UDC 624.04.012.4.044
DOI: 10.15587/1729-4061.2023.292692

LAYOUT AND CROSS-SECTIONAL SIZE OPTIMIZATION OF TRUSS STRUCTURES WITH MIXED DESIGN VARIABLES BASED ON GRADIENT METHOD

The object of the study was truss-type rod structures, which were investigated for the purpose of finding the optimal design solution in a mixed (continuous and discrete) space of variables. The parameters of the geometric scheme of the truss, as well as the dimensions of the cross-sections of its elements, were considered as design variables. The stated optimization problem is represented as a nonlinear programming task, in which the objective function and nonlinear constraints of the mathematical model are continuously differentiable functions of the design variables. The system of constraints includes strength and stability inequalities, formulated for the design cross-sections of the rod elements of the structure, which is subject to the effect of the design load combinations of the ultimate limit states. As a part of the system of constraints, the displacement constraints formulated for the specified structural nodes, which is subject to the action of design load combinations of the serviceability limit states, are considered. To solve the stated optimization problem, a method of the objective function gradient on the surface of active constraints was used, with the simultaneous elimination of residuals in the violated restrictions. For design variables, the variation of which must be performed according to a given set of possible discrete values, a discretization procedure for the optimal solution obtained in the continuous space of design variables is proposed. A comparison of the proposed optimization methodology with alternative metaheuristic methods and algorithms reported in the literature was performed. On the considered problem of parametric optimization of a 47-span tower structure, a design solution with a weight of 835,403 kg was obtained, which is 1.53...4.6 % better than the optimal solutions obtained by other authors

Keywords: shape optimization, mixed variables, gradient method, finite element method, sensitivity analysis

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Received date 20.09.2023
Accepted date 28.11.2023
Published date 21.12.2023

How to Cite: *Peleshko, I., Yurchenko, V., Rusyn, P. (2023). Layout and cross-sectional size optimization of truss structures with mixed design variables based on gradient method. Eastern-European Journal of Enterprise Technologies, 6 (7 (126)), 6–18. doi: https://doi.org/10.15587/1729-4061.2023.292692*

1. Introduction

Applied problems on the optimal design of rod structures are usually stated as searching problems for unknown parameters of the structure, which ensure the extreme value of the specified optimality criterion in the search space determined by a set of given constraints. In this case, the optimization of structures is performed by varying the values of the structural parameters. At the same time, the topology of the structure, the types of cross-sections of structural members, the types of conjugation of elements to the structural nodes, the support conditions for the structure, as well as the scheme and magnitudes of loads are predefined and constant. A mathematical model of parametric optimization problems of rod structures includes a set of design variables, an objective function, as well as constraints that reflect the requirements for ensuring the necessary load-carrying capacity of the structure and its structural members [1].

In optimal design problems of rod structures, the unknown dimensions of the cross-sections of structural elements are often considered as design variables, which must change discretely, that is, take values according to a defined finite set of possible options. Among the tools used to solve the applied problems of discrete optimization of rod structures, a special place is occupied by methods that implement a purposeful selection of a finite set of design decision options, in particular, metaheuristic methods [2, 3].

However, when using metaheuristic methods (stochastic search methods, evolutionary algorithms), despite their high efficiency, it is possible to obtain, as is known, design solutions of structures that are only close to the optimum [4]. This allows us to state that it is appropriate to conduct a study aimed at finding a global optimum for the class of problems under consideration. Further scientific research on the issue of discrete optimization of rod structures is considered relevant.

2. Literature review and problem statement

A large number of studies consider the problem of finding optimal sizes of cross-sections and parameters of the geometric scheme of truss structures in the presence of discrete variables. During the last decades, many metaheuristic methods have been proposed and developed by scientists and researchers. Work [3] provides an overview of various statements of optimization problems for the considered class of structures and optimization algorithms based on metaheuristic methods. At the same time, the desired parameters are set both on numerical sets and on finite sets of an arbitrary nature. The search strategy in such algorithms is based on the calculation and comparison of the values of some function of evaluating design decision at the points of the search space under consideration [4]. At the same time, requirements regarding unimodality, continuity, and differentiability of such a function are not put forward. This determines the possibility of using metaheuristic methods for a wide class of functions of the quality criterion and constraints of the mathematical model, including for functions that do not have an analytical description.

In work [5], which tackles the issue of finding the optimum in the tasks of optimizing the cross-sectional areas of elements and the node coordinates of truss-type structures, the authors used a modified genetic algorithm. This made it possible to avoid obtaining local optima during the optimization search. In [6], Jaya algorithm was applied to the class of problems under consideration. The authors developed its modification, which improves the speed of convergence to the optimal solution and reduces the number of structural analyzes during the search for the optimal point. Work [7] reports a new metaheuristic optimization method based on group search, which is applied to solving problems of optimizing the cross-sectional areas of elements and parameters of the geometric scheme for this type of structures. For the considered class of problems, the annealing simulation algorithm was also successfully applied in [8]. In papers [9, 10], the particle swarm method was used for truss optimization problems stated in a mixed space of variables and its modifications were proposed, which allowed the authors to obtain a better convergence to the optimum. In [11], the problem of minimizing the weight of the truss structure using the artificial bee colony algorithm is considered. At the same time, code-based constraints are considered in the mathematical model of the optimization problem. For the considered class of problems, the teaching-learning based optimization algorithm was also successfully applied in [12].

However, when applying the metaheuristic methods listed above, despite their high efficiency and productivity, the authors obtained design solutions for structures that are only close to the optimum [4]. This allows us to state that it is appropriate to conduct a study aimed at finding a global optimum for optimization problems stated in a mixed space of design variables.

On the other hand, one of the effective methods of solving parametric optimization problems of rod structures are gradient nonlinear optimization methods. Such methods are used in the event that the objective function and nonlinear constraints of the mathematical model represent continuously differentiable functions of the design variables vector. The problems of parametric optimization of rod structures with variable parameters of the geometric scheme and the dimensions of the cross-sections of its elements belong to this class of problems.

Thus, in work [13], gradient methods were successfully used to solve optimization problems of structural members made of cold-formed profiles. In [14], the authors proposed a methodology for parametric optimization based on gradient methods and applied it to steel structural systems. Work [15] solved the problem of parametric optimization of lattice frame structures, the elements of which are made of rectangular and square hollow sections. In these works, all design variables were assumed to be continuous, and the issue of the presence of discrete design variables and, accordingly, a mixed search space was not considered.

Thus, a critical review of the above works proved that the issue of finding the global optimum in the problems of parametric optimization of rod structures in the presence of discrete design variables can be improved.

3. The aim and objectives of the study

The purpose of our study is to solve the problem of optimal design for rod structures in the presence of design variables of continuous and discrete types using gradient optimization methods. This will make it possible to construct a mathematical apparatus for application software in the field of nonlinear programming and to solve applied problems of structural optimization with its application.

The set goal is accomplished on the basis of solving the following research tasks:

– to state the problem and develop an algorithm for solving the problem of parametric optimization of rod structures in the presence of continuous and discrete design variables based on the gradient method;

– to develop a procedure of discretization for a continuous optimal design solution for design variables, the variation of which should be carried out in accordance with a defined set of possible variants;

– to demonstrate the effectiveness of the proposed approach by comparing the results of optimization calculations obtained using the proposed numerical algorithm with the results reported in the literature.

4. The study materials and methods

The object of this study is to consider truss-type rod structures, which are investigated for the purpose of finding the optimal design solution in a mixed (continuous and discrete) space of variables. At the same time, the parameters of the geometric scheme of the structure and the dimensions of the cross sections of its elements are considered as design variables.

In the current study, the problem of optimal design of truss structures is stated as a nonlinear programming problem, in which the constraints and the objective function are continuously differentiable functions of the design variables. To solve the stated problem, gradient methods are used, the choice of which depends on the nature of the constraint functions and the objective function of the mathematical model.

The hypothesis of the study assumes that in the presence of a mixed space of design variables, for the solution of the considered class of problems, along with metaheuristic methods, gradient optimization methods can also be effectively applied. At the same time, the search for the optimal design solution is performed in a continuous space of design variables. When moving along the direction of the search for the optimum point, hitting the nodes of the discrete grid of variables is associated with a significant complication of the optimization algorithm, which can lead to deterioration of its convergence. That is why, when using gradient methods, the search for the optimum is performed only in the continuous space of design variables. After obtaining a continuous optimum, the task of its discretization arises, which can be solved by developing a special procedure.

5. Results of optimization of cross-section sizes and parameters of the geometric scheme of truss structures

5. 1. Statement of the problem and algorithm of parametric optimization of truss structures

The problem of parametric optimization of truss structures was considered. Such a problem was stated as a search for the optimal parameters of the geometric scheme of the structure and the dimensions of the cross sections of its elements, which ensure the extreme value of the determined criterion of optimality. At the same time, the constraints of the load-bearing capacity and rigidity of the structure must meet the requirements of the design codes. The topology of the structure, the types of cross-sections of its elements, the types of conjugation of elements to the structural nodes, the support conditions for the truss structure, as well as the scheme and the magnitude of the external loads are specified and do not change during optimization.

The stated optimization problem can be represented in the form of a nonlinear programming problem [16], namely, the problem of finding such values of unknown parameters of the system, $\iota=\overline{1,N_X}$:

$$\vec{X}=\{X_\iota\}^T, \tag{1}$$

which provide the smallest (or largest) value of the specified objective function:

$$f^*=f\left(\vec{X}^*\right)=\min_{\vec{X}\in\Im_N} f\left(\vec{X}\right), \tag{2}$$

in the area of admissible design solutions $\Im$ determined by the system of constraints-inequalities:

$$\varphi\left(\vec{X}\right)=\left\{\varphi_\eta\left(\vec{X}\right)\le 0 \mid \eta=\overline{1,N_{IC}}\right\}, \tag{3}$$

where $\vec{X}$ is a vector of design variables (searched design parameters); N_X is the number of unknown system parameters (design variables); f, φ_η are continuous functions of the vector argument; $\vec{X}^*$ is the optimal solution (vector of optimal values of variable parameters); f^* is the largest value of the objective function; N_{IC} is the number of constraints-inequalities $\varphi_\eta\left(\vec{X}\right)$, which determine the areas of admissible design solutions in the search space $\Im$.

The vector of design variables contains as components the unknown parameters of the geometric scheme of the structure $\vec{X}_G=\{X_{G,\alpha}\}^T$, $\alpha=\overline{1,N_{X,G}}$, as well as the unknown dimensions of the cross-sections of its elements $\vec{X}_{CS}=\{X_{CS,\beta}\}^T$, $\beta=\overline{1,N_{X,CS}}$ (Fig. 1):

$$\vec{X}=\left\{\vec{X}_G,\vec{X}_{CS}\right\}^T, \tag{4}$$

where $N_{X,CS}$ is the total number of unknown sizes of the cross-sections of the rod elements of the structure, $N_{X,G}$ is the total number of unknown parameters of the geometric scheme of the structure, $N_{X,G}+N_{X,CS}=N_X$.

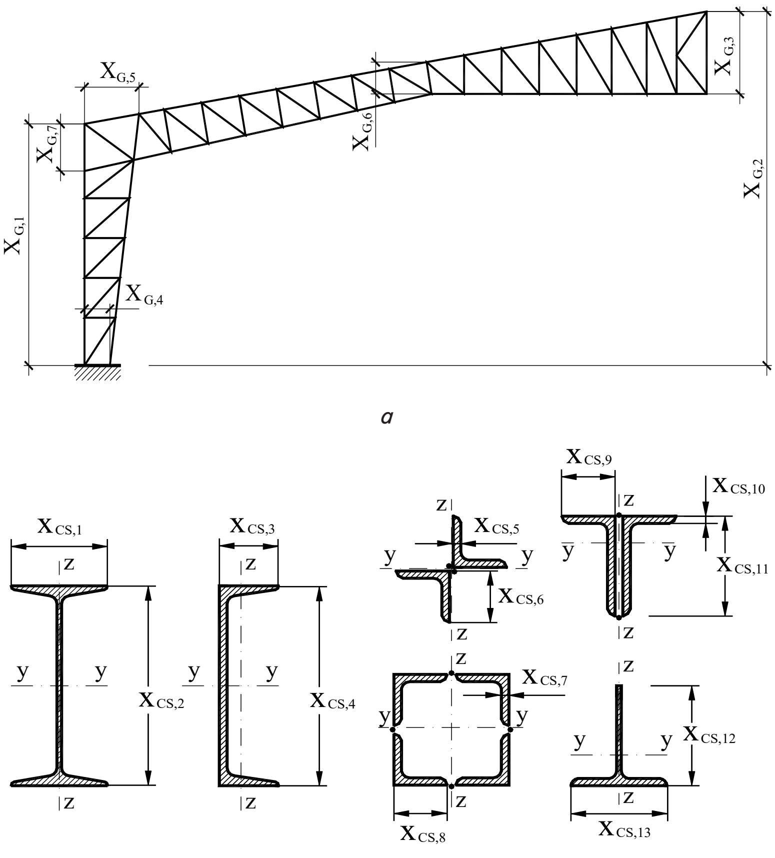


Fig. 1. Unknown parameters of the rod structure, which are considered as design variables: *a* – variable parameters of the geometric scheme; *b* – variable dimensions of cross-sections

As an objective function, a defined technical and economic indicator of the structure (material weight, material cost, manufacturing cost, construction cost, etc.) is considered. At the same time, they take into account the possibility of formulating an analytical expression for the objective function depending on the values of the design variables $\vec{X}$. In some cases, the weight of the construction material is taken as the objective function (2) of the optimization problem:

$$M^*=M\left(\vec{X}^*\right)=\min_{\vec{X}\in\Im_N} M\left(\vec{X}_G,\vec{X}_{CS}\right). \tag{5}$$

In the system of constraints (3), it is necessary to include load-carrying capacity constraints (strength and stability verifications) for all design cross-sections of structural members of the truss, which is subject to the effect of design load combinations of the ultimate limit states. In addition, as part of the system of constraints, it is necessary to consider displacement constraints (stiffness verifications) for nodes of the truss structure, which is subject to the action of design load combinations of the serviceability limit states. Additional requirements that describe the structural, technological, and operational features of the structure under consideration can also be included in the system of constraints (3).

The calculated internal forces in the structural members of the truss structure, which are used in the strength and stability verifications of the system (3) and depend on the design variables $\vec{X}$. were considered as state variables. These forces can be determined from the system of linear equations of the finite element method, $k=\overline{1,N_{LCC}^{ULS}}$:

$$\mathrm{K}\left(\vec{X}_G,\vec{X}_{CS}\right)\times\vec{z}_{ULS,k}=\vec{p}_{ULS,k}\left(\vec{X}_G\right), \quad (6)$$

where $\mathrm{K}\left(\vec{X}_G,\vec{X}_{CS}\right)$ is the stiffness matrix of the finite-element model of the truss structure, which is formed depending on the design variables $\vec{X}=\left(\vec{X}_G,\vec{X}_{CS}\right)^T$ of the optimization problem (1) to (3); $\vec{p}_{ULS,k}\left(\vec{X}_G\right)$ – a column vector of external nodal loads for the k-th design combination of loads of the ultimate limit states, which is formed depending on the unknown (variable) parameters of the geometric scheme $\vec{X}_G$ of the truss structure under consideration; $\vec{z}_{ULS,k}$ is the resulting column-vector of nodal displacements for the k-th design combination of loads of the ultimate limit states $\vec{z}_{ULS,k}=\mathrm{Z}_{FEM,k}^{ULS}\left(\vec{X}_{PS},\vec{X}_{CS}\right)$; N_{LCC}^{ULS} is the total number of design combinations of loads of the ultimate limit states. Depending on the obtained column-vector of nodal displacements $\vec{z}_{ULS,k}$, for each i-th design cross-section of the j-th structural element, it is possible to compute the design values of internal forces for the corresponding combination of loads.

As state variables, the nodal displacements of the truss structure were also considered, which are used in the stiffness constraints of system (3) and depend on the design variables $\vec{X}$. These displacements can be determined from the system of linear equations of the finite element method, $k=\overline{1,N_{LCC}^{SLS}}$:

$$\mathrm{K}\left(\vec{X}_G,\vec{X}_{CS}\right)\times\vec{z}_{SLS,k}=\vec{p}_{SLS,k}\left(\vec{X}_G\right), \quad (7)$$

where $\vec{p}_{SLS,k}\left(\vec{X}_G\right)$ is the column vector of external nodal loads for the k-th design combination of loads of the serviceability limit states, which is formed depending on the unknown (variable) parameters of the geometric scheme $\vec{X}_G$ of the truss structure under consideration; $\vec{z}_{SLS,k}$ is the resulting column-vector of nodal displacements for the k-th design combination of loads of the serviceability limit states $\vec{z}_{SLS,k}=\mathrm{Z}_{FEM,k}^{SLS}\left(\vec{X}_G,\vec{X}_{CS}\right)$; N_{LCC}^{SLS} is the total number of design combinations of loads of the serviceability limit states. In this way, for each m-th node of the finite-element model, it is possible to calculate the design values of vertical and horizontal displacements depending on the column-vector of nodal displacements $\vec{z}_{SLS,k}$. At the same time, the displacements were calculated only for the design load combinations of the serviceability limit states.

The system of constraints of mathematical model (3) should contain strength and stability verifications, stated for all structural members of the structure, which is subject to the action of design load combinations of the ultimate limit states. In the case of parametric truss optimization, normal stresses verifications in the design sections of its elements are involved in the system of constraints, $\forall j=\overline{1,N_B}$, $\forall k=\overline{1,N_{LCC}^{ULS}}$:

$$\frac{N_{t,jk}\left(\vec{X}\right)}{A_j\left(\vec{X}_{CS}\right)\sigma_{t,ult}}-1\le 0; \quad (8)$$

$$\frac{N_{c,jk}\left(\vec{X}\right)}{A_j\left(\vec{X}_{CS}\right)\sigma_{c,ult}}-1\le 0, \quad (9)$$

where N_B is the total number of rod elements; $A_j\left(\vec{X}_{CS}\right)$ is the cross-sectional area of the j-th element of the truss structure; $\sigma_{t,ult}$ and $\sigma_{c,ult}$ are allowable normal tensile and compressive stresses, respectively; $N_{t,jk}\left(\vec{X}\right)$ and $N_{c,jk}\left(\vec{X}\right)$ are the axial internal tensile and compressive forces, respectively, occurred in the j-th elements of the truss structure under the action of the k-th design combination of loads of the ultimate limit states and are calculated using the finite element method (6). In the case of statically indeterminate truss structures, the axial forces in the elements of the structures depend on the unknown parameters of the structural geometric scheme $\vec{X}_G$ and the unknown dimensions of the cross sections of its elements $\vec{X}_{CS}$.

In the system of constraints (3) it is also necessary to involve stability verifications for the compressed truss structural members due to flexural and flexural-torsional buckling. Such verifications are formulated for all rod elements in which axial compression occurs under the action of the design load combinations of the ultimate limit states, $\forall j=\overline{1,N_B}$, $\forall k=\overline{1,N_{LCC}^{ULS}}$:

$$\frac{N_{c,jk}\left(\vec{X}\right)}{\varphi_{y,j}\left(\vec{X}_G,\vec{X}_{CS}\right)A_j\left(\vec{X}_{CS}\right)\sigma_{c,ult}}-1\le 0; \quad (10)$$

$$\frac{N_{c,jk}\left(\vec{X}\right)}{\varphi_{z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)A_j\left(\vec{X}_{CS}\right)\sigma_{c,ult}}-1\le 0; \quad (11)$$

$$\frac{N_{c,jk}\left(\vec{X}\right)}{\varphi_{c,j}\left(\vec{X}_G,\vec{X}_{CS}\right)A_j\left(\vec{X}_{CS}\right)\sigma_{c,ult}}-1\le 0, \quad (12)$$

where $\varphi_{y,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$ and $\varphi_{z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$ are buckling coefficients that correspond to flexural buckling relative to the main axes of inertia and are calculated depending on the design lengths $l_{ef,y,j}$, $l_{ef,z,j}$, the type, and geometric characteristics of the cross section for the j-th structural member; $\varphi_{c,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$ is the buckling coefficient, which corresponds to the flexural-torsional buckling of the structural member and is calculated depending on the design lengths $l_{ef,y,j}\left(\vec{X}_G\right)$, $l_{ef,z,j}\left(\vec{X}_G\right)$, $l_{ef,T,j}\left(\vec{X}_G\right)$, the type, and geometric characteristics of the cross section for the j-th structural member. Buckling coefficients $\varphi_{y,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$, $\varphi_{z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$ and $\varphi_{c,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$ depend both on the variable parameters of the geometric scheme of the structure $\vec{X}_G$, and the variable dimensions of the cross sections of its elements $\vec{X}_{CS}$.

In some cases, the system of stability constraints (10) to (12) can be simplified when only the Euler elastic buckling constraints for the rod elements are considered. Such constraints are formulated for all compressed structural member, in which axial compression occurs under the action of design load combinations of the ultimate limit states, $\forall j=\overline{1,N_B}$, $\forall k=\overline{1,N_{LCC}^{ULS}}$:

$$\frac{N_{c,jk}\left(\vec{X}\right)}{A_j\left(\vec{X}_{CS}\right)\sigma_{cr,\min,j}\left(\vec{X}_G,\vec{X}_{CS}\right)}-1\le 0, \quad (13)$$

where $\sigma_{cr,\min,j}$ – minimum critical stresses, which correspond to the Euler elastic buckling:

$$\sigma_{cr,\min,j}\left(\vec{X}_G,\vec{X}_{CS}\right)= =\min\left\{\sigma_{cr,y,j}\left(\vec{X}_G,\vec{X}_{CS}\right),\ \sigma_{cr,z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)\right\}; \quad (14)$$

$$\sigma_{cr,z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)=\frac{\pi^2 E\cdot i_{y,j}^2\left(\vec{X}_{CS}\right)}{l_{ef,y,j}^2\left(\vec{X}_G\right)}; \quad (15)$$

$$\sigma_{cr,z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)=\frac{\pi^2 E\cdot i_{z,j}^2\left(\vec{X}_{CS}\right)}{l_{ef,z,j}^2\left(\vec{X}_G\right)}, \quad (16)$$

where E is the modulus of elasticity of the structural material; $i_{y,j}\left(\vec{X}_{CS}\right)$ and $i_{z,j}\left(\vec{X}_{CS}\right)$ – radii of inertia of the cross-section of the j-th structural member relative to the main axes of inertia, calculated depending on the variable dimensions of this cross-section $\vec{X}_{CS}$.

In the case when the design lengths of the rod elements are the same in both main planes of inertia $l_{ef,j}\left(\vec{X}_G\right)=l_{ef,y,j}\left(\vec{X}_G\right)=l_{ef,z,j}\left(\vec{X}_G\right)$, the minimum critical stresses of elastic buckling according to Euler can be calculated as:

$$\sigma_{cr,\min,j}\left(\vec{X}_G,\vec{X}_{CS}\right)=\frac{\pi^2 E\cdot i_{\min,j}^2\left(\vec{X}_{CS}\right)}{l_{ef,j}^2\left(\vec{X}_G\right)}, \quad (17)$$

where $i_{\min,j}\left(\vec{X}_{CS}\right)$ is the minimum radius of inertia of the cross-section of the j-th rod element, calculated depending on the variable dimensions of this cross-section $\vec{X}_{CS}$. Taking into account (17), the stability constraint (13) in the form of the Euler elastic buckling constraint can be rewritten as:

$$\frac{N_{c,jk}\left(\vec{X}\right)l_{ef,j}^2\left(\vec{X}_G\right)}{\pi^2 E\cdot A_j\left(\vec{X}_{CS}\right)i_{\min,j}^2\left(\vec{X}_{CS}\right)}-1\le 0, \quad (18)$$

or

$$\frac{N_{c,jk}\left(\vec{X}\right)}{\kappa E}\cdot\frac{l_{ef,j}^2\left(\vec{X}_G\right)}{A_j^2\left(\vec{X}_{CS}\right)}-1\le 0, \quad (19)$$

where κ is a coefficient that depends on the type of cross section.

To the system of constraints (3), it is also necessary to include movement displacement constraints for certain nodes of the rod structure, which is subject to the action of design combinations of loads of the serviceability limit states, $\forall m=\overline{1,N_N}$, $\forall k=\overline{1,N_{LC}^{SLS}}$:

$$\frac{\delta_{x,mk}\left(\vec{X}\right)}{\delta_{ux,m}}-1\le 0; \quad (20)$$

$$\frac{\delta_{z,mk}\left(\vec{X}\right)}{\delta_{uz,m}}-1\le 0, \quad (21)$$

where $\delta_{x,mk}\left(\vec{X}\right)$ and $\delta_{z,mk}\left(\vec{X}\right)$ are, respectively, the horizontal and vertical displacements of the m-th node of the rod structure, which is subject to the action of the k-th design combination of loads of the serviceability limit states, calculated from the system of linear equations of the finite element method (7); $\delta_{ux,m}$ and $\delta_{uz,m}$ are allowable horizontal and vertical displacement of the m-th structural node; N_N is the total number of structural nodes.

Additional constraints that describe the structural, technological, and operational features of the truss structure under consideration can also be included to the system of constraints (3). In particular, using additional constraints of the mathematical model, it is possible to describe the requirements for the cross-sectional dimensions of the rod elements that are adjacent to the same structural node [12]. Such constraints can be represented in the form of constraints on the upper and lower limits for the design variables values, $\forall\iota=\overline{1,N_X}$:

$$1-\frac{X_\iota}{X_\iota^L}\le 0; \quad (22)$$

$$\frac{X_\iota}{X_\iota^U}-1\le 0, \quad (23)$$

where X_ι^L and X_ι^U are the lower and upper bounds for the value of the ι-th design variable X_ι.

The parametric optimization problem of truss structures, stated as a nonlinear programming problem (4), (5), (8) to (12) or (19) to (23), can be solved using the gradient projection method. The method of projecting the gradient of the objective function onto the surface of active constraints with the simultaneous elimination of residuals in the violated constraints provides an effective search for the optimal solution to the problem of nonlinear programming. This statement refers to the case when the objective functions and constraints of the mathematical model are continuously differentiable functions of the vector argument (design variables). The gradient method operates only with the first derivatives or gradients of the objective functions (5) and constraints (8) to (12) or (19) to (23) of the mathematical model. It is based on the iterative construction of such a sequence (24) of approximations of the design variables $\vec{X}=\left\{X_\iota\right\}^T$, $\iota=\overline{1,N_X}$, which ensures convergence to the optimum point $\vec{X}^*$ (to the optimal values of the variable (unknown) design parameters) [17]:

$$\vec{X}_{t+1}=\vec{X}_t+\Delta\vec{X}_t, \quad (24)$$

where $\vec{X}_t=\left\{X_\iota\right\}^T$, $\iota=\overline{1,N_X}$ – current approximation to the optimal solution $\vec{X}^*$; $\Delta\vec{X}_t=\left\{\Delta X_\iota\right\}^T$, $\iota=\overline{1,N_X}$ – the increment of the vector of design variables for the current approximation $\vec{X}_t$; t – iteration number.

An algorithm for solving the parametric optimization problem stated above for truss structures has been developed, which is given below.

Step 1. Determination of the initial design solution (set of design variables) and initial data for the optimization calculation.

A vector of design variables $\vec{X}_k=\left(\vec{X}_G,\ \vec{X}_{CS}\right)_k^T$, is determined, where k is the iteration index, k=0. The topology of the truss structure, the types of cross-sections of its elements, the types of conjugation of elements to the structural nodes, the support conditions for the truss structure, as well as the scheme and the magnitude of the external load are defined in advance and are constants during optimization.

The initial data for the optimization calculation of the truss structure are:

– design resistances of the structural material, namely the allowable stresses, taking into account the safety factor for the material;

– coefficients for determining the design lengths $l_{ef,y,j}$, $l_{ef,z,j}$ for all rod elements of the truss structure undergoing axial compression;

– allowable values of horizontal and vertical displacements $\delta_{ux,m}$ and $\delta_{uz,m}$ of the determined nodes of the considered truss structure;

– the lower $\vec{X}^L$ and upper $\vec{X}^U$ bounds for the design variables, as well as the determined objective function $f\left(\vec{X}_k\right)$.

Step 2. Calculation of the geometric and design lengths of the structural members of the truss structure.

The geometric lengths l_j of the rod elements of the truss structure are calculated based on the coordinates of its nodes, which depend on the unknown (variable) parameters of the structural geometric scheme $\vec{X}_G$. The design lengths $l_{ef,y,j}$, $l_{ef,z,j}$ of the structural members are calculated using the geometric lengths l_j and the initial data – coefficients of design lengths, which are assumed to be constant during the optimization process. The variation of the geometric lengths l_j and the corresponding design lengths $l_{ef,y,j}$, $l_{ef,z,j}$ in subsequent iterations is performed on the basis of the current values of the variable parameters $\vec{X}_G$ of the structural geometric scheme.

Step 3. Calculation of the dimensions and geometric characteristics of the design cross-sections of the structural members.

The geometric characteristics of the design cross-sections (areas, moment of inertia, elastic moments of resistance, radii of inertia, etc.) are calculated depending on the current values of the variable dimensions of the cross-sections $\vec{X}_{CS}$.

Step 4. Linear static analysis of the considered truss structure.

Using the finite element method (7), linear and angular displacements $\vec{z}_{SLS,k}$ are calculated for each m-th node of the finite-element model of the truss, which is subject to the action of the k-th combination of loads of the serviceability limit states. Depending on the column-vector of nodal displacements $\vec{z}_{SLS,k}$, the corresponding calculated horizontal $\delta_{x,mk}\left(\vec{X}\right)$ and vertical $\delta_{z,lk}\left(\vec{X}\right)$ displacements are determined. Using the finite element method (6), linear and angular displacements $\vec{z}_{ULS,k}$ are calculated for each i-th design cross-section of the j-th structural member of the truss, which is subject to the action of the k-th design combination of loads of the ultimate limit states. Depending on the column-vector of nodal displacements $\vec{z}_{ULS,k}$, the corresponding calculated values of internal forces $N_{t,jk}\left(\vec{X}\right)$ and $N_{c,jk}\left(\vec{X}\right)$ are determined.

Step 5. Calculation of state variables (normal stresses, buckling coefficients or critical buckling stresses, etc.).

The values of normal stresses $\sigma_{x,ijk}\left(\vec{X}\right)$ at a specified point of the cross-section are calculated depending on the axial force occurring in the i-th design cross-section of the j-th structural member of the truss, which is subject to the action of the k-th design combination of loads of the ultimate limit states. Buckling coefficients $\varphi_{y,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$, $\varphi_{z,j}\left(\vec{X}_G,\vec{X}_{CS}\right)$ are determined depending on the corresponding design lengths, types of cross-sections, and geometric characteristics of the cross-sections of the structural members of the truss in accordance with the design codes.

Step 6. Verification of constraints and construction of a set of numbers (indices) of active constraints [18].

The verification of the constraints-inequalities (8) to (12) or (19) is performed for all design load combinations of the ultimate limit states and all design sections of the structural members of the truss. In addition, a verification of the constraints-inequalities (20), (21) is performed for the nodes of the truss structure under the action of design load combinations of the serviceability limit states. Additional constraints (22), (23) on the upper and lower limits of the variation of the design variables values are also checked.

Step 7. Determination of the increment for the current vector of design variables and calculation of the best approximation to the optimal solution. The increment vector $\Delta\vec{X}_k$ for the current values of design variables $\vec{X}_k$ is calculated according to the resolving system of equations of the method of the objective function gradient projecting onto the surface of active constraints with the simultaneous elimination of residuals in the violated constraints, described in [18]. The improved approximation $\vec{X}_{k+1}$ to the optimal solution is determined by (24).

Step 8. Checking the stopping criteria for the iterative optimal point search procedure. If all the constraints of the mathematical model are satisfied with acceptable accuracy, and at least one of the stopping criteria described in [18] is met, then the transition is made to step 9. In the opposite case, it is necessary to return to step 1 at $k \leftarrow k+1$.

Step 9. Discretization of the optimal solution $\vec{X}_k$, obtained in the continuous space of design variables.

Step 10. Optimal values for the unknown parameters of the truss design – $\vec{X}_k$, the optimal value of the optimality criterion – $f\left(\vec{X}_k\right)$.

5. 2. Discretization procedure for the optimal solution obtained in a continuous space of variables

The problem of nonlinear programming, stated in the mixed space of design variables, was considered. In such problems, some design variables must take values from given finite sets of possible values. For example, if the cross-sectional dimensions of the rods are considered as variables, then, as a rule, they should correspond to certain assortments of profiles and steel sheets. If you find the optimal solution of this problem in a continuous search space, then after obtaining a continuous optimum, the question of its discretization will arise. The discretization of the optimal solution obtained in the continuous space of design variables can be performed by a purposeful selection of possible solutions to the problem of nonlinear programming around the point of the obtained continuous optimum.

Step 1. Let $\vec{X}^*$ be the optimal design solution of the structure obtained in the continuous space of design variables. At the same time, the set of design variables includes continuous and discrete design variables:

$$\vec{X}^* = \left\{\vec{X}_C^*, \vec{X}_D^*\right\}, \quad \iota = \overline{1, N_X}, \tag{25}$$

where $\vec{X}_C^*$ – continuous variables (for example, variable parameters of the geometric scheme of the truss structure):

$$\vec{X}_C^* = \left\{X_{C,\chi}^*\right\}^T, \quad \chi = \overline{1, N_{XC}}; \tag{26}$$

$\vec{X}_D^*$ – design variables that must vary according to a given finite set of options for possible values (for example, variable cross-sectional dimensions of structural elements):

$$\vec{X}_D^* = \left\{X_{D,\delta}^*\right\}^T, \quad \delta = \overline{1, N_{XD}}; \tag{27}$$

N_{XC} – total number of continuous design variables; N_{XD} is the total number of discrete design variables, $N_X = N_{XC} + N_{XD}$.

Step 2. For each discrete design variable X_i^*, $X_i^* \in \vec{X}_D^*$, which must vary according to some finite set of possible values $X_i=\{\chi_n\}$, two possible variants of its value $X_{i,D}^{*L} \in \mathbf{X}_i$ and $X_{i,D}^{*U} \in \mathbf{X}_i$ can be determined, such that:

$$\begin{cases} X_{i,D}^{*L} \le X_i^* \le X_{i,D}^{*U}; \\ X_i^* - X_{i,D}^{*L} \to \min; \\ X_{i,D}^{*D} - X_i^* \to \min; \end{cases} \qquad (28)$$

here $X_{i,D}^{*L}$ is the nearest smaller possible value of the variable X_i^* from the set $\mathbf{X}_i=\{\chi_n\}$ and $X_{i,D}^{*U}$ is the nearest larger possible value of variable X_i^* from this set.

Step 3. Among all the design variables $\vec{X}_D^* = \left\{X_{D,\delta}^*\right\}^T$, $\delta = \overline{1, N_{XD}}$, which must vary discretely, the one and X_p^*, $X_p^* \in \vec{X}_D^*$, is chosen for which the length of the gradient vector of the objective function is the largest $\forall\delta = \overline{1, N_{XD}}$:

$$\ell_{\nabla,p} = \max\left\{\frac{\partial f}{\partial X_{D,\delta}^*}\right\}, \qquad (29)$$

and its further discretization is performed at the level of the nearest smallest possible value from the set $\mathbf{X}_i=\{\chi_n\}$: $X_p^* \leftarrow X_{p,D}^{*L}$. After that, the p-th variable is removed from the set of design variables: $\vec{X}^* \leftarrow \vec{X}^* - \left\{X_p^*\right\}$, the length of the vector of design variables decreases accordingly: $N_X \leftarrow N_X - 1$. The total number of N_{XD} variables that must vary discretely also decreases accordingly: $N_{XD} \leftarrow N_{XD} - 1$.

Step 4. The search for the optimal solution is performed with a reduced vector of design variables $\vec{X}^*$. If at the same time (when $X_p^* \leftarrow X_{p,D}^{*L}$) there is no optimal solution, then the discretization of the selected variable must be performed at the level of the nearest larger value from the set $\mathbf{X}_i=\{\chi_n\}$: $X_p^* \leftarrow X_{p,D}^{*U}$ followed by the search for the optimal solution.

The third and fourth steps are performed until the design variables $\vec{X}_D^* = \left\{X_{D,\delta}^*\right\}^T$, $\delta = \overline{1, N_{XD}}$, which must vary according to the defined set of possible values, are completely fixed, that is, until $N_{XD}>0$.

5. 3. Results of optimization of cross-section sizes and parameters of the geometric scheme of the tower design

The methodology for solving the problem of parametric optimization of rod structures in the presence of continuous and discrete design variables, presented above, is implemented in the OptCAD software [14, 15], which is designed for solving a wide range of problems, in particular:

– linear static analysis of the rod structure;

– solution of a nonlinear programming problem, stated in an explicit form;

– verification the load-carrying capacity of structural members in accordance with the specified design codes;

– search for the values of the parameters of the rod structure, according to which the structure meets the specified requirements (requirements of the design codes and/or requirements of the designer);

– parametric optimization of rod structures according to criteria defined by the designer.

To evaluate the effectiveness of new methods, algorithms, and procedures, comparisons should be made with alternative methods, algorithms, and optimization procedures that are widely reported in the literature. It is advisable to perform such a comparison on verification examples used for testing new methods, algorithms, and procedures. The evaluation of the effectiveness of the proposed search methodology will be based on the comparison of the results of optimization calculations obtained when applying the proposed algorithm with the results reported in the literature. The initial data and the mathematical model of the optimization problem discussed below were taken similar to those given in the literature.

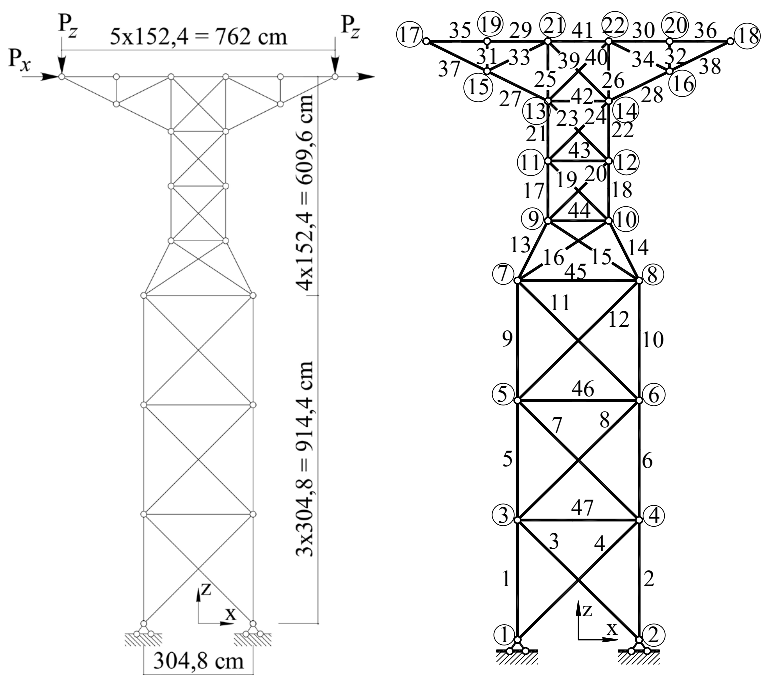
Fig. 2 shows the initial design solution for the construction of a plane 47-bar tower, which is designed for the action of several design combinations of loads. Initial data for the optimization calculation of the tower: material density – ρ=0.3 lb/inch^3=8303.97 kg/m^3, modulus of elasticity – E=30×10^4 ksi=2.068427×10^5 MPa. The absolute values of the allowable normal tensile and compressive stresses are $\sigma_{t,ult}$=20 ksi=137.895 MPa and $\sigma_{c,ult}$=15 ksi=103.42 MPa, respectively.

In the optimization process, three independent load combinations of the tower structure were considered, given below:

– combination I – concentrated force in the positive direction of the x–x axis P_x=6 kips=26.689 kN and concentrated force in the negative direction of the z–z axis P_z=–14 kips=–62.275 kN in nodes 17 and 18 (Fig. 3);

– combination II – concentrated force in the positive direction of the x–x axis P_x=6 kips=26.689 kN and concentrated force in the negative direction of the z–z axis P_x=–14 kips=–62.275 kN at node 17;

– combination III – concentrated force in the positive direction of the x–x axis P_x=6 kips=26.689 kN and concentrated force in the negative direction of the z–z axis P_z=–14 kips=–62.275 kN in the node (Table 1).



a *b*

Fig. 2. 47-bar truss structure of the tower: *a* – initial design solution and external load application scheme; *b* –design scheme with the numbers of nodes and rods

Table 1

Design load combinations of the tower structure

Load	Nodes	P_x	P_z
Load I	17, 18	26.689 kN	–62.275 kN
Load II	17	26.689 kN	–62.275 kN
Load III	18	26.689 kN	–62.275 kN

The parametric optimization problem of a 47-bar tower structure was stated as the problem of searching to the optimal values of the nodal coordinates of the tower and the optimal values of the cross-sectional areas of all its structural members. Minimization of the structural weight (5) was considered as the objective function.

During the optimization calculation, the *x* and *z* coordinates of the 15th, 16th, 17th, and 18th nodes (Fig. 2), as well as the *z* coordinates of the 1st and 2nd nodes, were taken as fixed. As design variables (4), the unknown (variable) areas of the cross-sections of the structural members of the tower were considered, as well as the unknown (variable) *x* and *z* coordinates for all unfixed nodes of the tower structure:

$$\vec{X}_{CS}=\{A_1,A_2,A_3,\ \dots A_{47}\};$$

$$\vec{X}_{G}=\{x_1,x_2,x_3,z_3,\ \dots x_{22},z_{22}\}.$$

In addition, the variation of the unknown cross-sectional areas $\vec{X}_{CS}=\{A_1,A_2,A_3,\ \dots A_{47}\}$ of structural members of the tower must be consistent with the defined numerical set of possible values: **A**={0.1, 0.2, ..., 4.9, 5.0)} in^2 or **A**={0.64516, 1.29032, ..., 31.61284, 32.258)} cm^2.

The lattice tower structure was designed to be symmetrical about the *z–z* axis. Accordingly, the vector of design variables was reduced using the following symmetry conditions:

$A_1=A_2$; $A_3=A_4$; $A_5=A_6$; $A_7=A_8$; $A_9=A_{10}$; $A_{11}=A_{12}$;

$A_{13}=A_{14}$; $A_{15}=A_{16}$; $A_{17}=A_{18}$; $A_{19}=A_{20}$; $A_{21}=A_{22}$;

$A_{23}=A_{24}$; $A_{25}=A_{26}$; $A_{27}=A_{28}$; $A_{29}=A_{30}$; $A_{31}=A_{32}$;

$A_{33}=A_{34}$; $A_{35}=A_{36}$; $A_{37}=A_{38}$; $A_{39}=A_{40}$;

$-x_1=x_2$; $z_1=z_2$; $-x_3=x_4$; $z_3=z_4$; $-x_5=x_6$; $z_5=z_6$; $-x_7=x_8$; $z_7=z_8$;

$-x_9=x_{10}$; $z_9=z_{10}$; $-x_{11}=x_{12}$; $z_{11}=z_{12}$; $-x_{13}=x_{14}$; $z_{13}=z_{14}$;

$-x_{19}=x_{20}$; $z_{19}=z_{20}$; $-x_{21}=x_{22}$; $z_{21}=z_{22}$.

The system of constraints of the mathematical model included strength conditions (8), (9), which were formulated for all structural members (*j*=1...47) of the tower structure, which was subjected to three independent combinations of loads (*k*=1...3). Buckling constraints in the form of elastic buckling according to Euler (19) with a coefficient of κ=3.96 were also involved in the system of constraints.

The dimensionality of the considered optimization problem was 44 design variables and 374 constraints.

The search for the optimal solution was initially implemented in the continuous space of design variables using the improved gradient method [17]. The obtained optimal design solution of the structure is shown in Fig. 3, *a*, and Fig. 4, *a*. As a result, the optimal project solution of the tower with a structural weight of 830,736 kg was obtained (6th column of Tables 2, 3). The continuous optimum point was characterized by the presence of 38 active constraints, namely:

– constraints on the lower limit of the variable cross-sectional area for the 43rd, 44th, 45th, 46th, and 47th bar elements;

– tensile strength constraints for the 30th, 34th, and 36th structural members under the action of the 1st combination of loads;

– tensile strength constraints for the 30th, 34th, and 36th structural members under the action of the 3rd combination of loads;

– compressive strength constraints for the 2nd, 6th, 10th, 14th, 28th, 32nd, 37th and 42nd structural members under the action of the 1st combination of loads;

– compressive strength constraints for the 37th structural members under the action of the 2nd combination of loads;

– compressive strength constraints for the 14th, 18th, 22nd, 28th and 32nd structural members under the action of the 3rd combination of loads;

– Euler buckling constraints in compression for the 16th, 23rd, 33rd, 44th and 46th structural members under the action of the 1st combination of loads;

– Euler buckling constraints in compression for the 3rd, 7th, 11th, 16th, 19th and 33rd structural members under the action of the 2nd combination of loads;

– Euler buckling constraints in compression for the 26th and 39th structural members under the action of the 3rd combination of loads.

The maximum residual in the violated constraint was 3.066×10^{-6} and was observed in the Euler buckling constraint for the 39th structural member under the action of the 3rd combination of loads.

The search for the optimal design solution was subsequently implemented in the mixed space of design variables using the improved gradient method [17] and the proposed discretization procedure. The obtained optimal design solution of the tower structure is shown in Fig. 3, *b*, and Fig. 4, *b*. As a result, the optimal project solution of the tower with the structural weight 835.403 kg was obtained (7th column of Tables 2, 3). The optimum point in the mixed space of variables was characterized by the presence of 23 active constraints, namely:

– constraints on the lower limit of variable cross-sectional areas for the 43rd, 44th, 45th, 46th and 47th structural members of the tower;

– tensile strength constraints for the 36th structural member under the action of the 1st load combination;

– tensile strength constraints for the 36th structural member under the action of the 3rd load combination;

– compressive strength constraints for the 2nd, 6th, 32nd and 42nd structural member under the action of the 1st load combination;

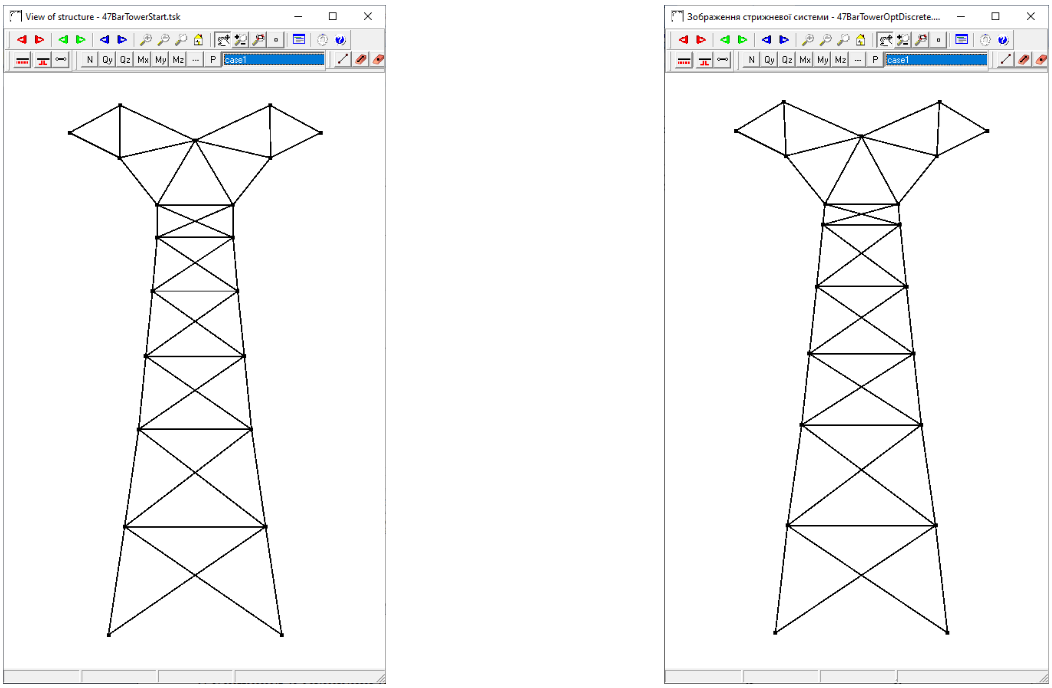
– compressive strength constraints for the 18th and 32nd structural members under the action of the 3rd load combination;

– Euler buckling constraints in compression for the 16th, 23rd, 33rd, 44th and 46th structural members under the action of the 1st load combination;

– Euler buckling constraints in compression for the 3rd, 7th, 11th, 19th and 33rd structural members under the action of the 2nd load combination.

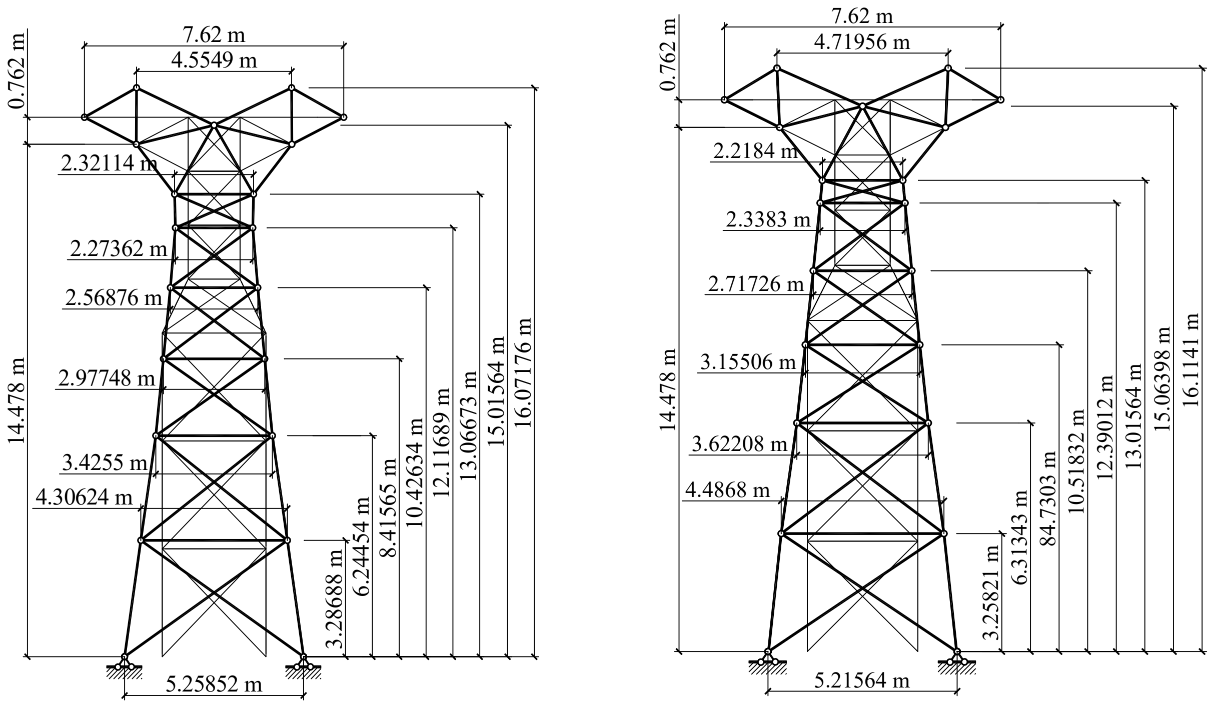
The maximum residual in the violated constraint was 1.061×10^{-6} and was observed in the Euler buckling constraint for the 44th structural member under the action of the 1st load combination.

The optimal solution to the considered problem in the continuous space of design variables (continuous optimum point) is characterized by the structural weight 830.736 kg. In the event that the space of design variables is mixed, that is, when along with continuous variables there are discrete design variables, the optimal solution will always be worse compared to the point of the continuous optimum. The application of the gradient method in combination with the proposed discretization procedure for solving the optimization problem in a mixed space of variables made it possible to obtain an optimal solution located closer to the point of a continuous optimum.



a *b*

Fig. 3. Optimal configuration of the 47-bar lattice tower structure (OptCAD: screenshot): *a* – in the continuous space of design variables; *b* – in the mixed space of design variables



a *b*

Fig. 4. Optimal values of the nodal coordinates of the 47-bar lattice tower structure, when the search for the optimum was performed: *a* – in the continuous space of design variables; *b* – in the mixed space of design variables

Table 2

Optimal values of the variable nodal coordinates (m) and cross-sectional areas (cm^2) of the elements of the 47-bar tower structure

Design variables	Work [19]	Work [20]	Work [21]	Work [5, 22]	This work	
					In the space of variables	
					Continuous	Mixed
A_1=A_2	20.19351	19.3548	20.6451	21.2903	20.88492	20.64512
A_3=A_4	7.225792	7.74192	7.09676	7.09676	6.92424	7.09676
A_5=A_6	19.7419	18.0645	19.3548	20.6451	20.03136	19.3548
A_7=A_8	6.709664	8.38708	7.74192	7.09676	6.84839	7.09676
A_9=A_{10}	18.45158	17.4193	18.7096	19.3548	18.78865	18.06448
A_{11}=A_{12}	5.935472	5.16128	5.16128	5.16128	5.65248	5.80644
A_{13}=A_{14}	16.70964	15.4838	16.129	17.4193	16.71602	16.129
A_{15}=A_{16}	5.419344	7.09676	6.4516	5.80644	5.55334	5.80644
A_{17}=A_{18}	16.83868	16.129	16.129	18.0645	16.84554	16.129
A_{19}=A_{20}	4.451604	4.51612	5.16128	4.51612	4.74041	5.16128
A_{21}=A_{22}	16.5161	14.1935	16.129	16.129	15.72144	16.129
A_{23}=A_{24}	5.16128	8.38708	4.51612	6.4516	5.78871	4.51612
A_{25}=A_{26}	4.967732	3.2258	4.51612	4.51612	4.54050	5.16128
A_{27}=A_{28}	11.03224	11.6129	11.6129	11.6129	11.08176	10.96772
A_{29}=A_{30}	6.645148	6.4516	5.80644	6.4516	5.76587	5.80644
A_{31}=A_{32}	6.258052	7.74192	8.38708	6.4516	7.04009	7.09676
A_{33}=A_{34}	2.322576	2.58064	2.58064	2.58064	1.93811	1.93548
A_{35}=A_{36}	6.4516	5.80644	5.80644	7.09676	5.98377	5.80644
A_{37}=A_{38}	8.645144	7.74192	7.74192	9.6774	7.95810	7.74192
A_{39}=A_{40}	7.032244	5.16128	5.80644	5.16128	5.45067	5.80644
A_{41}	5.677408	23.2258	4.51612	32.258	5.28297	5.16128
A_{42}	7.290308	8.38708	8.38708	7.09676	6.81529	7.74192
A_{43}	3.032252	0.64516	0.64516	0.64516	0.64516	0.64516
A_{44}	3.54838	0.64516	0.64516	0.64516	0.64516	0.64516
A_{45}	1.6129	0.64516	0.64516	0.64516	0.64516	0.64516
A_{46}	4.322572	0.64516	0.64516	0.64516	0.64516	0.64516
A_{47}	0.64516	1.29032	0.64516	0.64516	0.64516	0.64516
$-x_1$=x_2	2.737104	2.8956	2.6416	2.5537058	2.62926	2.60782
$-x_3$=x_4	2.26441	2.4638	2.2098	2.0581087	2.15312	2.24340
z_3=z_4	3.504692	3.175	3.2512	3.4848876	3.28688	3.25821
$-x_5$=x_6	1.69545	1.9304	1.778	1.6213684	1.71275	1.81104
z_5=z_6	6.463538	6.6294	6.5786	6.4562685	6.24454	6.31343
$-x_7$=x_8	1.457452	1.7526	1.5748	1.4260703	1.48874	1.57753
z_7=z_8	8.690864	8.0264	8.2804	8.3287616	8.41565	8.47303
$-x_9$=x_{10}	1.26619	1.4224	1.3462	1.2260783	1.28438	1.35863
z_9=z_{10}	10.596118	10.5156	10.4648	10.350835	10.42634	10.51832
$-x_{11}$=x_{12}	1.134364	1.27	1.1938	1.0781233	1.13681	1.16915
z_{11}=z_{12}	12.07389	11.7602	12.3444	11.908198	12.11689	12.39012
$-x_{13}$=x_{14}	1.043686	1.3716	1.143	1.1650777	1.16057	1.10920
z_{13}=z_{14}	13.03401	13.3096	12.8016	13.088384	13.06673	13.01564
$-x_{19}$=x_{20}	2.375916	2.5146	2.2606	2.0506715	2.27745	2.35978
z_{19}=z_{20}	15.848076	16.0274	16.1798	15.788053	16.07176	16.11410
$-x_{21}$=x_{22}	0.45466	0.0254	0.0508	2.54e-5	0.00001	0.00001
z_{21}=z_{22}	15.187168	14.9098	14.8336	14.908385	15.01564	15.06398
Weight, [kg]	861.826	873.524	848.990	848.161	830.736	835.403

Table 3

Optimal values of variable nodal coordinates (m) and cross-sectional areas (cm^2) of the elements of the 47-bar tower structure (continued)

Design variables	Work [23]	Work [24]	Work [9]	Work [25]	This work	
					In the space of variables	
					Continuous	Mixed
$A_1=A_2$	19.3548	20.0	21.29028	21.29028	20.88492	20.64512
$A_3=A_4$	7.74192	7.09676	7.09676	5.16128	6.92424	7.09676
$A_5=A_6$	18.06448	19.3548	21.29028	19.99996	20.03136	19.3548
$A_7=A_8$	8.38708	7.09676	5.80644	5.80644	6.84839	7.09676
$A_9=A_{10}$	17.41932	18.0645	19.3548	18.70964	18.78865	18.06448
$A_{11}=A_{12}$	5.16128	7.09676	6.4516	9.6774	5.65248	5.80644
$A_{13}=A_{14}$	15.48384	16.7742	17.41932	18.06448	16.71602	16.129
$A_{15}=A_{16}$	7.09676	5.80644	5.80644	2.58064	5.55334	5.80644
$A_{17}=A_{18}$	16.129	17.4193	16.77416	15.48384	16.84554	16.129
$A_{19}=A_{20}$	4.51612	4.51612	4.51612	7.09676	4.74041	5.16128
$A_{21}=A_{22}$	14.19352	16.7742	16.129	14.19352	15.72144	16.129
$A_{23}=A_{24}$	8.38708	5.16128	7.74192	7.74192	5.78871	4.51612
$A_{25}=A_{26}$	3.2258	5.16128	5.16128	3.87096	4.54050	5.16128
$A_{27}=A_{28}$	11.61288	10.9677	10.32256	10.96772	11.08176	10.96772
$A_{29}=A_{30}$	6.4516	6.4516	6.4516	6.4516	5.76587	5.80644
$A_{31}=A_{32}$	7.74192	6.4516	5.16128	5.80644	7.04009	7.09676
$A_{33}=A_{34}$	2.58064	1.93548	1.93548	1.93548	1.93811	1.93548
$A_{35}=A_{36}$	5.80644	6.4516	6.4516	7.74192	5.98377	5.80644
$A_{37}=A_{38}$	7.74192	8.38708	8.38708	10.32256	7.95810	7.74192
$A_{39}=A_{40}$	5.16128	5.80644	7.09676	5.16128	5.45067	5.80644
A_{41}	23.22576	5.80644	5.80644	6.4516	5.28297	5.16128
A_{42}	8.38708	7.74192	7.09676	8.38708	6.81529	7.74192
A_{43}	0.64516	0.64516	1.93548	0.64516	0.64516	0.64516
A_{44}	0.64516	0.64516	0.64516	3.87096	0.64516	0.64516
A_{45}	0.64516	0.64516	0.64516	0.64516	0.64516	0.64516
A_{46}	1.29032	0.64516	0.64516	0.64516	0.64516	0.64516
A_{47}	0.64516	0.64516	1.29032	3.87096	0.64516	0.64516
$-x_1=x_2$	2.8956	2.784094	2.5238202	2.63160002	2.62926	2.60782
$-x_3=x_4$	2.4638	2.3641812	2.11947506	2.07012032	2.15312	2.24340
$z_3=z_4$	3.175	3.21691	3.2101663	3.6335335	3.28688	3.25821
$-x_5=x_6$	1.9304	1.7971008	1.76567592	1.70222926	1.71275	1.81104
$z_5=z_6$	6.6294	6.256528	5.54231302	6.42230364	6.24454	6.31343
$-x_7=x_8$	1.7526	1.4267688	1.47321016	1.38481562	1.48874	1.57753
$z_7=z_8$	8.0264	9.049004	8.18457088	9.49992004	8.41565	8.47303
$-x_9=x_{10}$	1.4224	1.2318492	1.3055981	1.01149404	1.28438	1.35863
$z_9=z_{10}$	10.5156	11.083798	10.19969004	11.27623094	10.42634	10.51832
$-x_{11}=x_{12}$	1.27	1.076198	1.1902567	0.78606396	1.13681	1.16915
$z_{11}=z_{12}$	11.7602	12.462764	11.64087334	12.49665014	12.11689	12.39012
$-x_{13}=x_{14}$	1.3716	1.056894	1.1909679	0.93369638	1.16057	1.10920
$z_{13}=z_{14}$	13.3096	13.234416	13.4075805	12.954	13.06673	13.01564
$-x_{19}=x_{20}$	2.5146	2.4209248	2.49742706	1.97271894	2.27745	2.35978
$z_{19}=z_{20}$	16.0274	15.900146	2.49742706	15.74523394	16.07176	16.11410
$-x_{21}=x_{22}$	0.0254	0.03562604	0.41237916	0.44897802	0.00001	0.00001
$z_{21}=z_{22}$	14.9098	15.172944	15.51557984	15.21183394	15.01564	15.06398
Weight, kg	873.524	848.990	865.831	849.054	830.736	835.403

6. Discussion of the obtained results of the optimization calculation

The reliability of the obtained results of the optimization calculation (Tables 2, 3) is confirmed by the rigor and correctness of the mathematical model of the optimal design problem for the class of structures under consideration. In addition, the reliability of the obtained results is confirmed by the stability of the obtained numerical solutions in relation to the initial data and the analysis of the convergence of the iterative search process.

The considered problem of optimal design of a 47-bar tower structure was also solved in works [9, 19–25]. At the same time, various optimization methods and procedures were used (genetic algorithms and their modifications; Jaya algorithm; optimization method based on group search; particle swarm method and its modifications; teaching-learning based optimization algorithm, etc.). Tables 2, 3 show a comparison of the results of the optimization calculation of the 47-bar tower structure. As can be seen from the results, the optimal project solution of the tower, obtained using the proposed approach, is better compared to the optimization results reported in [9, 19–25]. Therefore, when the search for the optimum is performed in a mixed space of design variables, gradient methods in connection with the proposed discretization procedure provide a better result compared to the results obtained using metaheuristic optimization methods.

The discretization of the design variables, the variation of which must be performed according to the given numerical sets of possible values, was performed step by step. With each fixation of a discrete variable, residuals in the constraints accumulates in the structure (reserve of load-carrying capacity – strength, stability, etc.). After each fixation of some discrete variable, further variation of the continuous type variables (in the considered problem of variable parameters of the structural geometric scheme) made it possible to reduce these residuals to a minimum. This explains the best optimal solution obtained in the optimization problem of the 47-bar tower structure compared to the results obtained using metaheuristic optimization methods.

It should be noted that the presented methodology for optimizing the parameters of the geometric scheme and cross-sectional dimensions applies only to the case of truss structures. However, the proposed procedure of discretization of the optimal design solution, obtained in a continuous space of design variables, is universal and can be applied to other types of rod structures. Its only limitation is the need to differentiate the objective function according to (29), which requires the continuity of this function around the point of a continuous optimum.

The disadvantage of the proposed discretization procedure of the optimal design solution, obtained in a continuous space of design variables, is the need to perform numerical differentiation, which requires a significantly greater number of repeated static calculations of the structure. On the other hand, the proposed gradual discretization of the variables, the variation of which should be carried out according to the given numerical sets of possible values, significantly increases the number of iterations when searching for the optimum point. However, at today's stage of computer technology development, when applying technologies of parallel computing and multi-core processors, it can be seen that this shortcoming will not need to be eliminated.

The further development of our study may consist in expanding the mixed space of design variables by assigning binary variables, which during the search for the optimum point can take only binary values. This will make it possible to consider, in particular, the topology structural optimization problem.

7. Conclusion

1. We have stated the problem of finding the optimal parameters of the geometric scheme of the truss structures and the dimensions of the cross-sections of their elements. At the same time, the vector of design variables is of a mixed type, i.e., along with variables of a continuous type, there are variables whose variation must be performed according to a given set of possible values.

2. For the stated class of problems, it is proposed to search for the optimal solution in a continuous space of design variables. An algorithm for solving the problem of parametric optimization of rod structures has been developed using the method of projecting the objective function gradient onto the surface of active constraints with the simultaneous elimination of residuals in the violated constraints.

3. For design variables, the variation of which must be performed in accordance with a given set of possible values, a discretization procedure of the optimal design solution obtained in the continuous space of design variables has been developed. A feature of the proposed methodology is the step-by-step discretization of variables, the order of which is determined by the components of the gradient vector of the objective function.

4. The comparison of the results of the optimization calculations reported in the current paper proved that in the case of a mixed space of variables, gradient methods in combination with the proposed discretization method provide a better optimal solution – located closer to the point of a continuous optimum. At the same time, the obtained optimum was compared with the results reported by other authors when applying various metaheuristic algorithms. On the considered problem of parametric optimization of a 47-bar tower structure, a design solution with a weight of 835,403 kg was obtained, which is 1.53...4.6 % better than the optimal solutions obtained by other authors.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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