of the interpolation nodes.

A comprehensive analysis of interpolation and extrapolation techniques is conducted in [7]. Such criteria as applicability, complexity reduction, and logic are considered. The authors propose a standard for the analysis of existing techniques based on a unified set of criteria and a framework for classification and comparison. A single set of criteria may not always be universal and applicable to different types of data and tasks. Conducting an analysis based on a single set of criteria can be complex and time-consuming, especially when analyzing a large amount of data and different methods.

A series of studies on rule interpolation under high levels of uncertainty were proposed in [8–10], and other works. For example, the main motivation of work in [8] relies on the difficulty to construct precise membership functions. As a result, a systematic approach to interpolation for rules with rough fuzzy sets is proposed. Rough fuzzy sets can be limited in representing complex and dynamic data, and in the ability to interpolate data, especially in non-linear scenarios. Several benchmark problems are used to illustrate the efficacy of the proposed approach. In [9], a new approach to fuzzy interpolation is presented, which uses fuzzy sets of interval type 2 (second order). By calculating the ranking values of the upper and lower membership functions of these fuzzy sets, the authors propose to efficiently process fuzzy interpolation reasoning in sparse fuzzy systems. This approach provides a more flexible and intelligent way to work with fuzzy data, allowing to better account for uncertainty and fuzziness in the data. Note that fuzzy sets of the second order may be less universal and applicable to some types of data and tasks.

In [10], an approach to fuzzy rule interpolation based on scale and displacement transformation (T-FRI) is presented, which supports sparse rule bases (transformation-based rule interpolation). T-FRI provides the ability to get a ballpark inference when an observation does not match any of the rules in the existing rule base. The authors also popularize the FRI approach, which allows interpolation and extrapolation with multiple rules and antecedents. However, the difficulty lies in determining the exact values of the membership functions required to represent fuzzy rules or observations, which limits its application. To solve this problem, the authors propose to use fuzzy sets of interval type 2, since the membership functions of such fuzzy sets are fuzzy in themselves. This provides more flexible modeling tools and solves the problem of determining the exact membership functions. However, the use of second-order fuzzy sets may be less able to generalize data to unknown regions or beyond interpolation nodes.

All of the above allows to assert the expediency of conducting a study for Z-numeric if-then rules. This is supported by the presence of relevant factors and arguments that point to the potential benefits and relevance of such research. In particular, giving the combination of various aspects and requirements associated with Z-numbers and numerical rules, it is expected that research in this area may lead to the development of more efficient interpolation methods, contributing to more accurate and reliable results when working with such numerical rules.

3. The aim and objectives of the study

The aim of the study is to develop a method for interpolation with Z-number-valued if-then rules (Z-rules). The considered concept of distance forms the basis for the processing of imperfect information in decision making and inference.

To achieve this aim, let's accomplish the following objectives:

– to formalize a way to measure the 'closeness' of new Z-valued input with rules antecedents;

– to develop a technique for Z-valued output as the weighted mean of rule consequents (based on the measured 'closeness');

– to present the decision of practical problem on reasoning with Z-rules on the example of job satisfaction evaluation problem.

4. Materials and methods of research

The object of research is a theoretical analysis of Z-value if-then rules and an experimental study on real data in order to better understand the logical relationships and applicability of Z-value rules in various contexts.

The main hypothesis of the study is the assumption that the uncertainty of information can be adequately described using Z-numbers. The article discusses aspects of using Z-numbers for modeling and assessing the degree of uncertainty, which can contribute to the development of theory and practice in this area.

This article is supposed to identify probabilistic and fuzzy categories that are suitable for the subsequent formation of understandable rules for a person.

When using linear interpolation, intermediate values are calculated based on already-known data or conditions. In the context of reasoning and inferring with Z-value if-then rules, this can mean using linear interpolation to predict inferences or outcomes based on known conditions, creating smoother transitions between different scenarios or states.

The concept of Z-numbers, proposed by L. Zadeh, plays an important role in the theory of fuzzy logic and fuzzy systems, allowing more flexible and adaptive work with fuzzy data and uncertainty, which is used in various fields such as artificial intelligence, management, decision making, control systems, and others.

The use of Z-number distance in approximate reasoning with if-then rules allows more accurate and flexible data analysis, which makes this approach promising for further research and application in various fields where the processing of fuzzy and uncertain information is important.

For the computational implementation of the problem presented in the work, the MATLAB package was used.

5. Results of the study on developing a method of general interpolation for Z-number-valued if-then rules

5. 1. Formalization of a way to measure the 'closeness' of new Z-valued input with rules antecedents

Definition 1. A discrete Z-number.

A discrete Z-number is an ordered pair $Z=(A, B)$ where A is a discrete fuzzy number playing a role of a fuzzy constraint on values that a random variable X may take:

X is A,

and B is a discrete fuzzy number with a membership function μ_B: $\{b_1,...,b_m\} \rightarrow [0, 1]$, $\{b_1,...,b_m\} \subset [0, 1]$, playing the role of a fuzzy constraint on the probability measure $P(A)=\sum_{x\in X}\mu_A(x)p(x)$ of A:

$P(A)$ is B.

This implies that information on probability distribution (or probability density function in continuous case) p is imprecise [11].

Definition 2. A distance between Z-numbers.

As a Z-number $Z=(A, B)$ is characterized by fuzzy number A, fuzzy number B, and an underlying set of probability distributions G, let's propose to define the distance between Z-numbers $D(Z_1, Z_2)$ as follows.

Distance between A_1 and A_2 is computed as:

$$D(A_1,A_2)=\sup_{\alpha\in(0,1]} D(A_1^\alpha,A_2^\alpha), \quad (1)$$

$$D(A_1^\alpha,A_2^\alpha)=\left|\frac{A_{11}^\alpha+A_{12}^\alpha}{2}-\frac{A_{21}^\alpha+A_{22}^\alpha}{2}\right|, \quad (2)$$

where A_1^α and A_2^α denote α-cuts of A_1 and A_2 respectively, A_{11}^α, A_{12}^α denote lower and upper bounds of A_1^α (A_{21}^α, A_{22}^α are those of A_2^α). The distance between B_1 and B_2 is computed analogously.

It is also have to find the distance between the sets G_1 and G_2 of probability density functions p_1 and p_2 underlying Z_1 and Z_2. The distance between p_1 and p_2 can be expressed as:

$$D(G_1,G_2)=\inf_{p_1\in G_1,p_2\in G_2}\left\{(1-\int_R\left((p_1p_2)^{\frac{1}{2}}dx\right)^{\frac{1}{2}}\right\}. \quad (3)$$

In (3), the expression in figure brackets is the Heellinger distance between two pdfs p_1 and p_2. The inf operator is used to determine distance between the closest two pdfs $p_1\in G_1$ and $p_2\in G_2$. In other words, the pair of the closest $p_1\in G_1$ and $p_2\in G_2$ is found among all the possible pairs of distributions to define distance $D(G_1, G_2)$.

Given $D(A_1, A_2)$, $D(B_1, B_2)$ and $D(G_1, G_2)$, the distance between Z-numbers is defined as:

$$D(Z_1,Z_2)=\beta D(A_1,A_2)+(1-\beta)D_{total}(B_1,B_2), \quad (4)$$

$D_{total}(G_1, G_2)$ is computed as:

$$D_{total}(B_1,B_2)=wD(B_1,B_2)+(1-w)D(G_1,G_2), \quad (5)$$

where β, $w\in(0, 1)$ are the user's assigned degrees used to measure the importance of A, B, and G sets for the computation of the distance between Z-numbers [12].

Definition 3. Ranking of Z-numbers.

For Z-numbers Z, Z' it holds:

$$Z\le Z', \text{ if } D(Z,Z^*)\ge D(Z',Z^*),$$

where D is distance defined as in Definition 2 by equations (1)–(5). The components of Z-number $Z^*=(A, B)$ are fuzzy singletons:

1) $A=\bar{a}, \bar{a}$ is the upper bound of the universe of discourse $X\in$ R, R is the set of real numbers;

2) $B=1$. Thus, Z^* is the ideal (highest) Z-number – A takes its extremal value and the reliability is 100 %. Thus, Z-number Z' is higher than Z if Z' is closer to the ideal Z-number Z^*.

The considered relation ≤ is a partial order [13].

The problem of interpolation of Z-rules is formulated below.

Given the following Z-rules:

– rule 1: if X_1 is $Z_{X_1,1}=(A_{X_1,1},B_{X_1,1})$ and, ..., and X_m is $Z_{X_m,1}=(A_{X_m,1},B_{X_m,1})$ then Y is $Z_Y=(A_{Y,1}, B_{Y,1}$;

– rule 2: if X_1 is $Z_{X_1,2}=(A_{X_1,2},B_{X_1,2})$ and, ..., and X_m is $Z_{X_m,2}=(A_{X_m,2},B_{X_m,2})$ then Y is $Z_Y=(A_{Y,2}, B_{Y,2}$;

– rule n: if X_1 is $Z_{X_1,n}=(A_{X_1,n},B_{X_1,n})$ and, ..., and X_m is $Z_{X_m,n}=(A_{X_m,n},B_{X_m,n})$ then Y is $Z_Y=(A_{Y,n}, B_{Y,n})$.

And a current observation:

$$X_1 \text{ is } Z'_{X_1}=(A'_{X_1},B'_{X_1}) \text{ and, ..., and } X_m \text{ is } Z'_{X_m}=(A'_{X_m},B'_{X_m}),$$

find the Z-value of Y.

Let's propose an extension of the method of general interpolation for fuzzy rules to the case of Z-valued rules. The main idea remains the same: if components of the current observation vector $\mathbf{Z}'=(Z'_{X_1},...Z'_{X_m})$ are «in-between» the components of vectors of antecedents $\mathbf{Z}_1=(Z_{X_1,1},...,Z_{X_m,1})$, $\mathbf{Z}_2=(Z_{X_1,2},...,Z_{X_m,2})$ (unfortunately, it is impossible to write this complex index by tools of MS Word) of two rules then the corresponding output is computed as a linear combination of consequents. Coefficients of this combination reflect influence of each rule consequent on resulting output [14].

5. 2. Development of a technique for Z-valued output as the weighted mean of rule consequents (based on the measured 'closeness')

The proposed method of general interpolation for Z-number-valued rules includes the stages described below:

Stage 1. It is needed to check if the ordering conditions are satisfied for current observation $\mathbf{Z}'=(Z'_{X_1},...Z'_{X_m})$ and vectors of rules antecedents $\mathbf{Z}_1=(Z_{X_1,1},...,Z_{X_m,1})$, $\mathbf{Z}_2=(Z_{X_1,2},...,Z_{X_m,2})$ (unfortunately, it is impossible to write this complex index by tools of MS Word):

$$Z_{X_1,1}\le Z'_{X_1}\le Z_{X_1,2}\left(\text{or } Z_{X_1,2}\le Z'_{X_1}\le Z_{X_1,1}\right),...,$$

$$Z_{X_m,1}\le Z'_{X_m}\le Z_{X_m,2}\left(\text{or } Z_{X_m,2}\le Z'_{X_m}\le Z_{X_m,1}\right).$$

By using definition 3, these conditions are described as follows:

$$\begin{pmatrix}\text{or } D\left(Z_{X_1,1},Z_1^*\right)\le D\left(Z'_{X_1},Z_1^*\right)\le D\left(Z_{X_1,2},Z_1^*\right),...,\\ D\left(Z_{X_m,1},Z_m^*\right)\ge D\left(Z'_{X_m},Z_m^*\right)\ge D\left(Z_{X_m,2},Z_m^*\right)\end{pmatrix}$$

$$\left(\text{or } D\left(Z_{X_m,1},Z_m^*\right)\le D\left(Z'_{X_m},Z_m^*\right)\le D\left(Z_{X_m,2},Z_m^*\right)\right), \quad (6)$$

references to the formulas in the text have the form (1), (2)–(4), where Z_1^*, ..., Z_m^* are ideal Z-numbers.

Stage 2. If conditions of (6) are satisfied then the values of distance between the current observation vector Z' and the vectors of the antecedents of two rules, $D_v(Z', Z_1)$ and $D_v(Z', Z_2)$ are computed:

$$D_v\left(\mathbf{Z}',\mathbf{Z}_1\right)=\sqrt{D^2\left(Z'_{X_1},Z_{X_1,1}\right)+...+D^2\left(Z'_{X_m},Z_{X_m,1}\right)},$$

$$D_v\left(\mathbf{Z}',\mathbf{Z}_2\right)=\sqrt{D^2\left(Z'_{X_1},Z_{X_1,2}\right)+...+D^2\left(Z'_{X_m},Z_{X_m,2}\right)}, \quad (7)$$

where D is distance between Z-numbers.

Stage 3. Given $D_v(Z', Z_1)$ and $D_v(Z', Z_2)$ computed at Stage 2, it is needed to compute the interpolation coefficients (weights) w_j, $j=1, 2$. The lower the distance of current observation vector Z' to rule antecedents vector Z_j, the higher weight w_j, $j=1, 2$ is. At the same time, weights w_j, $j=1, 2$ should satisfy $w_1, w_2\in[0, 1]$ and $w_1+w_2=1$. In view of this, the following formula can be used:

$$w_j=1-\frac{D_v\left(\mathbf{Z}',\mathbf{Z}_j\right)}{D_v\left(\mathbf{Z}',\mathbf{Z}_1\right)+D_v\left(\mathbf{Z}',\mathbf{Z}_2\right)}, j=1,2. \quad (8)$$

Surely, $w_1, w_2\in[0,1]$ and $w_1+w_2=1$.

Stage 4. A resulting output is computed as the weighted sum of the consequents of rules 1 and 2:

$$Z'=w_1Z_{Y,1}+w_2Z_{Y,2}, \quad (9)$$

where Z_{Yj} is the Z-valued consequent of the j-th rule, w_j, $j=1, 2$ are weights of linear interpolation computed at Stage 3.

Further let's consider two examples to illustrate the proposed approach.

Example 1. Consider the following Z-rules:

– rule 1: if X_1 is $Z_{X_1,1}=(L,P)$ and X_2 is $Z_{X_2,1}=(L,U)$ and X_3 is $Z_{X_3,1}=(H,P)$ then Y is $Z_{Y,1}=(L,U)$;

– rule 2: if X_1 is $Z_{X_1,2}=(M,U)$ and X_2 is $Z_{X_2,2}=(H,R)$ and X_3 is $Z_{X_3,2}=(L,U)$ then Y is $Z_{Y,2}=(VH,P)$.

The codebooks of the linguistic terms described by triangular fuzzy numbers (TFN) are given in Tables 1, 2.

Table 1

The encoded linguistic terms A part of Z-number

Level of Satisfaction	Linguistic value
Very low (*VL*)	{0/1, 1/1, 0/2}
Low (*L*)	{0/1, 1/2, 0/3}
Medium (*MS*)	{0/2, 1/3, 0/4}
High (*H*)	{0/3, 1/4, 0/5}
Very high (*VH*)	{0/4, 1/5, 1/5}

Table 2

The encoded linguistic terms for B part of Z-number (Reliability)

Level of Satisfaction	Linguistic value
Rare (*R*)	{0/0.05, 1/0.25, 0/0.5}
Plausible (*P*)	{0/0.25, 1/0.5, 0/0.85}
Usual (*U*)	{0/0.5, 1/0.85, 0/1}

Let the current observation be described by Z-numbers using TFN-based parts:

$$X_1 \text{ is } Z'_{X_1}=\left(\left(1.5 \quad 2.5 \quad 3.5\right)\left(0.35 \quad 0.65 \quad 0.95\right)\right),$$

$$X_2 \text{ is } Z'_{X_2}=\left(\left(2.5 \quad 3.5 \quad 4.5\right)\left(0.2 \quad 0.4 \quad 0.6\right)\right),$$

and

$$X_3 \text{ is } Z'_{X_3}=\left(\left(2.5 \quad 3.5 \quad 4.5\right)\left(0.35 \quad 0.65 \quad 0.95\right)\right).$$

Let the current observation vector be:

$$\mathbf{Z}'=\begin{pmatrix}Z'_{X_1}=\left(\left(1.5 \quad 2.5 \quad 3.5\right)\left(0.35 \quad 0.65 \quad 0.95\right)\right),\\ Z'_{X_2}=\left(\left(2.5 \quad 3.5 \quad 4.5\right)\left(0.2 \quad 0.4 \quad 0.6\right)\right),\\ Z'_{X_3}=\left(\left(2.5 \quad 3.5 \quad 4.5\right)\left(0.35 \quad 0.65 \quad 0.95\right)\right)\end{pmatrix},$$

the vectors of rules antecedents are:

$$\mathbf{Z}_1=\left(Z_{X_1,1}=(L,P),Z_{X_2,1}=(L,U),Z_{X_3,1}=(H,P)\right),$$

$$\mathbf{Z}_2=\left(Z_{X_1,2}=(M,U),Z_{X_2,2}=(H,R),Z_{X_3,2}=(L,U)\right).$$

Let's find the Z-value of Y.

According to Stage 1, satisfaction of the ordering conditions is verified by using (6):

$$D\left(Z_{X_1,2},Z^*\right)=1.17\le D\left(Z'_{X_1},Z^*\right)= =1.6\le D\left(Z_{X_1,1},Z^*\right)=1.99,$$

$$D\left(Z_{X_2,2},Z^*\right)=0.69\le D\left(Z'_{X_2},Z^*\right)= =1.6\le D\left(Z_{X_2,1},Z^*\right)=1.87,$$

$$D\left(Z_{X_3,1},Z^*\right)=0.59\le D\left(Z'_{X_3},Z^*\right)= =0.9\le D\left(Z_{X_3,2},Z^*\right)=1.87.$$

Thus, the ordering conditions are satisfied.

At Stage 2, distance values $D_v(\mathbf{Z}',\mathbf{Z}_1)$ and $D_v(\mathbf{Z}',\mathbf{Z}_2)$ are computed by (7):

$$D_v(\mathbf{Z}',\mathbf{Z}_1)= =\sqrt{D\left(Z_1',Z'_{X_1,1}\right)^2+D\left(Z_2',Z'_{X_2,1}\right)^2+D\left(Z_3',Z'_{X_3,1}\right)^2}= =\sqrt{0.43^2+1.24^2+0.43^2}=1.38,$$

$$D_v(\mathbf{Z}',\mathbf{Z}_2)= =\sqrt{D\left(Z_1',Z'_{X_1,2}\right)^2+D\left(Z_2',Z'_{X_2,2}\right)^2+D\left(Z_3',Z'_{X_3,2}\right)^2}= =\sqrt{0.43^2+0.45^2+1.13^2}=1.29.$$

At Stage 3, interpolation weights are found by (8):

$$w_1=1-\frac{D_v(\mathbf{Z}',\mathbf{Z}_1)}{D_v(\mathbf{Z}',\mathbf{Z}_1)+D_v(\mathbf{Z}',\mathbf{Z}_2)}=1-\frac{1.38}{1.38+1.29}=0.48,$$

$$w_2=1-\frac{D_v(\mathbf{Z}',\mathbf{Z}_2)}{D_v(\mathbf{Z}',\mathbf{Z}_1)+D_v(\mathbf{Z}',\mathbf{Z}_2)}=1-\frac{1.29}{1.38+1.29}=0.52.$$

At Stage 4, the resulting output is found by using (9):

$$Z_y=w_1Z_{y,1}+w_2Z_{y,2}=0.48*(L,U)+0.52*(VH,P)= =0.48*\left((1\quad 2\quad 3)(0.5\quad 0.85\quad 1)\right)+ +0.52*\left((4\quad 5\quad 5)(0.25\quad 0.5\quad 0.85)\right)= =(0.48\quad 0.96\quad 1.44)(0.5\quad 0.85\quad 1)+ +(2.08\quad 2.6\quad 2.6)(0.25\quad 0.5\quad 0.85)= =(2.56\quad 3.56\quad 4.04)(0.43\quad 0.7\quad 0.79).$$

This Z-number can be labeled as (M, B_U).

Example 2. Let Z-number-based rules be given as follows:

If X_1 is $Z_{X_1,1}=(M,U)$ and X_2 is $Z_{X_2,1}=(H,R)$ and X_3 is $Z_{X_3,1}=(L,U)$ then Y_1 is $Z_{Y,1}=(VH,P)$.

If X_1 is $Z_{X_1,2}=(H,U)$ and X_2 is $Z_{X_2,2}=(L,U)$ and $Z_{X_3,2}=X_3$ is (M,U) then Y_2 is $Z_{Y,2}=(M,U)$.

The codebooks of the linguistic terms are given in Tables 1, 2.

The current observation:

$$X_1 \text{ is } Z'_{X_1}=\left((2.5\quad 3.5\quad 4.5)(0.5\quad 0.85\quad 1)\right),$$

$$X_2 \text{ is } Z'_{X_2}=\left((2\quad 3\quad 4)(0.2\quad 0.4\quad 0.6)\right),$$

$$X_3 \text{ is } Z'_{X_3}=\left((1.5\quad 2.5\quad 3.5)(0.5\quad 0.85\quad 1)\right).$$

Then find the Z-value of Y.

Analogously to what is done in Example 1, satisfaction of ordering conditions in (6) is verified (Stage 1), distance values between current observation vector and vectors of rule antecedents are computed (Stage 2) and interpolation weights are found (Stage 3):

$w_1=0.52$, $w_2=0.48$.

At Stage 4, the resulting output is computed by using (9):

$$Z_y=w_1Z_{y,1}+w_2Z_{y,2}=0.52*(VH,R)+0.48*(M,U)= =(3.04\quad 4.04\quad 4.52)(0.44\quad 0.7\quad 0.79).$$

This Z-number can be labeled as (H, U).

5. 3. Presentation of the decision of practical problem on reasoning with Z-rules on the example of job satisfaction evaluation problem

A job satisfaction evaluation is an essential problem. It is characterized by imprecise and partially reliable information related to dependence between overall job satisfaction level and its facets. The issue is that information reflects psychological, perceptional, mental and other aspects. As a result, such information is usually described linguistically. Let's consider the following Z-number valued If-Then rules describing influence of 20 factors on job satisfaction [10] (Table 3). As the source for construction of a Z-valued If-Then rules base, the questionnaires completed by experts are used.

The codebooks of the terms used in the rules are given in Tables 4, 5.

Assume the following observation of facets is given:

$$X_1 \text{ is } Z'_{X_1}=\left((3.5\quad 4.5\quad 5)(0.5\quad 0.75\quad 1)\right),$$

$$X_2 \text{ is } Z'_{X_2}=(3\quad 4\quad 5)(0.5\quad 0.75\quad 1),$$

$$X_3 \text{ is } Z'_{X_3}=(3.5\quad 4.5\quad 5)(0.5\quad 0.75\quad 1),$$

$$X_4 \text{ is } Z'_{X_4}\left((3.5\quad 4.5\quad 5)(0.5\quad 0.75\quad 1)\right),$$

$$X_5 \text{ is } Z'_{X_5}=\left((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1)\right),$$

$$X_6 \text{ is } Z'_{X_6}=\left((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1)\right),$$

$$X_7 \text{ is } Z'_{X_7}\left((2\quad 3\quad 4)(0.5\quad 0.75\quad 1)\right),$$

$$X_8 \text{ is } Z'_{X_8}=\left((2\quad 3\quad 4)(0.5\quad 0.75\quad 1)\right),$$

$$X_9 \text{ is } Z'_{X_9}=\left((3.5\quad 4.5\quad 5)(0.5\quad 0.75\quad 1)\right),$$

$$X_{10} \text{ is } Z'_{X_{10}}=\left((3\quad 4\quad 5)(0.5\quad 0.75\quad 1)\right),$$

$$X_{11} \text{ is } Z'_{X_{11}}=\left((3.5\quad 4.5\quad 5)(0.5\quad 0.75\quad 1)\right),$$

$$X_{12} \text{ is } Z'_{X_{12}}=\left((1.5\quad 2.5\quad 3.5)(0.35\quad 0.65\quad 0.95)\right),$$

X_{13} is $Z'_{X_{13}} = ((2.5 \quad 3.5 \quad 4.5)(0.35 \quad 0.65 \quad 0.95)),$

X_{14} is $Z'_{X_{14}} = ((2.5 \quad 3.5 \quad 4.5)((0.5 \quad 0.75 \quad 1)),$

X_{15} is $Z'_{X_{15}} = ((2.5 \quad 3.5 \quad 4.5)(0.5 \quad 0.75 \quad 1)),$

X_{16} is $Z'_{X_{16}} = ((2.5 \quad 3.5 \quad 4.5)(0.5 \quad 0.75 \quad 1)),$

X_{17} is $Z'_{X_{17}} = ((2.5 \quad 3.5 \quad 4.5)(0.35 \quad 0.65 \quad 0.95)),$

X_{18} is $Z'_{X_{18}} = ((2 \quad 3 \quad 4)(0.5 \quad 0.75 \quad 1)),$

X_{19} is $Z'_{X_{19}} = ((2.5 \quad 3.5 \quad 4.5)(0.5 \quad 0.75 \quad 1)),$

X_{20} is $Z'_{X_{20}} = ((2.5 \quad 3.5 \quad 4.5)(0.5 \quad 0.75 \quad 1)).$

Let's compute the corresponding value of job satisfaction by using the method described in the previous section.

At Stage 1, conditions of (6) are checked. The results show that Rules 1 and 2 can be used for interpolation.

The values of distance computed at Stage 2 are as follows:

$$D_v(\mathbf{Z}',\mathbf{Z}_1) = = \sqrt{D\left(Z'_1, Z'_{X_1,1}\right)^2 + D\left(Z'_2, Z'_{X_2,1}\right)^2 + \ldots + D\left(Z'_{20}, Z'_{X_{20},1}\right)^2} = = \sqrt{0.23^2 + 0.23^2 + 0.23^2 + \ldots + 0.43^2} = 1.93,$$

$$D_v(\mathbf{Z}',\mathbf{Z}_2) = = \sqrt{D\left(Z'_1, Z'_{X_1,2}\right)^2 + D\left(Z'_2, Z'_{X_2,2}\right)^2 + \ldots + D\left(Z'_{20}, Z'_{X_{20},2}\right)^2} = = \sqrt{0.23^2 + 0.23^2 + 0.23^2 + \ldots + 1.15^2} = 1.96.$$

The corresponding interpolation weights computed at Stage 3 are $w_1 \approx 0.5$, $w_2 \approx 0.5$.

Table 3

The encoded linguistic terms for Job Satisfaction Inscriptions in the table are horizontal only (unfortunately, in horizontal presentation)

	JOB FACTORS/FACETS	1	2	3	4	5	6
1	Activity	(*VS*, *H*) $B_H H$)	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)
2	Independence	(*S*, *H*)	(*S*, *H*)	(*S*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*QS*, *H*)
3	Variety	(*VS*, *H*)	(*S*, *H*)	(*S*, *H*)	(*LS*, *M*)	(*QS*, *H*)	(*LS*, *M*)
4	Social status	(*VS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)
5	Supervision-Human relations	(*VS*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*US*, *M*)	(*S*, *H*)	(*S*, *H*)
6	Supervision-technical	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*US*, *M*)	(*S*, *H*)	(*S*, *H*)
7	Moral values	(*QS*), *H*)	(*QS*, *H*)	(*S*, *H*)	(*US*, *M*)	(*QS*, *H*)	(*LS*, *M*)
8	Security	(*QS*, *H*)	(*QS*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*LS*, *M*)
9	Social service	(*VS*, *H*)	(*S*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*QS*, *H*)	(*S*, *H*)
10	Authority	(*S*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*US*, *M*)	(*QS*, *H*)	(*S*, *H*)
11	Ability	(*VS*, *H*)	(*S*, *H*)	(*S*, *H*)	(*LS*, *M*)	(*QS*, *H*)	(*S*, *H*)
12	Company policies and practices	(*QS*, *H*)	(*LS*, *M*)	(*QS*, *H*)	(*LS*, *M*)	(*LS*, *M*)	(*S*, *H*)
13	Compensation	(*S*, *H*)	(*LS*, *M*)	(*QS*, *H*)	(*QS*, *H*)	(*LS*, *M*)	(*LS*, *M*)
14	Advancement	(*VS*, *H*)	(*QS*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*QS*, *H*)
15	Responsibility	(*VS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*LS*, *M*)	(*S*, *H*)	(*S*, *H*)
16	Creativity	(*VS*, *H*)	(*S*, *H*)	(*S*, *H*)	(*LS*, *M*)	(*QS*, *H*)	(*S*, *H*)
17	Working conditions	(*S*, *H*)	(*LS*, *M*)	(*QS*, *H*)	(*QS*, *H*)	(*LS*, *M*)	(*QS*, *H*)
18	Co-workers	(*QS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*S*, *H*)	(*S*, *H*)
19	Recognition	(*VS*, *H*)	(*QS*, B_h)	(*S*, *H*)	(*VS*, *H*)	(*QS*, *H*)	(*S*, *H*)
20	Achievement	(*VS*, *H*)	(*QS*, *H*)	(*QS*, *H*)	(*S*, *H*)	(*QS*, *H*)	(*S*, *H*)
21	Overall Job Satisfaction	(*S*, *H*)	(*QS*, *H*)	(*QS*, *H*)	(*LS*, *M*	(*QS*, *H*)	(*S*, *H*)

Table 4

The encoded linguistic terms for Job Satisfaction

Level of Satisfaction	Linguistic value
Unsatisfied (U)	{0/1, 1/1, 0/2}
Less Satisfied (LS)	{0/1, 1/2, 0/3}
Quite Satisfied (QS)	{0/2, 1/3, 0/4}
Satisfied (S)	{0/3, 1/4, 0/5}
Very Satisfied (VS)	{0/4, 1/4, 0/5}

Table 5

The encoded linguistic terms for B (Reliability)

Level of Satisfaction	Linguistic value
Low (L)	{0/0.05,1/0.25, 0/0.05}
Medium (M)	{0/0.25, 1/0.5, 0/0.75}
High (H)	{0/0.5, 1/0.75, 0/1}

At Stage 4, the overall level of job satisfaction is computed by using (9):

$$Z_y = w_1 Z_{y,1} + w_2 Z_{y,2} = 0.5*(S,H) + 0.5*(QS,H) =$$
$$= 0.5*((3\quad 4\quad 5)(0.5\quad 0.75\quad 1)) +$$
$$+0.5*((2\quad 3\quad 4)(0.5\quad 0.75\quad 1)) =$$
$$= (1.5\quad 2\quad 2.5)(0.5\quad 0.75\quad 1) + (1\quad 1.5\quad 2)(0.5\quad 0.75\quad 1) =$$
$$= (2.5\quad 3.5\quad 4.5)(0.36\quad 0.62\quad 0.96).$$

According to the codebooks, the overall level can be labeled as (*QS*, *H*).

Consider another case of information concerning the facets:

X_1 is $Z'_{X_1} = ((3\quad 4\quad 5)(0.5\quad 0.75\quad 1))$,
X_2 is $Z'_{X_2} = ((3\quad 4\quad 5)(0.5\quad 0.75\quad 1))$,
X_3 is $Z'_{X_3} = ((2.5\quad 3.5\quad 4.5)(0.35\quad 0.65\quad 0.95))$,
X_4 is $Z'_{X_4} = ((3\quad 4\quad 5)(0.5\quad 0.75\quad 1))$,
X_5 is $Z'_{X_5} = ((1.5\quad 2.5\quad 3.5)(0.35\quad 0.65\quad 0.95))$,
X_6 is $Z'_{X_6} = ((1.5\quad 2.5\quad 3.5)(0.35\quad 0.65\quad 0.95))$,
X_7 is $Z'_{X_7} = ((1.5\quad 2.5\quad 3.5)(0.35\quad 0.65\quad 0.95))$,
X_8 is $Z'_{X_8} = ((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1))$,
X_9 is $Z'_{X_9} = ((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1))$,
X_{10} is $Z'_{X_{10}} = ((2.5\quad 3.5\quad 4.5)(0.35\quad 0.65\quad 0.95))$,
X_{11} is $Z'_{X_{11}} = ((2.5\quad 3.5\quad 4.5)(0.35\quad 0.65\quad 0.95))$,
X_{12} is $Z'_{X_{12}} = ((1\quad 2\quad 3)(0.25\quad 0.5\quad 0.75))$,
X_{13} is $Z'_{X_{13}} = ((1.5\quad 2.5\quad 3.5)(0.35\quad 0.65\quad 0.95))$,
X_{14} is $Z'_{X_{14}} = ((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1))$,
X_{15} is $Z'_{X_{15}} = ((2.5\quad 3.5\quad 4.5)(0.35\quad 0.65\quad 0.95))$,
X_{16} is $Z'_{X_{16}} = ((2.5\quad 3.5\quad 4.5)(0.35\quad 0.65\quad 0.95))$,
X_{17} is $Z'_{X_{17}} = ((1.5\quad 2.5\quad 3.5)(0.35\quad 0.65\quad 0.95))$,
X_{18} is $Z'_{X_{18}} = ((3\quad 4\quad 5)(0.5\quad 0.75\quad 1))$,
X_{19} is $Z'_{X_{19}} = ((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1))$,
X_{20} is $Z'_{X_{20}} = ((2.5\quad 3.5\quad 4.5)(0.5\quad 0.75\quad 1))$.

According to condition of (6), Rules 2 and 4 can be used for interpolation. The values of distance:

$$D_v(\mathbf{Z}', \mathbf{Z}_1) = 1.58,\ D_v(\mathbf{Z}', \mathbf{Z}_2) = 3.76.$$

The interpolation weights:

$w_1 = 0.7$, $w_2 = 0.3$.

The overall level of job satisfaction:

$$Z_y = w_1 Z_{y,1} + w_2 Z_{y,2} = 0.7*(QS,H) + 0.3*(LS,M) =$$
$$= 0.7*((2\quad 3\quad 4)(0.5\quad 0.75\quad 1)) +$$
$$+0.3*((1\quad 2\quad 3)(0.25\quad 0.5\quad 0.75)) =$$
$$= (1.4\quad 2.1\quad 2.8)(0.5\quad 0.75\quad 1) +$$
$$+(0.3\quad 0.6\quad 0.9)(0.25\quad 0.5\quad 0.75) =$$
$$= (1.7\quad 2.7\quad 3.7)(0.19\quad 0.41\quad 0.73).$$

According to the codebooks, the overall level can be labeled as (*QS*, *M*). By using ranking of Z-numbers, one can see that in this case, job satisfaction level is lower than that in the first one, (*QS*, *H*).

6. Discussion of the results of the study on developing method of general interpolation for Z-number-valued if-then rules

Under conditions of high uncertainty, information is often characterized as vague or with partial certainty. One way to reduce this uncertainty is to use Z-numbers to express the degree of confidence in the value of a fuzzy variable, which is a combination of fuzziness and probability to describe the degree of confidence in the data. In the work, the main results are the obtained distances between the elements of a given rule and the elements of the current rule (1), (2). Practical applications of these formulas are shown in Tables 1, 2 (here are parts A and B of Z-numbers describing linguistic variables).

Compared to existing methods using fuzzy logic and fuzzy sets [15], Z-numbers more effectively describe imperfect information, which fuzzy numbers are paired in such a way that information is presented with a high degree of reliability [13, 14]. This article discusses the use of Z-numbers in decision making. Without detracting from the achievements of the authors in [16] (this work is also devoted to the representation of a measure of job satisfaction by means of fuzzy sets), let's believe that the method proposed in this article gives the better results.

However, this study is limited to using only one type of distance between probability distributions. This limitation may reduce the scope and generalizability of the study, as some types of distances may be more or less suitable for specific applications. If the data is complex or contains different types of distributions, distance alone may not be enough to adequately assess the similarities or differences between them.

The main disadvantage of the presented study is that the approach is based on linear interpolation. Linear interpolation assumes that there is a linear relationship between two known points. This limits its ability to capture complex trends or non-linear changes in data. In some cases, linear interpolation can lead to oscillations or «jumps» in the interpolated function, especially if the distance between the known points is small [17].

This research focuses on the application of classical Z-numbers in decision-making. However, future research on Z-numbers may include extensions of MCDM (multi-criteria decision-making) models based on these generalized Z-numbers: intuitionistic Z-numbers, neutrosophic Z-numbers, Pythagorean Z-numbers, and Fermatean Z-numbers. These generalized Z-numbers are considered the best representation of Z-numbers and have advantages that deserve further study [18]. This will effectively identify the strengths of these generalized Z-numbers and their application in various decision-making contexts. Such an extended study can greatly enrich the understanding of Z-numbers and increase their practical value in decision-making in various fields.

7. Conclusions

1. A way to measure the 'closeness' of new Z-valued input with rules antecedents is formalized. The basics of a general interpolation method for Z-valued If-Then rules are proposed. Ordering conditions for interpolation are based on distance

between Z-numbers. Given a current observation in form of a vector of Z-numbers, a resulting output is computed as weighted sum of Z-number-valued consequents. The components of the current observation vector are «between» the components of the antecedent vectors of the two rules, and the corresponding result was calculated as a linear combination of consequents. The coefficients of this combination reflect the influence of each subsequent rule on the resulting result.

2. A technique for Z-valued output as the weighted mean of rule consequents (based on the measured 'closeness') is developed. The proposed method of general interpolation for Z-number-valued rules includes certain stages. At the first stage, it is necessary to check whether the ordering conditions are met for the current observation and rule-antecedent vectors. At the second stage, with a positive outcome, the values of the distance between the current observation vector and the vectors of the antecedents of the rules are calculated. At the next stage, it is necessary to calculate the interpolation coefficients (weights). At the final stage, the resulting result is calculated as a weighted sum of the following rules.

3. As an application of the proposed method, job satisfaction evaluation problem is considered. The problem is characterized by imprecise and partially reliable information related to influential factors of job satisfaction. The obtained results show efficiency of the method for Z-valued If-Then rules with high number of antecedents.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

Data will be made available on reasonable request.

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