MATHEMATICAL MODEL OF A RAILROAD GRAIN CARGO RIDESHARING SERVICE IN THE FORM OF COALITIONS IN CONGESTION GAMES

The object of this study is the processes of transportation of grain cargoes based on the principles of ridesharing in railroad systems without observing the traffic schedule for freight trains. In order to study the influence of the model of railroad transportation of grain based on the principles of joint use on the operation of the railroad system, it is proposed to formalize this process under the conditions of the peak load period. It is proposed to formalize the transportation of grain using the ride-sharing service in the form of coalitions in congestion games. It is proposed to turn the game setup into a nonlinear optimization problem.

As part of the research, mathematical modeling of the ridesharing service of railroad transportation of grain cargoes was carried out. Adequacy of the mathematical model was proven. It was established that compliance with the traffic schedule leads to an increase in non-productive downtime of railroad cars after loading, which reduces incentives for the formation of coalitions by shippers. However, according to the results of the simulation, under the conditions of traffic according to the schedule, taking into account the coordination of the shippers and carrier, the transportation indicators are significantly improved. This encourages shippers to form coalitions. It was found that the average duration of shipment transportation decreased by 14.9 % from the indicator according to the scenario of the current transportation model – without observing the schedule.

A feature of the results within the framework of the study is that the proposed mathematical model makes it possible to adequately simulate the ride-sharing service of grain transportation in the railroad system.

The field of practical application of the results is the railroad industry. The conditions for the practical application of the research results are the importance of implementing digital platforms of aggregators for the coordination of shippers and carriers.

Current research will contribute to devising the improvements for grain logistics in railroad transport

Keywords: rail freight, grain transportation, ridesharing, coalition games, congestion games

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1. Introduction

One of the directions for providing the system of transportation of grain cargo with significant flexibility of operations and the speed of movement of shipments is the hybridization of the existing transportation model «hub-and-spoke» with «point-to-point». This approach can be based on transportation technologies that make it possible at the first stages of the transportation process to combine groups of cars to form block trains destined for the unloading station according to the ridesharing principle [1–3]. This will speed up movement and reduce the cost of transporting grain batches in small groups of cars [4]. This is achieved due to the exclusion of irrational transformations at large sorting stations. Such transformations occur on the route following the option of following these cars as car or group shipments – Single Wagonload (SWL) shipments [5]. One of the well-known transportation technologies is the formation of so-called staged-route trains (SRT) [6]. However, the construction of a service product on railroads for the transportation of grain cargo by staged-route trains with the use of a ridesharing service is new and unexplored. This model of transportation corresponds to the principles of the sharing economy [7]. The analysis of transportation services based on the principles of ridesharing proved its effectiveness in the fields of motor transport and aviation [8, 9]. There are no results of the influence of change in the model of transportation of grain cargoes on the performance of a railroad system. Therefore, research into improving grain logistics based on the formalization of the ridesharing service of the association of shippers in the organization of rail transportation of grain and the study of the impact of such a service on the operation of the railroad system are relevant.

2. Literature review and problem statement

Many studies have tackled the issue of building mathematical models of the ridesharing service. Quite a number of works [10–12] consider the modeling of a joint trip of passengers on road transport. Each of the studies takes into account the specifics of the use of ridesharing for various

transport services – public transport (buses), taxi services, private cars. For example, some studies take into account the dynamic nature of the problem statement. In [10], a simulation model of shared transportation is presented, which takes into account the dynamic supply of drivers and the demand of passengers, as well as the complex interaction between drivers and passengers. Papers [11, 12] report approaches to ridesharing service modeling, but the drawback of such studies is the impossibility of building a mathematical model for the railroad system where the behavioral mechanisms of shippers are completely different. In [13], a dynamic tourist network approach was used to model and predict the potential use of shared transport over time. The simulation results proved the possibility of predicting the potential use of shared transport, demonstrating its high instability over time. In all the works listed above [10–13], the research results confirm the effectiveness of the formalization of the ridesharing service for its study and further development in real transport systems.

The analysis of approaches to ridesharing service modeling in transport networks proved the importance of combining different mathematical methods and approaches when building a mathematical model. It is common to formalize the ridesharing process based on an optimization model in the form of posing the problem as the Vehicle Routing Problem (VRP) or its special case the Dial A Ride Problem (DARP) [14–16]. However, the authors did not conduct research on the modeling of the ridesharing service under the conditions of overloading of the transport system and the mutual influence of traffic flows on the duration of the journey. In [14], agent-oriented modeling (ABS) was used, the main drawback of which is an increase in calculation time under the conditions of an increase in the dimensionality of the problem. Papers [15, 16] did not address the issue of formalization of technical and technological capabilities in the transportation of grain cargoes in the railroad system. In [17], a modification of the DARP problem statement was used to develop the «dynamic dispatching» function, which solves the issue of distribution of vehicles among passengers in an operational mode. A significant difference of «dynamic dispatching» among other methods is that passengers can use both private and public transport for full or partial movement along the route. According to the results, the new concept can significantly improve the efficiency of transportation. A rather similar statement, called the pickup and delivery problem with transfers (PDPT), was used in [18]. In the work, a mathematical model has been developed to provide orders for the delivery of cargo parcels taking into account the shared use of vehicles. This model takes into account restrictions on the number of available vehicles; time constraints taking into account the delivery of various cargoes to several destinations. The models built cannot be used for the formalization of the ridesharing service on railroads for the transportation of grain cargo, but taking into account these limitations and setting the problem is quite similar to the conditions of transportation on railroad transport. The drawback of research [17, 18] is the lack of an opportunity to take into account the specifics of the organization of rail transport. An analogy regarding the technology of railroad transport can be found in work [19], where two approaches were used within the scope of the problem of cargo removal and delivery. The first is the pickup and delivery problem with time window and transfer (PDPTWT). The second is the problem of export and delivery of goods with temporary windows and scheduled routes (PDPTW-SL). Mathematical

models were built based on the use of a software product called CPLEX, and the impact of various transportation models on transportation efficiency was studied. According to calculations, the statement according to PDPTW-SL leads to a reduction of total operating costs by 20 % due to the use of public transport for the transportation of small cargo. However, the authors note that the proposed model still needs research and does not take into account the influence of additional external factors on it (for example, traffic jams). Like previous studies, the statement of PDPTW-SL does not take into account the specifics of railroad freight transportation, but accounting for time windows and the presence of constant schedules on traffic routes can be used to simulate the movement of freight trains according to the schedule. Similar shortcomings are inherent in study [20].

Taking into account the opposite interests of the two parties within the limits of supply and demand, which are combined with the help of a ridesharing service, an approach that formalizes ridesharing in transport systems using game theory is quite interesting and promising. For example, in [21], a cooperative game-theoretic approach was applied to the problem of car sharing. The problem of combining passengers through a social network in a short time for a one-time trip is presented and considered by the authors as a problem of forming coalitions in games. The study is focused on the problem of optimizing the formation of passenger coalitions that minimize travel costs in the overall system and consideration of the aspect of payment distribution. The results proved that the optimization of coalitions reduces travel costs up to 36.22 % compared to the scenario without ride sharing.

A game-theoretic approach that models the distribution of traffic in a transport system taking into account the interdependence of flows as a problem of cooperative congestion games [22, 23]. The dependence of the duration of movement of grain trains on traffic jams on the railroad network, which often occur during peak load periods, is a common problem in railroad systems. Setting the problem of simulating ridesharing on railroads by analogy with a cooperative congestion game will allow us to take into account this feature and investigate the behavior in the system. Paper [24] investigates the issue of the influence of collusion on the quality of the decision made in congestion games. The results showed that in some non-atomic games, increasing cooperation among players always improves the quality of the solution. The study of coalition games in the field of communications and network technologies is reported in [25]. The mathematical models described above [22–25] were not built for simulating ridership in railroad systems, however, by their analogy, it is possible to construct an effective mathematical model.

The review of the literature [10–25] allows us to conclude that there are no mathematical models that make it possible to simulate the ridesharing service of grain cargo transportation in the railroad system. However, the identified advantages and results of research in other fields allow us to conclude that the construction of such a mathematical model should be based on a combination of problem statement for the following problems: DARP, PDPTWT, RIDESHARING, CONGESTION GAMES, COALITIONAL GAMES.

The combination of the game-theoretic approach with classical optimization transport problems will make it possible to build a new mathematical model based on the hybridization of mathematical methods. Such a mathematical model takes into account the complex interaction of shippers in the railroad system, taking into account their

behavior and interests, which often do not coincide with the interests of the effective functioning of the railroad system under conditions of congestion. Therefore, research into the formalization of the ride-sharing service for the transportation of grain cargoes based on the hybridization of modeling methods is promising.

3. The aim and objectives of the study

The aim of this work is to construct a mathematical model of the ride-sharing service of grain transportation, which will allow simulating various scenarios of the operation of the railroad network and investigating the behavioral mechanisms of the functioning of the transportation system.

To achieve the goal, the following research tasks were solved:

– formalizing the process of transportation of grain cargoes based on the principles of ridesharing in railroad systems without observing the timetable for freight trains;

– conducting simulations to study the impact of the ride-sharing service of grain transportation under conditions of congestion and compliance with traffic schedules on the current operation of the railroad system.

4. The study materials and methods

The object of our study is the processes of transportation of grain cargoes based on the principles of ridesharing in railroad systems without observing the traffic schedule for freight trains.

The main hypothesis of the study assumes that improving the speed of grain transportation and reducing costs in the railroad system is possible due to the introduction of the ridesharing service of the association of shippers in the organization of railroad transportation. This is achieved due to the formalization of the ride-sharing service and the study of the impact of such a service on the operation of the railroad system. It is possible to solve this issue by developing a mathematical model for the theoretical substantiation of the effectiveness of the use of ridesharing under the conditions of compliance with the schedule of freight trains and under the current conditions of traffic – without compliance.

The railroad system was studied without observing the schedule of freight trains. Such systems include the railroad system of Ukraine where significant volumes of grain cargo are transported. Under such conditions, it is proposed to formalize the process of transportation of grain cargoes on the example of the specifics of the functioning of railroads in Ukraine.

Considering the competitiveness of the environment and the opposite interests of many shippers and the operator of the railroad infrastructure (JSC Ukrzaliznytsia) as the body that dispatches the system, it is proposed to conduct a study based on the basics of game theory. It is proposed to formalize the formation of a staged-route trains and its movement in the form of coalitions in congestion games [26].

Coalition congestion game $\Gamma = <N,\{S_i\}_{i\in N}, \{\varphi(N_v(s),v)\}_{v\in V}>$, where $N=\{1, 2, ..., n\}$ is a finite set of players – shippers, S_i is a set of strategies for player i, $i\in N$ and the set $V=\{v_1,v_2,...,v_m\}, v_j:2^N\to\Re, v_j(\varnothing)=0\forall j\in\{1, 2, ..., m\}$ is a set of cooperative games – combinations of requests from different shippers to form a tiered route or send according to the current transportation model, and $\varphi(N_v(s),v)$ is the price of

the cooperative subgame $\varphi(N_v(s),v)$ of the game (N_v, v), $v\in V$, $s\in S$ [26, 27].

$S = S_1 \times S_2 \times ... \times S_n, u_i : S \to \Re$ is the payoff function of player i, $i\in N$, where $\forall i\in N$, $\forall s_i\in S_i$: $s_i,..., \subseteq V$.

In normal form, a cooperative congestion game with cost φ can be written as:

$$\Gamma = \left\langle N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\right\rangle_{|V,\varphi},$$

in which the payoff functions take the form:

$$u_i(s) = \sum_{v\in s_i}\varphi_i(N_v(s),v), i\in N,$$

where $N_v(s)=\{i|v\in s_i, i\in N\}$ is the set of players who chose game v from the strategy profile s, $s\in S$.

Each player, the «shipper», has two possible strategies:

– $s=1$ – do not join the coalition but send own freight car shipment according to the usual model of transportation in district trains with re-formations at sorting stations (for simplification, it is proposed to denote this strategy as SWL in the work);

– $s=2$ – enter into a coalition with other shippers to combine requests and send their own freight car shipment using a staged-route train. This will speed up transportation (in case any body reads this, it is proposed to label this strategy as SRT in the paper).

Within the formalization of the game, it is important to take into account the peculiarities of operation of the railroad system. It is proposed to consider this game on the railroad network of Ukraine, which is divided into load regions with core stations where there are requests from shippers to load different groups of cars at different stations within the desired time window. Shipment routes lead to the largest seaports and pass through the «bottleneck» of the railroad network – an overloaded railroad section. The «bottleneck» connecting the railroad system of Ukraine with the largest seaports of Ukraine is the railroad section of the regional branch «Odesa Railroad» of JSC «Ukrzaliznytsia» [28]. All routes are known in advance and defined by a normative document – the Train Formation Plan (TFP), which is adopted once a year [6].

The destinations of shipments are port stations whose access tracks are connected to the largest seaports where grain is transshipped onto ships for further export. This statement of the problem corresponds to the actual conditions of transportation of grain cargoes in Ukraine. However, even if we consider a different situation, for example, after the large-scale Russian invasion of Ukraine, an overloaded railroad section appeared in the direction of the western crossings, the research methodology will be unchanged. This methodology can be applied to freight flows following in the direction of border stations where the width of the track changes (1520 mm – 1435 mm) and reloading is carried out in other cars or there is a change of bogies of freight cars.

Restrictions on the formation of coalitions are the technical and technological features of the formation of graded route trains on the railroad network of Ukraine:

– freight and group shipments of different consignors must coincide in time with the desired time window of loading;

– loading stations must be next to each other within the same network region, and the destinations must match;

– loading can be carried out around the clock at stations open for cargo operations. These include both stations with access tracks on which it is possible to quickly load through discharge pipes from the storage tanks of the elevator into

the car, and stations with minor development of railroad infrastructure and loading mechanization;

– the number of cars in a freight train should not exceed the length and mass of trains on the route specified by the regulations;

– the processing capacity of the unloading fronts at the destination station must correspond to the volume of grain transportation arriving in trains of graded routes.

This game is categorized as an atomic game [27]. A co-operative congestion game can be reduced to a potential game [29–31]. A potential game implies the existence of a potential function – a function whose critical property for the game is that if one player changes the strategy, it affects the other players. In addition, it changes the potential function, which evaluates the interdependence of the players in the system taking into account the use of the system resource – the use of the throughput capacity of the congested railroad section of a railroad network [31]. In the game simulating the ride-sharing service of grain transportation, the potential function can be understood as the cost function. Such a function evaluates the duration of the shipper's shipment according to the selected transportation strategy, taking into account the interdependence of the intensity of train traffic on the congested railroad section. Such interdependence was found in [28] and proposed to be used in this study.

Under such conditions, the statement of the game can be considered as a routing game in the form of a cost function and distribution of flows on the network. It is proposed to turn this game into a nonlinear optimization problem. To solve the optimization mathematical model, a genetic algorithm [32] was used, within the fitness function of which it is proposed to use an artificial neural network as a mathematical model of nonlinear regression. Such an artificial neural network allows simulating a potential function that predicts a change in the duration of train traffic when choosing different player strategies [28]. The mathematical model was solved in the MATLAB software environment (USA).

task of organizing tiered routes precisely by departure from any station of the network to four destination stations. These destination stations correspond to the main port stations of the busiest railroad section of the regional branch of Odesa Railroad, JSC Ukrzaliznytsia. A practical analysis of the movement of car flows according to TFP in the direction of the port stations of this railroad section revealed that the network can be divided into loading railroad sections. Such railroad sections have a core station, main routes, and a limiting element of the network – a load-intensive railroad section within which all train flows are united before the destination stations [28]. Thus, each loading site has core stations lying on trunk routes and connecting the loading site core station with the first core station adjacent to the unloading site. It is proposed to separate the unloading railroad section into a separate subgraph and consider it in the statement of the problem as a sub-network of interdependent train routes. In order to simplify the statement of the problem, it is proposed to allocate core stations $u=ij$, $u \in A$ on the network, which for a specified load railroad section correspond to the station that is the input for following the train flow along the destination route to the station $t \in A$.
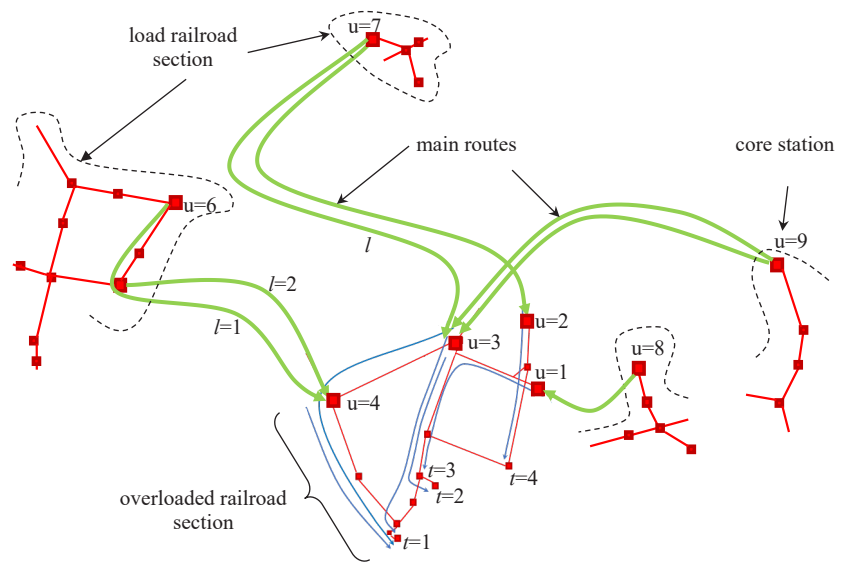

Fig. 1. Simplified physical graph of the railroad network $G(A, E)$

5. Results of the construction of a mathematical model of the ride-sharing service of railroad transportation of grain cargoes

5. 1. Formalization of the ride-sharing service of rail transportation of grain in the form of coalitions in congestion games

To simulate various scenarios of behavior in the railroad system, in particular, the formation of coalitions for the formation of graded route trains, the mathematical model described below was used in this study.

Consider a simplified physical graph of the railroad network $G(A, E)$, where the set of vertices A represent the set of stations of a railroad network, and the set of edges E, where e are the railroad lines connecting the stations. Station index is $ij \in A$ (Fig. 1). Taking into account that almost all train traffic on the railroad network of Ukraine is directed towards the seaports of Great Odesa, it is proposed to consider the

Each trunk route has an index l, $l \in L$, and runs through the main railroad routes on the network, the capacity of which is quite intensively utilized. Each route l is connected to a subroute μ, where $\mu = \{1,2,3,..6\}$ on the graph of the port adjacency railroad section. Given that the proposed technology for organizing staged routes based on ridesharing principles corresponds to the transportation market at the operational level of planning, the so-called Ad hoc market. When it is possible to create a route that does not have a regular departure within the limits of the current PTF, for each trunk route l there are predetermined free time periods within the train schedule from the core station u – unused capacity $[t_p^{ul}, \overline{t_p^{ul}}]$, where p is the ordinal index of the time window (slot) in within the limits of the entire planning period Tmax, min. Each slot p is intended for movement of only one stage route.

According to the statement of the problem, it is proposed to assign a request index r, where $r \in R$, to each request for a freight car or group shipment at the load yards. Each request

has the following information: the index of the departure station (load) s ($s=ij$)), the index of the destination station t ($t=ij$)), the group of cars for departure q_r, the time window within which the consignor wishes to send own cargo $[a_r, b_r]$, where a_r is the beginning of the time window; b_r is the end of the time window (Fig. 2).
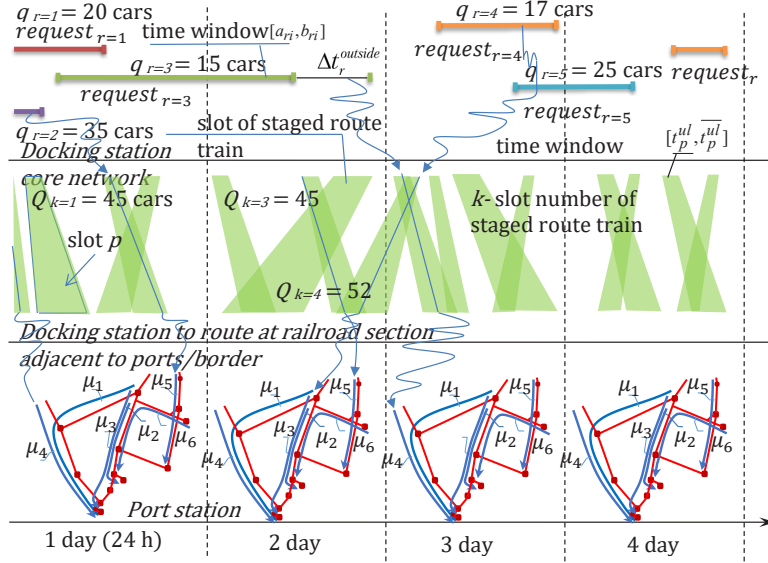

Fig. 2. Spatial-temporal diagram of the movement of cars with grain cargo according to the requests of consignors to destination stations

Under the condition of setting the problem, it is necessary to determine the set of available train locomotives $K=\{1, 2, 3, ..., k\}$ on the network. These locomotives are located at the specified stations of the load railroad sections $ij \in A$.

Let's introduce a variable X_{rp}^k if 1, then the locomotive k processes the request r on the slot p, otherwise it is 0, $X_{rp}^k = \{1,0\}$. This variable allows us to simulate two strategies of the player – the shipper $X_{rp}^k = 1$ – the formation of a coalition to form a staged route train (SRT), in the case $X_{rp}^k = 0$ – sending a freight car shipment according to the current transportation system through reformation at sorting stations (SWL).

To simulate the moment of dispatch of the request cars r within the time window of the slot p, a variable tt_p^k is introduced. The variable can take values within the time window of the slot during which the train is allowed to be dispatched:

$$tt_p^k \to \left[\underline{tt_p^{ul}}, \overline{tt_p^{ul}}\right],$$

after receiving the value tt_p^k the modeling algorithm should provide for the reverse deployment of the route according to the diagram in Fig. 3.

The duration of shipment tracking r is determined by the expression:

$$t^r = \begin{cases} \text{If } \sum_r X_{rp}^k > 1, \\ \text{then } t^{ridscher} = t_{moving} + t_{coupling} \cdot \gamma_{coupling} + \\ +t_l + t_\mu(N + N_{add}) \Big|^d, \\ \text{otherwise,} \\ t^{single} = t_{moving} + t_{resorting} \cdot \delta_{resorting} + t_l + t_\mu(N + N_{add})\Big|^d, \end{cases}$$

where $d=1, 2, 3, ..., D$ – days of the planning period $\frac{t_{moving} + t^{coupling} + t_l}{1440} \approx d$; t_{moving} – the duration of movement of the shipment from the departure station to the docking station u with route l, min.; (calculated according to the graph and accepted distances between stations) $t_{moving} = \sum_i \sum_j t_{ij}$;

$t_{coupling}$ – duration of stay at the group unification station, min (depends on the number of groups being combined into a stage train, 2 hours can be taken equally); $\Gamma_{coupling}$ – the number of stations of group unification is taken according to the variable X_{rp}^k; $t_{restoring}$ – the duration of the shipment's stay at technical stations under reshaping operations (we take the average duration of the freight car's stay at the technical station). We define $\delta_{restoring}$ as the number of freight car reshaping stations, we take it as given according to TFP; $t_\mu(N + N_{add})\big|^d$ – the duration of following on the sub-route μ sub-graph of the railroad section of the planned train flow. This travel duration can be determined by the dependence modeled by the regression model, which is proposed to be implemented on the basis of an artificial neural network. This artificial neural network models the dependence of the intensity of train movement on the duration of following on the main routes of an overloaded railroad section.

A change in the duration of tracking on the subgraph of the railroad section is modeled by the dependence:

$$t_\mu(N + N_{add})\big|^d = f\left(N^d + \left[\sum_r q_r X_{rp}^k / Q_p\right]^d\right),$$

where $f\left(N^d + \left[\sum_r q_r X_{rp}^k / Q_p\right]^d\right)$ is the function of the dependence of the duration of traffic on the routes of the subgraph on the intensity of train traffic, simulated by an artificial neural network, which is trained on the actual data of the «low» or «high» season of transportation of grain cargoes, depending on the simulation scenario, h; N^d – the number of train threads scheduled for the corresponding day of movement d, trains (taken before the start of the simulation – this allows you to simulate the real loading conditions of the railroad section, which is a subgraph of the network);

$N_{add}^d = \sum_r q_r X_{rp}^k / Q_p$ – the additional number of threads of train movement, which provides for the movement of staged routes (SRT) or district trains (SWL) on the corresponding day of movement d, trains. It should be noted that the simulation assumes that, in the case of the SWL shipment tracking strategy, grain cars reach the core stations according to the current TFP. At these stations, cars are waiting for the formation of a district train. Such a train consists of cars with goods to various stations of destination. Carriages in a district train go to the first oncoming sorting station, where the train breaks up. Under such conditions, the tracking of a shipment on sub-route μ can be considered as the tracking of a full-fledged freight train, which, in the same way as a staged routing train, affects the loading of the railroad section and the duration of the movement. Therefore, the calculated

number of cars in shipments is added up for each day d and rounded to a larger integer using a mathematical operation.
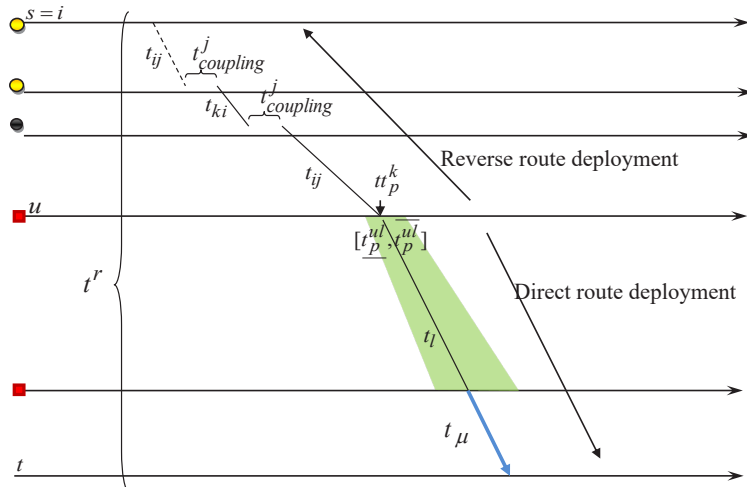

Fig. 3. The scheme of representation of the calculation of the duration of the tracking of the shipment according to the forward and reverse deployment of the route after setting the time of the shipment tt_p^k within the slot p

The limitation that makes it possible to model options for the formation of coalitions based on the combination of groups of cars to form a staged route can be written as:

$$\sum_r X_{rp}^k \geq 0. \tag{1}$$

Only one staged route is assigned to each slot:

$$\sum_p X_{rp}^k < 1. \tag{2}$$

However, it should be noted that the number of groups is limited according to the condition of the maximum length of the train:

$$\sum_r q_r^k X_{rp}^k \leq Q_p, \tag{3}$$

where Q_p is the length of the freight train, established in accordance with the standard train schedule for the time slot p on the route l, conditional cars.

The constraint ensures that each shipper's order is served only once:

$$\sum_k \sum_p X_{rp}^k = 1; r \in R. \tag{4}$$

It is important to take into account the logic of continuity and sequence of movement of shipments in the network. Therefore, the moment of departure from station j of cars of order r should be equal to the total idle time at this station and the time of travel to it from the previous station i. Idle time at station j takes into account the norms for the duration of technological operations with cars at the station, where j is the receiving station after maintenance. The condition can be written in the following form:

$$X_{rp}^k \left(tt_p^k - \sum t_{ij|ijrk\in A} - \sum t_{j|jrk\in A} + tt^r \right) = 0; \tag{5}$$

$$X_{rp}^k \left(tt_p^k - \sum t_{ij|ijrk\in A} + tt^r \right) = 0, \tag{6}$$

where tt_p^k is the moment of dispatch of the request cars r within the time window of the slot p; t_{ij} – duration of shipment movement between stations i and the next station j, h; $\sum t_j = \gamma_{coupling}^j \cdot t_{coupling} + \delta_{resorting}^j \cdot t_{resorting}$ – waiting time at station j, which depends on the sequence of carriage processing in accordance with transportation technology, h; tt^r – the moment of dispatch of the cars of the request r is determined by the expression $tt^r = tt_p^k - \sum t_{ij} - t_j^{coupling}$.

Restrictions on the border of the time windows set by the consignor:

$$a_{ri} \leq tt^r \leq b_{ri}, \tag{7}$$

where a_{ri} is the beginning of the time window; b_{ri} is the end of the time window.

Late order is allowed, early service is not allowed:

$$\Delta t_r^{outside} = \begin{cases} \text{If } tt^r - b_r > 0, \text{then } \Delta t_{ri} = tt^r - b_r, \\ \Delta t_r^{outside} = 0, \text{otherwise.} \end{cases} \tag{8}$$

The instant of time of dispatch of the request cars r is determined by the expression $tt^r = tt_p^k - \sum t_{ij} - t_j^{coupling}$.

The price of the game within the optimization problem can be calculated as the sum of two components of the cost function:

$$u\left(s\left\{ X_{rp}^k, tt_p^k \right\} \right) = F_1 + F_2 \to \min, \tag{9}$$

where F_1 is the value function that evaluates the profit from the position of the consignor's interests (selfish behavior); F_1 minimizes the cost of carriage-hours of the movement of its own shipments to the destination, taking into account the reduction of the expectation from the desired time period of the shipment, UAH:

$$F_1\left(X_{rp}^k, tt_p^k \right) = \sum_r \left(Cq_r t^r + C_{inside} q_r \Delta t_r^{inside} + C_{outside} q_r \Delta t_r^{outside} \right), \tag{10}$$

where C is the unit cost rate of 1 freight car-hour, y. o.; C_{inside} is a unit cost rate of 1 car-hour under the conditions of the desired time of departure of the shipment, y. o.; $C_{outside}$ – unit cost rate of 1 car-hour under conditions of violation of the desired time of departure of the shipment, y. o.; $\Delta t_r^{outside}$ – length of time after the end of the planned time window until the actual departure, hours; The calculation is performed according to the expression:

$$\Delta t_r^{outside} = \begin{cases} \text{If } tt^r - b_r > 0, \text{then } \Delta t_{ri} = tt^r - b_r, \\ \Delta t_r^{outside} = 0, \text{otherwise,} \end{cases} \tag{11}$$

Δt_r^{inside} – the length of time from the start of the desired departure to the actual departure within the planned time window, hours. The calculation is performed according to the expression:

$$\Delta t_r^{inside} =$$
$$= \begin{cases} \text{If } tt^r - a_r > 0, \text{ then } tt^r - b_r < 0, \text{ then } \Delta t_{ri} = tt^r - a_r, \\ \text{elseif } tt^r - b_r > 0, \text{ then } \Delta t_r^{inside} = b_r - a_r, \\ \Delta t_r^{inside} = 0, \text{otherwise,} \end{cases} \tag{12}$$

F_2 is a cost function that evaluates the gain from the perspective of the interests of the efficiency of the railroad system, y. o. Railroad efficiency refers to the planning of transportation with the coordination of shippers to ensure better use of the capacity of the railroad infrastructure and speed up the movement of shipments of all shippers. F_2 minimizes the train-hour cost of all freight trains on the main routes of the congested railroad section. The goal of system coordination is to balance the duration of movement within the limits of movement standards in the phase of following trains – «free flow» [33]. Such balancing should be performed on traffic routes throughout the planning period. An increase in the duration of traffic leads to the formation of traffic jams and negative consequences for the entire railroad network.

In this work, it is proposed to calculate F_2 as the total weighted average cost of the duration of movement for each day of the planning period for each route of the congested railroad section:

$$F_2\left(X_{rp}^k, tt_p^k\right) = \sum_\mu \frac{1}{\sum \mu} \sum_d \frac{C_{train-hour} \cdot (N + N_{add}) \cdot t_\mu\big|^d}{\sum d}, \qquad (13)$$

$C_{train-hour}$ – unit cost rate of 1 train-hour, UAH; $t_\mu\big|^d$ – duration of movement on the route μ, h.

An optimization mathematical model with objective function (9) and constraints is proposed. The following restrictions are taken into account:

– formation of coalitions (1);
– compliance with the logic of execution of transportation processes (2);
– limitation on the number of cars in freight trains (3);
– the importance of order fulfillment (4);
– continuity and sequence of shipment movement (5), (6);
– use of dedicated slots (7).

The essence of solving the problem boils down to finding a service plan for shippers by modeling SWL or SRT strategies taking into account the formation of coalitions and the efficiency of using the railroad infrastructure, which is the bottleneck of the railroad network.

5. 1. 1. A method for finding a solution based on a mathematical model of a ride-sharing service for railroad transportation of grain cargoes

Despite the approaches used to simplify the computational complexity of the mathematical model, this problem belongs to the problems of mixed-integer nonlinear programming (MINLP) [34]. These problems are NP-complete. To speed up calculations and increase the accuracy of solutions, a heuristic optimization method was used – the genetic algorithm [35].

The implementation of the first stage of the algorithm involves representing a set of variables of the mathematical model in the form of a chromosome of fixed length, taking into account the restrictions on their ranges and reducing them to one numerical vector C_h, consisting of four parts:

$$C_h = \left(C^1 C^2 C^3 C^4\right), \qquad (14)$$

where $h = \overline{1, K}$ is the number of chromosome C. The first part of the chromosome is represented by genes that are determined by the number of applications from shippers and make it possible to model coalitions based on heuristics. The other three component of the chromosome are designed to simulate the selection of a movement slot, the binding of the train departure schedule of the staged route within the given slot (variable tt_p^k). The scheme of encoding and decoding of the chromosome of decisions on the choice of strategies by shippers and the formation of coalitions is shown in Fig. 4.

For the software implementation of the fitness function of the genetic algorithm, it is necessary to reduce the proposed objective function (9) with constraints (1) to (7) to the objective function of unconditional optimization of the form:

$$F = F_1 + F_2 + \lambda \left(\sum_{\psi=1}^\varphi \left(h_\kappa(x) \right)^2 \right) \to \min, \qquad (15)$$

where λ is the penalty function parameter, λ>0; $h_\kappa(x)$ is the inequality constraint of the problem reduced to the form $h_\kappa(x) \le 0$, $\kappa \in K$.
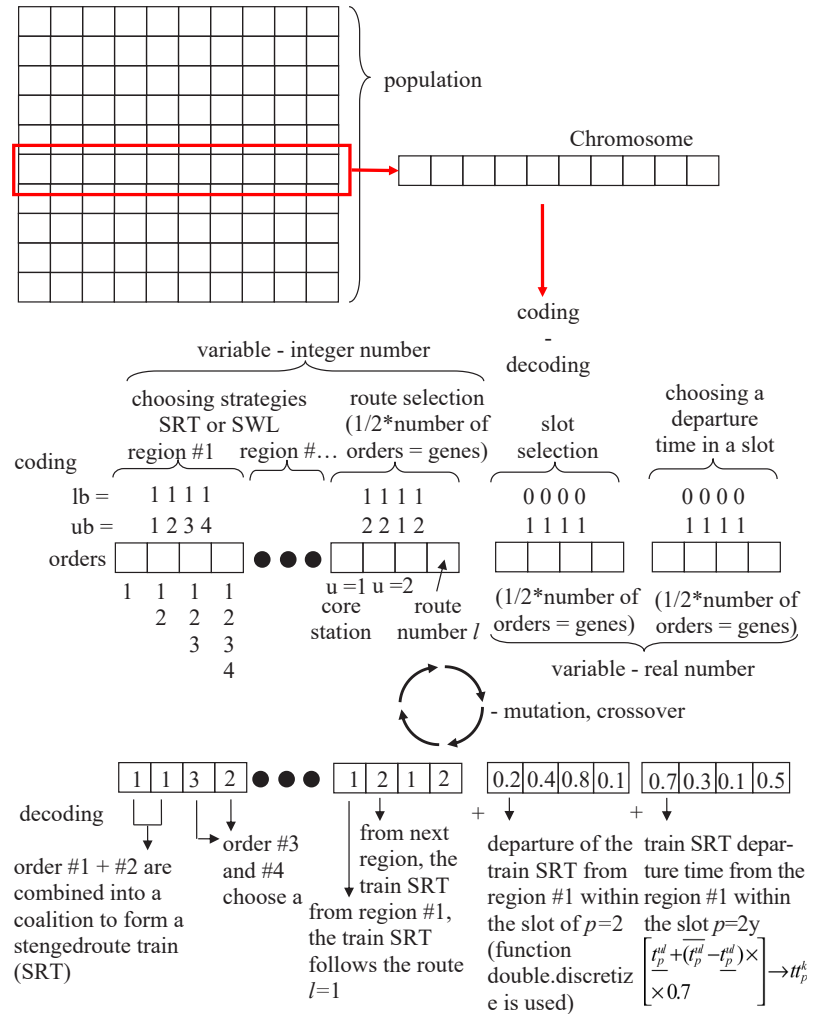

Fig. 4. Scheme of encoding and decoding of the chromosome for strategy selection decisions by shippers and formation of coalitions

According to the statement of the mathematical model within the fitness function of the genetic algorithm it is proposed to use an artificial neural network as a mathematical model of nonlinear regression. A feedforward neural network was used [36]. An artificial neural network allows you to simulate a potential function that determines, depending on the selected coalitions within the players' strategies, the total delay of train flows at the congested railroad section, taking into account their interdependence on main routes. The general sequence of stages of the implementation of the genetic algorithm for solving the proposed cooperative congestion game can be represented in the form of a block diagram in Fig. 5. According to the class of the problem to be solved the mixed-integer genetic algorithm was used, which, using the intCon vector, imposes an integer on the specified variable chromosomes – C^1.

This allows you to implement the solution of the mixed-integer nonlinear programming problem within the ga function in the Global Optimization Toolbox package. Vectorization of the GA fitness function was used to speed up the solution speed. By default, the special functions of creation, crossover, and mutation force the variables to be integers in the first part of the chromosome [35].
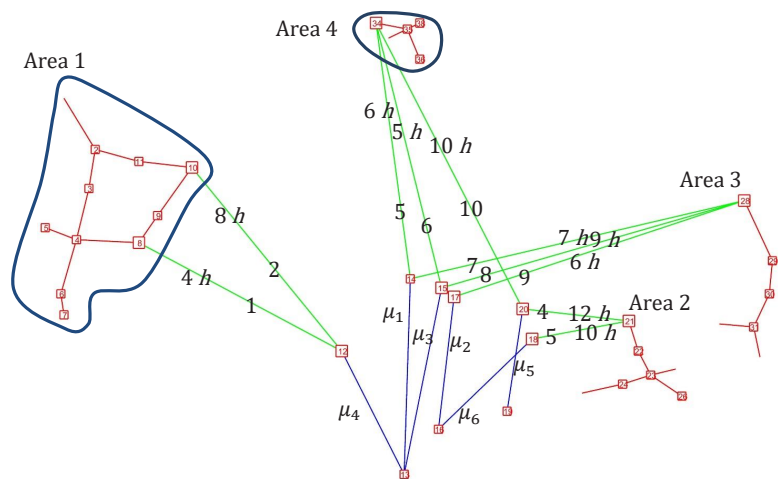
Fig. 6. Visualization of graph G of a network of 38 vertices

The network graph consists of four areas, each with one or more core stations. It should be noted that in order to reflect the real conditions of the topology of the railroad network of Ukraine, area 1 has two core stations – 8 and 10. From each area, different variants of main traffic routes (edges of the graph marked in green) to the congested railroad section are provided. The main routes on the congested railroad section, which is the bottleneck of the network, are marked in blue – the duration of movement on these routes is simulated by an artificial neural network. The three destination stations are 13, 16, and 19, simulating near-port stations.

In order to display the possible options for combining and simplifying the solution, $R=19$ requests for the transportation of grain (r is the request index, where $r \in R$) are assigned to the three stations of the above-mentioned stations. With the SWL strategy, for the possibility of calculating the duration of the movement of the shipment according to TFP [6], in the work for each core station according to the route of movement (green edges of the graph G), the number of transformations $\delta_{restoring}$ and the indicators of the average duration of the presence of a freight car at the sorting stations $t_{restorin} \times \delta_{restoring} = =[13.15\ 29.32\ 13.15\ 26.3\ 13.15\ 13.15\ 13.15\ 13.15\ 13.15\ 13.15]$ are set. These indicators are taken according to the static data from JSC «Ukrzaliznytsia» in 2019.

With the SRT strategy, $t_{coupling}=2$ hours is adopted. This duration includes maneuvering operations to connect groups of cars into a single train. To calculate the duration of movement of groups of cars on the first mile of transportation to the core station, a speed of 31.6 km/h was adopted, which
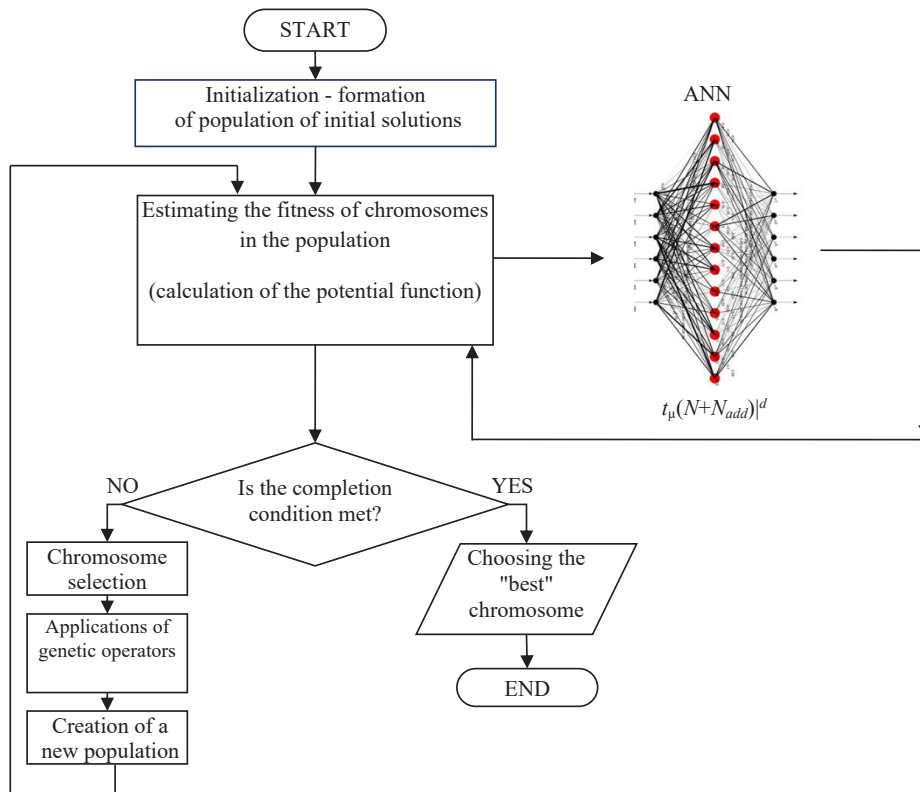

Fig. 5. Block diagram of the genetic algorithm for solving the proposed cooperative congestion game

5. 2. Simulation results of a cooperative congestion game under different scenarios of behavior in the railroad system

For modeling, a railroad network was built in the work in the form of a graph $G(A, E)$, which consists of 38 vertices (Fig. 6).

corresponds to the statistical data of the network in 2019. In order to simulate the conditions of network operation, taking into account seasonality, the planned load of the main routes of the network was formed. The simulation conditions assume that the planning period is seven days $D=7$ and the uneven loading of routes during the days of the week is taken into account.

A software interface has been developed that allows visualization of simulation results. In order to conduct experimental studies to investigate the influence of the model of railroad transportation of grain based on the principles of joint use on the operation of the railroad system, three scenarios are proposed:

– scenario 1 involves movement in the railroad system of SWL and SRT shipments without observing the schedule. This corresponds to the current system of freight transportation – shipments without observing the schedule of freight trains. It should be noted that for modeling according to the proposed methodology, the time windows of the slots were extended to the duration of the day;

– scenario 2 provides for the movement in the railroad system of SWL shipments without observing the schedule

(the current transportation model) and SRT according to the allocated slots (departure from the railroad section core station is possible according to the schedule);

– scenario 3 involves the movement in the railroad system of SWL shipments without observing the schedule (the current model of transportation) and SRT according to the allocated slots. Departures from the core station of the railroad section are possible according to the schedule, taking into account the acceleration of traffic on the routes by 20 % compared to the traffic of SWL shipments. It should be noted that the value of 20 % of the standard duration of movement was adopted by us for reasons of practical implementation of train movement according to the selected movement schedules. The movement of the train according to the schedule is planned, which makes it possible to reduce the duration due to the reduction of non-productive downtimes during the change of locomotives and the execution of unforeseen operations during the movement – overtaking or crossing with other trains. Visualization of the shipment movement plan according to three scenarios is shown in Fig. 7–9.
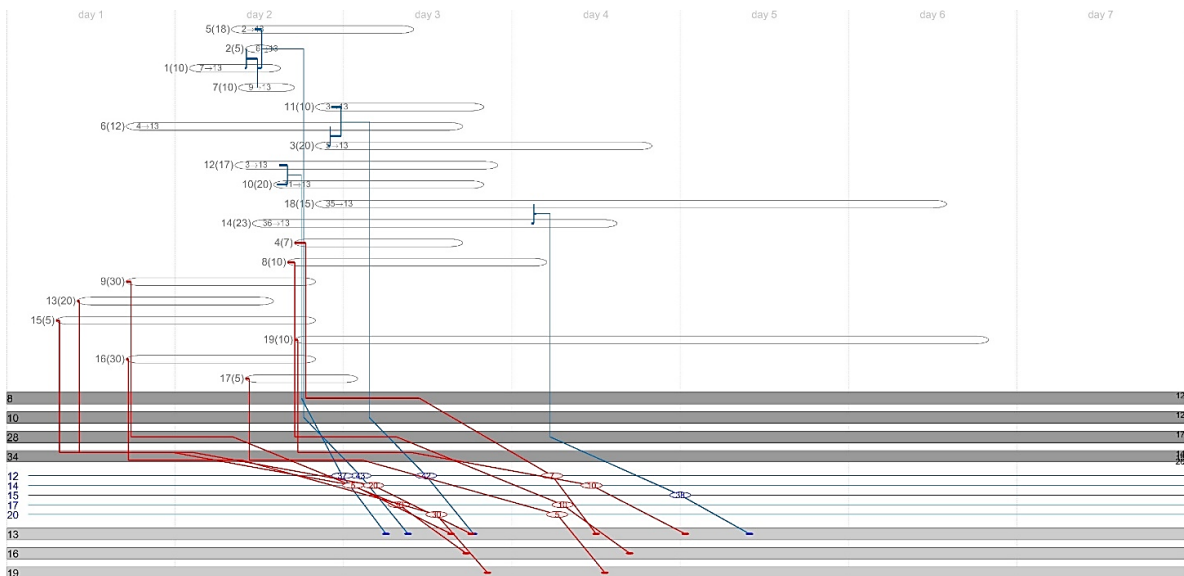

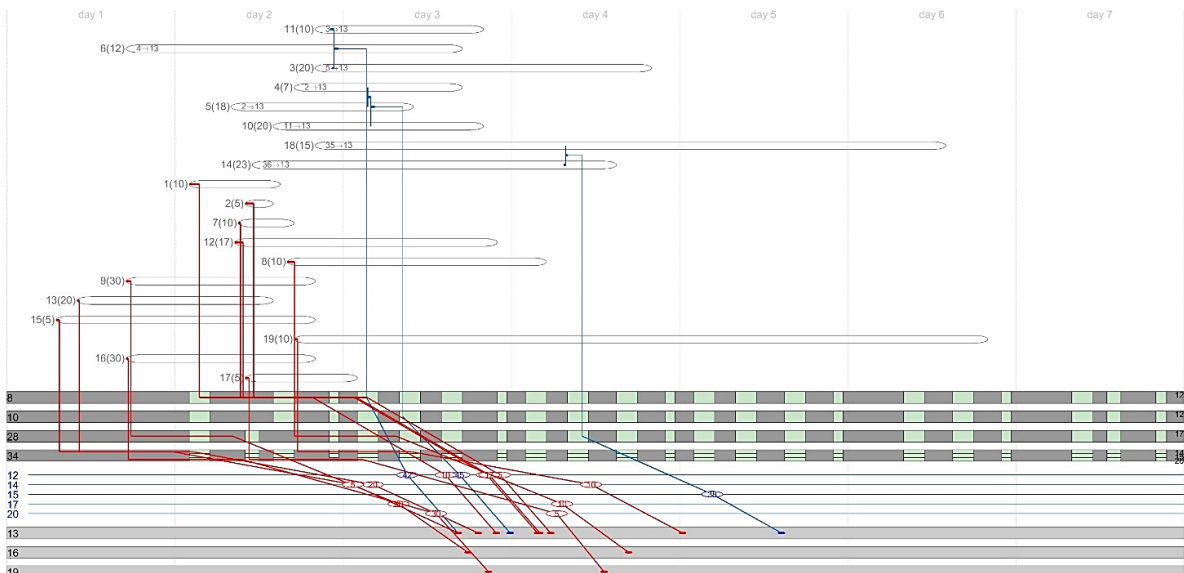Fig. 7. Visualization of the shipment movement plan according to scenario 1

Fig. 8. Visualization of the shipment movement plan according to scenario 2
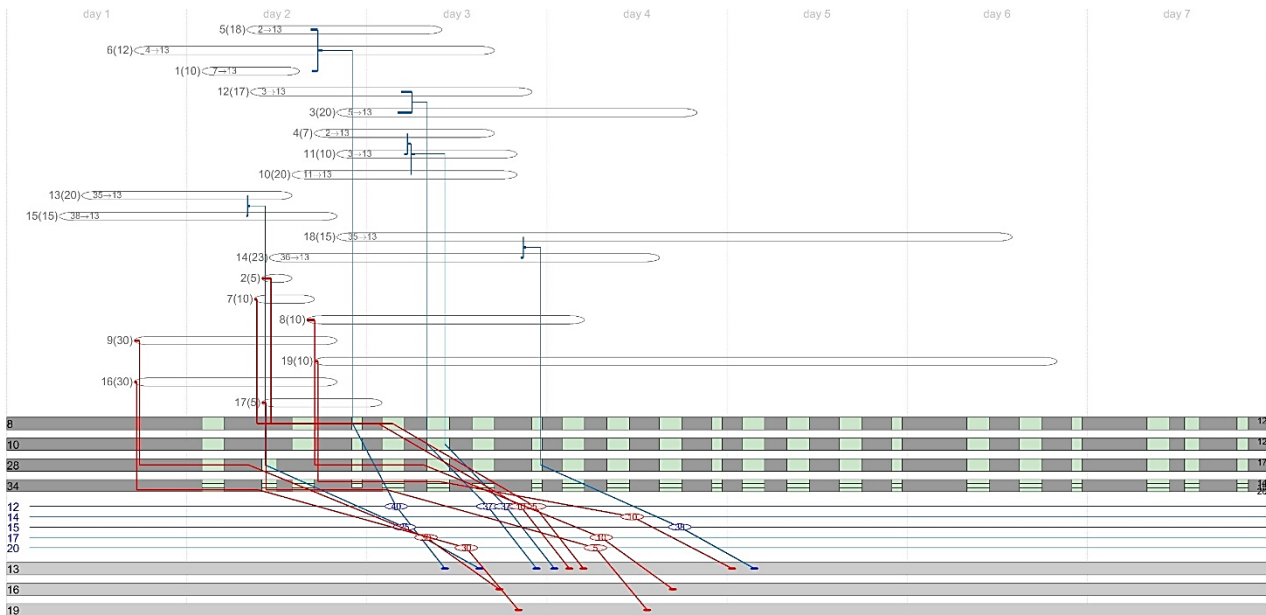
Fig. 9. Visualization of the shipment movement plan according to scenario 3

To analyze the average duration of traffic on each of the routes of the congested railroad section, taking into account the intensity of trains on each day of the planned planning period, it is suggested to visualize the diagrams for each of the scenarios in Fig. 12–14. From the diagrams, it can be seen that the increase in the number of trains reaching the core station of the overloaded railroad section on the third or fourth day leads to an increase in the duration of traffic on routes (Fig. 10–12). The delay in movement at the railroad section simulated by an artificial neural network reflects real traffic conditions during the peak period of grain cargo transportation, which confirms the adequacy of our results.

A comparative analysis of the diagrams of the duration of shipment movement under different conditions, shown in Fig. 10–12, was performed. It was found that the daily duration of traffic on each of the routes in scenario 3 is more evenly distributed throughout the entire planning period. The largest spikes are observed in scenarios 1 and 2, especially on the third day of planning, which shows the formation of traffic jams and slowdowns. The simulation results are summarized in Table 1.
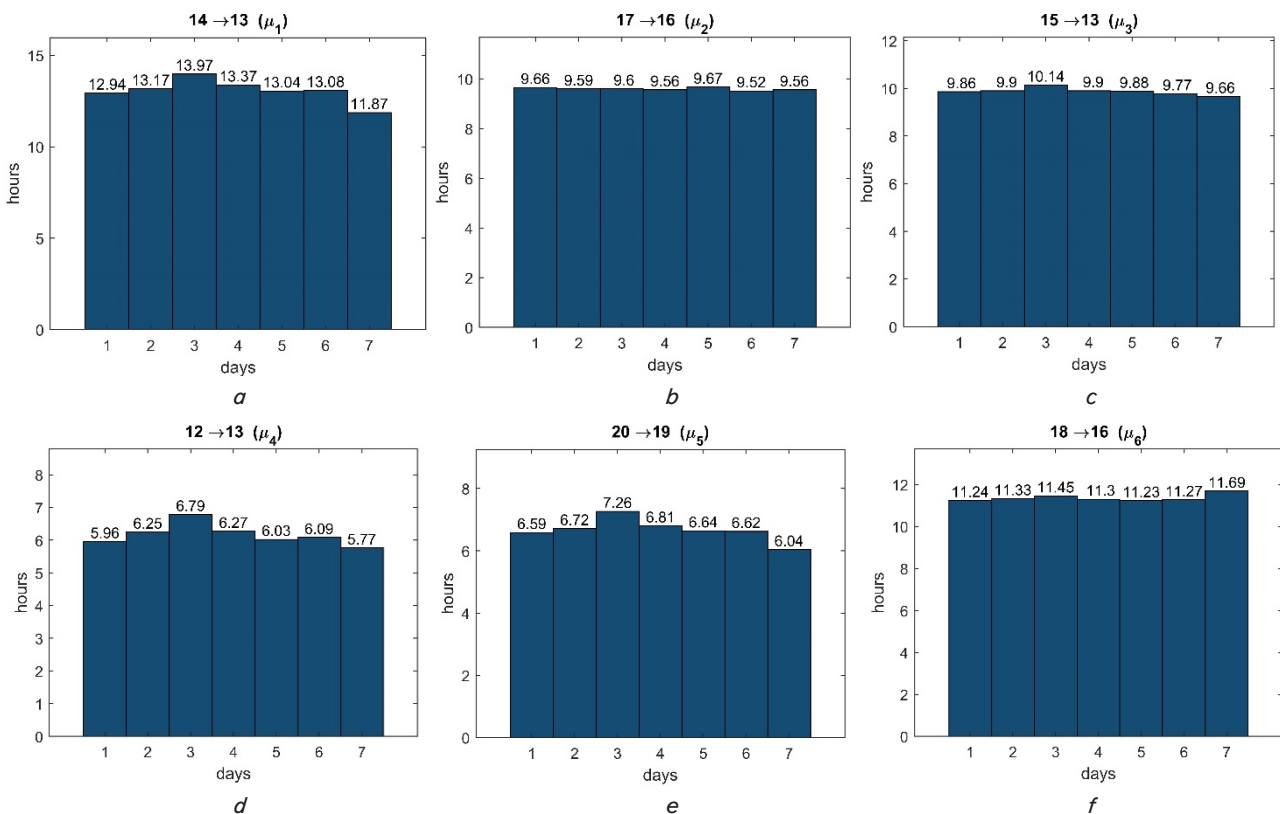

Fig. 10. Dependence charts of train movement duration on the planning day according to scenario 1:
a — route μ_1; b — route μ_2; c — route μ_3; d — route μ_4; e — route μ_5; f — route μ_6

Fig. 11. Dependence charts of train movement duration on the planning day according to scenario 2:
a — route μ_1; b — route μ_2; c — route μ_3; d — route μ_4; e — route μ_5; f — route μ_6
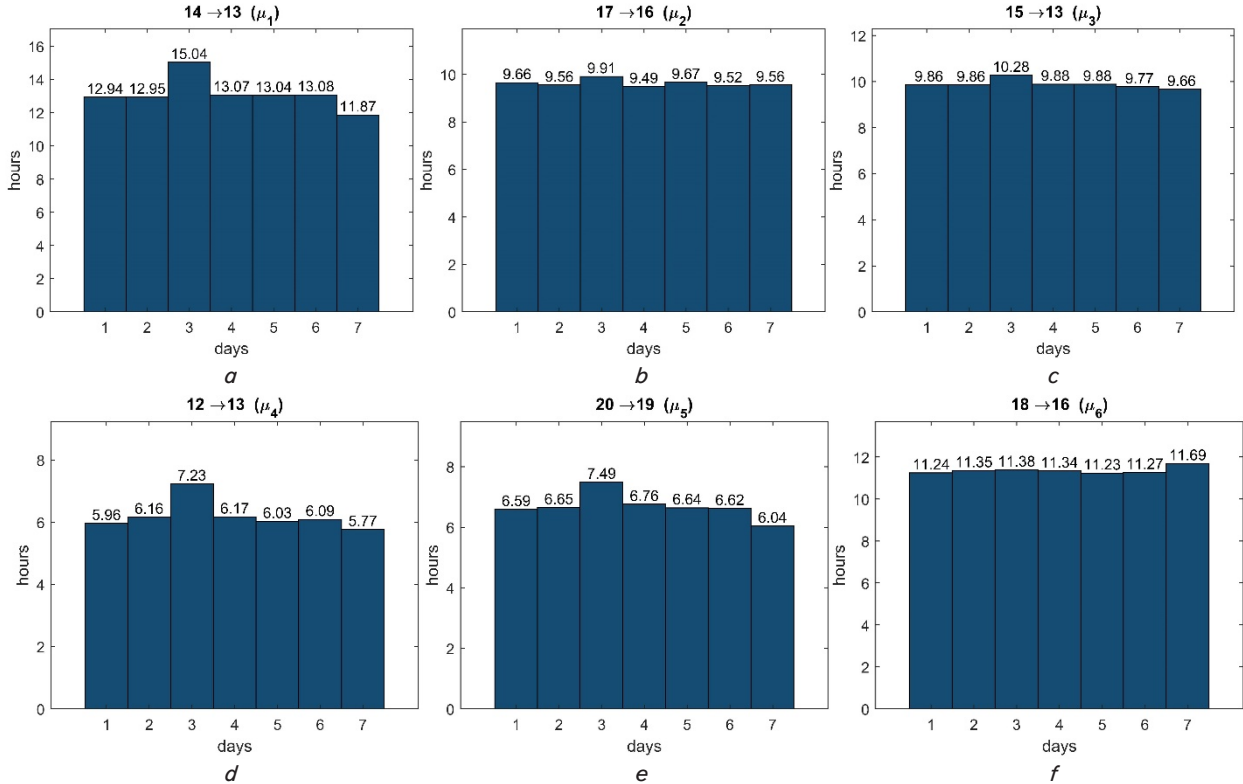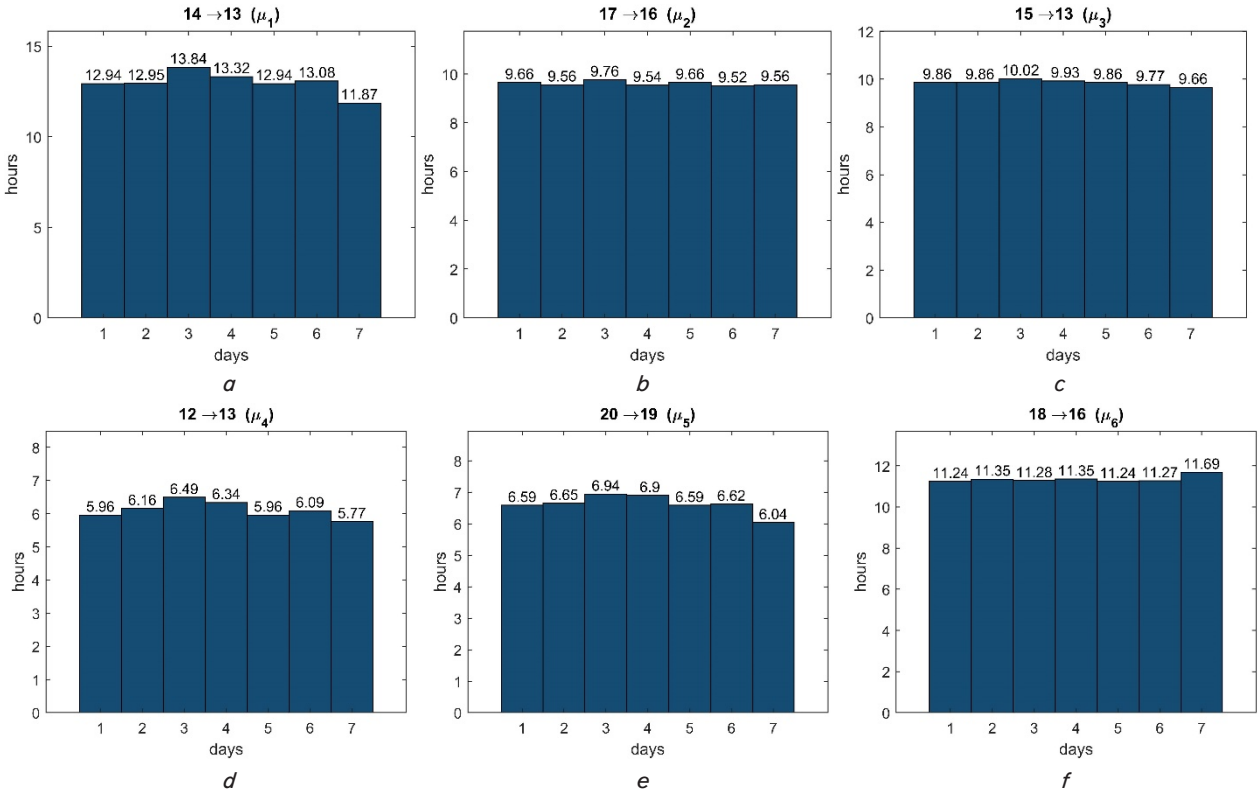

Fig. 12. Dependence charts of train movement duration on the planning day according to scenario 3:
a — route μ_1; b — route μ_2; c — route μ_3; d — route μ_4; e — route μ_5; f — route μ_6

Therefore, according to our results, which are given in Table 1, the implementation of the ride-sharing service with respect to the traffic schedule makes it possible to simulate traffic in the railroad system that is close to reality. Separate threads of the traffic schedule for freight trains make it possible to reduce the number of conflicts during movement with other trains. This corresponds to theoretical expectations for improving the efficiency of planning in the railroad system of Ukraine.

Table 1

Simulation results for different scenarios

Metric name	Scenario 1 SRT movement without schedule	Scenario 2 SRT movement by slots/schedule	Scenario 3 SRT movement by slots/schedule at accelerated speed
	the value of the indicator calculated as an average indicator for all orders in the system		
The average duration of shipment transportation, hours	34.50	37.93	29.37
Average car-hours per shipment	504.14	547.79	430.12
The average waiting time for the shipment within the desired window, hours	11.34	19.79	18.20
Average waiting time for a shipment outside the desired window, hours	0	0	0.14
Total loss of shipper's time, hours	45.84	57.71	47.70
Total losses of the consignor in wagon-hours	692.39	883.54	706.93
Average daily train hours	266.22	267.89	265.26

6. Discussion of results of the construction of a mathematical model of the ride-sharing service of railway transportation of grain cargoes

Our results of the construction of a mathematical model of the ridesharing service for the railroad transportation of grain cargoes have made it possible to study the impact of ridesharing on the current operation of the railroad system under various operating conditions. The simulations were carried out and an increase in the duration of shipment movement was found under the conditions of the scenario of shipment «on readiness» and movement according to allocated time slots without coordination. This proved that the mathematical model built makes it possible to adequately simulate various transport scenarios taking into account the formation of traffic jams and traffic slowdowns. The analysis of the results according to scenario 1 proved that the results correspond to the current transportation system. Under scenario 2, all research indicators deteriorate compared to the movement of shipments under scenario 1 (Table 1). This is explained by additional waiting for the dispatch of cars of the selected scheduled threads of the traffic schedule. Taking into account the same duration of freight car movement for different types of shipments specified in the model, there are no incentives for shippers to form coalitions for the formation of SRT trains.

The model situation according to scenario 3 made it possible to simulate the movement close to reality in the railroad system when the selected threads of the traffic schedule make it possible to reduce the number of conflicts during movement with other trains. It can be assumed that moving according to the schedule will reduce the probability of traffic jams at the congested railroad section and destination stations due to their planned use, and, accordingly, more uniform arrival of trains. According to the simulation results, the average duration of shipment transportation was 29.37 hours, which is 14.9 % of the indicator under scenario 1. It is believed that the acceleration of shipment movement when running on SRT trains stimulated shippers to choose the SRT strategy, which increased the number of trains to five. This reduced the number of freight trains to 6 in the third and fourth planning days in the congested yard compared to scenario 2. In scenario 2, the number of freight trains was 8 in the corresponding planning period, which increased the load and negatively affected the efficiency of the network.

The limitation of this study is that attention was paid to investigating the effectiveness of the ridesharing service only in the component duration of transportation and the impact on the load of the railroad network. Conditions of overloading and stoppage of operation of sorting stations on the network are not taken into account.

As a shortcoming of this research, we can note the need to simplify the statement of the problem, where the duration of movement varies depending on the intensity of train movement only on the congested railroad section, and not on the entire movement route. The duration of movement to the core stations of the test site was accepted as standard for all shipments. This led to a deterioration of modeling accuracy. Taking into account the interdependence of the movement of train flows on the entire network, and not on a separate railroad section, will allow creating a more accurate mathematical model.

The advantage of our research in comparison with works [10–25] is that, for the first time, a mathematical model has been built for simulating a ridesharing service for a railroad system without observing the schedule of freight trains, such as the railroads of Ukraine. In contrast to works [11–20], a hybrid approach to the formalization of the problem was applied. This helped increase the accuracy of the results without losing the complexity of the description of the most important technical and technological limitations in the organization of railroad transportation.

The advantages of the results are that not only the adequacy of the mathematical model was evaluated but also the influence of the introduction of the ride-sharing service of grain transportation, taking into account the scheduled movement, on the efficiency of the railroad system was proven. The obtained results are conditional but make it possible to theoretically justify the effectiveness of the application of the grain cargo transportation model based on the ridesharing service. This transportation technology, provided the creation of a digital platform and its interconnection with the carrier's current planning systems, will allow providing the planned start time of the load in advance. This will reduce unproductive downtime of loaded cars before departure. In addition, the costs of reforming cars at the sorting stations of the network and their probability of stopping due to overloading, which often occurs during peak loads, will be reduced. The predictability of traffic on the congested railroad section will increase and the number of traffic jams will decrease.

The conditions for the practical application of the research results are the importance of taking into account incentives for shippers in new service products of ride-sharing services for the transportation of grain by railroad. Such incentives will

contribute to a greater desire to abandon the selfish behavior of «loading the dispatch freight car» to a more rational one – the formation of coalitions for the formation of block trains. One such incentive could be the provision of a dedicated movement slot, which will speed up transport compared to normal ones.

The further development of this research is the determination of the behavior of shippers and the response of the transportation system under conditions of off-peak loads – in the low season of grain transportation. In addition, the mathematical model built allows conducting research into estimating the price of anarchy (PoA) [37] in order to estimate the cost of the lack of coordination in the railroad system during the transportation of grain cargoes with the quality of centrally optimal planning.

7. Conclusions

1. The process of transportation of grain cargoes has been formalized based on the principles of ridesharing in railroad systems without observing the traffic schedule for freight trains using game theory and optimization methods. This made it possible to simulate the behavioral mechanisms occurring in the railroad system under the conditions of implementation of the ride-sharing service of grain cargo transportation with sufficient accuracy and speed of resolution. It is proposed to consider the setting of the game as the formation of coalitions in games with congestion in the form of a cost function and the distribution of flows on the network. It is proposed to turn this game into a nonlinear optimization problem. To find the optimal solution according to the mathematical model, a genetic algorithm was used, within the fitness function of which it is proposed to use an artificial neural network as a mathematical model of nonlinear regression, which allows simulating the interdependence of the movement of train flows on the network. The results of the solution were compared with the actual data on the current transportation and proved its adequacy for the process under investigation.

2. Our experimental studies have made it possible to analyze the influence of the model of railroad transportation of grain in the peak period of load based on the principles of joint use on the operation of the railroad system. It was established that compliance with the traffic schedule leads to an increase in non-productive downtime of cars after loading, which reduces incentives for the formation of coalitions by shippers. However, movement within the allocated time corridors makes it possible to increase the stability of the movement of stage trains, to improve coordination in the railroad system, which makes it possible to reduce delays on the main routes of the congested railroad section, and to avoid traffic jams. According to the results of modeling under the conditions of traffic according to the schedule, taking into account the coordination of consignors and carriers in a single information environment, the average duration of shipment transportation was 29.37 hours. This indicator is 14.9 % lower than the indicator according to the scenario of the current transportation model – without observing the schedule. Acceleration of shipment movement when running on SRT trains is believed to have encouraged shippers to choose the SRT strategy, which increased the number of such trains and reduced congestion on the rail network. The proposed modeling approach can be used as a tool to study the impact of coordination in railroad systems without observing freight train schedules.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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