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PROBLEMS OF CALCULATION AND FOUNDATION OF MATHEMATICS

Problems of calculation and foundation of mathematics are discussed. Historical analysis of these questions is represented. Three basic concepts of foundation of mathematics (intuitionistic, logical and formal) are observed. Its shown that these concepts can't be theories of foundation of mathematics. Knowledge and science are open systems; therefore theory of its optimal formalizations must be open. This theory must include calculation and measure. Polymetric analysis is represented as example of this theory.

Key words: calculation, foundation of mathematics, open systems, polymetric analysis, functional numbers, hybrid theory of systems.

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ПРОБЛЕМИ ОБЧИСЛЕННЯ ТА ОСНОВ МАТЕМАТИКИ

Обговорюються проблеми обчислення та основ математики. Наведений історичний аналіз цих питань. Досліджено три основні концепції основ математики (інтуїціоністська, логічна та формальна). Показано, що ці концепції не можуть бути основами математики. Наука та знання є відкритими системами, тому теорія їх оптимальної формалізації має бути також відкритою. Ця теорія повинна включати обчислення та міру. Поліметричний аналіз наведено як приклад такої теорії.

Ключові слова: обчислення, основи математики, відкриті системи, поліметричний аналіз, функціональні числа, гібридна теорія систем.

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ПРОБЛЕМЫ ВЫЧИСЛЕНИЯ И ОСНОВАНИЙ МАТЕМАТИКИ

Обсуждаются проблемы вычисления и оснований математики. Проведен исторический анализ этих вопросов. Исследованы три основные концепции оснований математики (интуиционистская, логическая и формальная). Показано, что эти концепции не могут быть основаниями математики. Наука и знание – открытые системы, поэтому теория их оптимальной формализации должна быть также открытой. Эта теория должна включать вычисления и меру. Полиметрический анализ приведен в качестве примера такой системы.

Ключевые слова: вычисления, основания математики, открытые системы, полиметрический анализ, функциональные числа, гибридная теория систем.

Introduction

The problems of calculation and foundation of mathematics have long history [1 – 3]. This problem is connected with problem simplicity-complexity, which is central problem for each science. Roughly speaking it is the problem of optimal formalization of knowledge. This problem is beginning from famous Archimedes phrase: "Give me fulcrum and I'll reverse Universe" [1].

Therefore this problem is basic for the creation new science. It was used by Aristotle, Descartes, Newton and other researches [1]. But each science must be optimal system. This problem must be beginning from the Newtonian four rules of conclusions in the physics [1]:

Rule 1. Do not require natural reasons than those that are true and sufficient to explain the phenomena.

Rule 2. Therefore, as far as possible, the same reason we should attribute displays the same kind of nature.

Rule 3. Such properties of bodies which can not be either amplified or weaken and are all bodies over which you can do the test, must properties considered for all bodies in general case.

Rule 4. In experimental philosophy, propositions derived from phenomena through a common induction must be considered for accurate or approximately correct, despite the possibility of opposing their hypothesis until there are phenomena that are more accurate or are found to be invalid.

Determination of mathematics with numerical (measured) point of view was formulated by Ch. Volf [1]: "Mathematics is the science to measure everything that can be measured. Of course it is described as the science of numbers, the science of value, ie the things that can increase or decrease. Since all finite things can be measured in all that they have a finite, that is what they are, then nothing is, what can not be applied math, and because you can not have any more precise knowledge than when the properties of things can be measured, the math leads us to the most perfect knowledge of all possible things in the world".

This same concept of measured value was formulated by "king" of mathematics of XVIII-th century, L.Eyler [1]: "First value called everything that can increase or decrease, or something that you can add anything or subtract anything which can be ... There are many different kinds of values that can not be account, and from them come the various branches of mathematics, each of which has to do with their native values. However, it is impossible to determine whether a measured value except as known to take as another value of the same type and specify the ratio in which it is to her."

In determining whether the measurement values of any kind so we come to that first established some known value of the same kind, called the measure or unit and depends solely on our choice. Then determined in what respect is the value corresponding to the extent that it is always expressed in numbers. Thus measurement is no more than an attitude, which is one size to another, taken as a unit. Mathematics in general case is nothing but the science of the quantities involved finding ways to measure past.

But in XX century the three concepts of foundation of mathematics were observed. Logical concept (B. Russel and A. N. Whitehead) is based on logical formalization of knowledge; formal concept is based on axiomatic aspects of formalization and third, intuitionist, is based on constructive (analytic) aspects of mathematics [1 – 3]. First two concepts are based on closed theories and third concept is based only in analysis and therefore it isn't optimal. One of founder of logic concept A. N. Whitehead refused from his concept and express an opinion about "organism" concept of mathematics [4].

The theory of foundation of mathematics must be based on its nature: analysis, synthesis and formalization of whatever chapter of knowledge. All three concept of foundation of mathematics aren't corresponded to its nature in toto.

Therefore we must again to address to Pythagor, Newton, Volf and Euler. This problem now is one of central in modern cybernetics and computer science.

But it connected with problem of complexity, which is one of central problem in modern mathematics and cybernetics [1, 5 – 7]. This problem is caused in synthetically sciences. Roughly speaking it has two aspects: system (problem of century in cybernetics according S. Beer[1, 4 – 6]) and computational (problem of computational complexity[1, 5 – 7]. Last problem is included in basic problems of modern mathematics (Smale problems) [1, 5 – 7].

As variant of resolution system aspect of problem complexity in cybernetics may be problem simplicity – complexity, which is included in Polymetric Analysis (PA) (universal system of analysis, synthesis and formalization of knowledge) as principle simplicity [1]. Basic elements of this theory – functional numbers is generalizing of quadratic forms.

Hybrid theory of systems (HTS) as element of PA is created on the basis principles (criteria) of reciprocity and simplicity [1, 5 – 7]. Only 10 minimal types of formalization system may be used. But number of real systems may be infinite. These systems are differed by step of its complexity. It is may be represented as answer on the one of basic question of modern theory of systems [1, 5 – 7] about possible number of systems and its classification with point of simplicity – complexity[1, 5 – 7].

Therefore HTS may be represented as variant of resolution the problem of century in cybernetics according S. Beer and may be used for the resolution problem of computational complexity (theory of informative calculations, TIC) [1, 5 – 7].

Theory of informative calculations may be represented as variant of resolution of computational complexity[1, 5 – 7].

Basic results and discussions

Modern science is the realization of the R. Bacon – Descartian concept "Science is so science, how many mathematics is in her" [1]. Development of modern science practically isn't possible without computers. Basic motto of modern computing scince according A. Ershov is formalization of Canadian philosopher phrase "Everything, which go from head, is reasonable" [1]. Therefore we must created theory, which is included these peculiarities of modern science. This theory must include calculations in more widely sense, including measure and transformations, procedures of measurements and estimation, structures of modern knowledge and science and perspectives of its development. It must be theory of open system.

Polymetric analysis (PA) was created as alternative optimal concept to logical, formal and constructive conceptions of modern mathematics and theory of information [1]. This concept is based on the idea of triple minimum: mathematical, methodological and concrete scientific (Fig.1) [1].

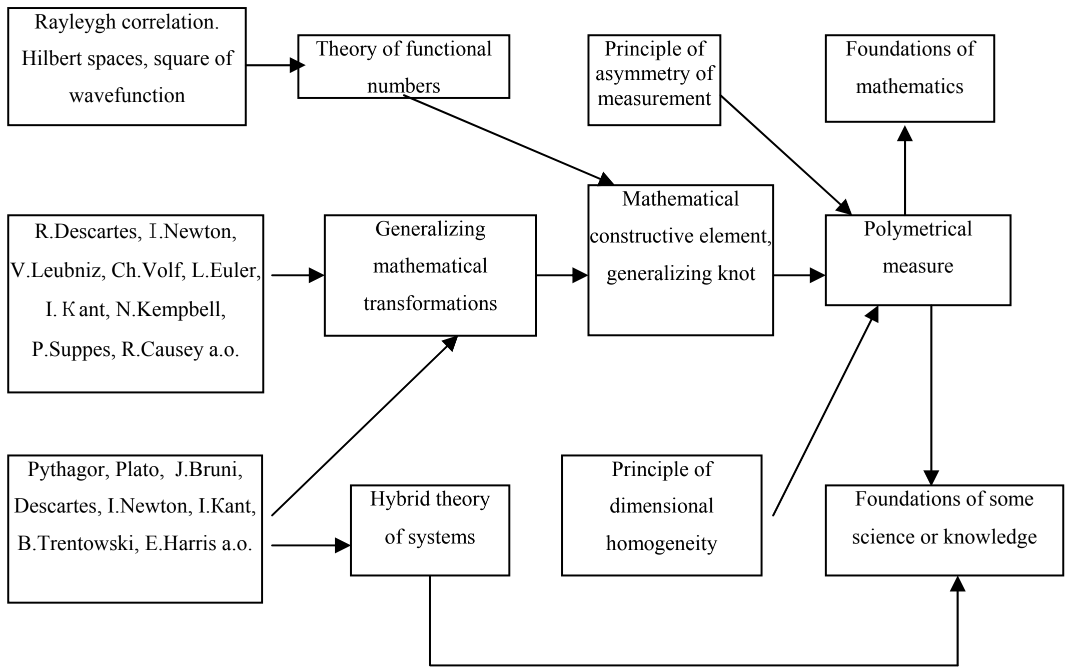

Fig. 1. Schema of polymetric method and its place in modern science [1].

The polymetric analysis may be represented as universal theory of synthesis in Descartian sense [1]. For resolution of this problem we must select basic notions and concepts, which are corresponded to PA. The universal simple value is unit symbol, but this symbol must be connected with calculation. Therefore it must be number. For the compositions of these symbols (numbers) in one system we must use system control and operations (mathematical operations or transformations). After this procedure we received the proper measure, which is corresponding system of knowledge and science.

Basic elements of PA were called the generalizing mathematical elements or its various presentations – informative knots [1, 5 – 7]. Generalizing mathematical element ${}^{stqo}_{nmab}M_{ijkp}$ is the composition of functional numbers (generalizing quadratic forms, including complex numbers and functions) and generalizing mathematical transformations, which are acted on these functional numbers in whole or its elements [1]. Roughly speaking these elements are elements of functional matrixes.

This element ${}^{stqo}_{nmab}M_{ijkp}$ may be represented in next form

$$ {}^{stqo}_{nmab}M_{ijkp} = A_i\bar{A}_jO_k\bar{O}_pA^r_s\bar{A}^r_tO^r_q\bar{O}^r_oA^l_n\bar{A}^l_mO^l_a\bar{O}^l_bN_{\phi_{ij}} \; . \tag{1} $$

Where $N_{\varphi_{ij}}$ – functional number; $O_k, O^r_q, O^l_a, \bar{O}_p, \bar{O}^r_o, \bar{O}^l_b$; $A_i, A^r_s, A^l_n, \bar{A}_j, \bar{A}^r_t, \bar{A}^l_m$ are quantitative and qualitative transformations, straight and inverse (with tilde), (r) – right and (l) – left..

We have only 15 minimal types of mathematical transformations, only six are mathematicval in classic sense.

Polyfunctional matrix, which is constructed on elements (1) is called informative lattice. For this case generalizing mathematical element was called knot of informative lattice[1, 5 – 7]. Informative lattice is basic set of theory of informative calculations. This theory was constructed analogously to the analytical mechanics [1]. This analogy is result of application de Broglie formula [8]

$$ \frac{S_e}{k_B} = \frac{S_a}{\hbar}, \tag{2} $$

which was obtained from the analysis of thermodynamics point [1], a measure of disordered physical information (number of photons) equally disorder structured information (where S_a – action; $\hbar$ – Planck constant, S_e – entropy, k_B – Boltzmann constant). Through (2) the ratio of the increase of entropy for nonequilibrium processes (open systems) can be expanded at the action, that is, in other words, the physics of open systems can be built and action functional .

Basic elements of this theory are [1, 5 – 7]:

1. Informative computability C is number of possible mathematical operations, which are required for the resolution of proper problem.

2. Technical informative computability $C_t = C\sum t_i$, where t_i – realization time of proper computation.

3. Generalizing technical informative computability $C_{t0} = k_{ac}C_t$, where k_{ac} – a coefficient of algorithmic complexity [1].

Basic principle of this theory is **the principle of optimal informative calculations**[1, 4 – 6]: any algebraic, including constructive, informative problem has optimal resolution for minimum informative computability *C,* technical informative computability C_t or generalizing technical informative computability C_{to}.

For classification the computations on informative lattices hybrid theory of systems was created [1]. This theory allow to analyze proper system with point of view of its complexity,

The basic principles of hybrid theory of systems are next: 1) **the criterion of reciprocity;** 2) **the criterion of simplicity**.

The criterion of reciprocity is the principle of the creation the corresponding mathematical constructive system (informative lattice). The criterion of simplicity is the principle the optimization of this creation.

The basic axiomatic of hybrid theory of systems is represented below.

The set of functional numbers and generalizing transformations together with principles reciprocity and simplicity (in more narrow sense the criterion of the reciprocity and principle of optimal informative calculations) may be represented with help **the hybrid theory of systems (HTS)**.

Criterion of the reciprocity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;
2) the equilibrium;
3) the equality of the number epistemological equivalent known and unknown notions.

Criterion of the simplicity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;
2) the equilibrium;
3) the principle of the optimal calculative transformations.

For more full formalization the all famous regions of knowledge and science the **parameter of connectedness** $\boldsymbol{\sigma_t}$ was introduced. This parameter is meant the number of different bounds the one element of mathematical construction with other elements of this construction. For example, in classic mathematics $\sigma_t = 1$, in linguistics and semiotics $\sigma_t > 1$. The parameter of connectedness is the basic element for synthesis in one system of formalization the all famous regions of knowledge and science.

Thus we can receive next 10 types of hybrid systems [1, 5 – 7]:

1. The system with conservation all positions the criteria of reciprocity and simplicity for all elements of mathematical construction ($N_{\varphi_{ij}}$ and transformations) is called the *simple system.*

2. The system with conservation the criterion of simplicity only for $N_{\varphi_{ij}}$ is called the *parametric simple system.*

Remark 1. Further in this classification reminder of criteria of reciprocity and simplicity is absented. It mean that these criteria for next types of hybrid systems are true.

3. The system with conservation the criterion of simplicity only for general mathematical transformations is called *functional simple system.*

4. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t = 1$ is called the *semisimple system.*

5. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ and with $\sigma_t = 1$ is called the *parametric semisimple system.*

6. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t = 1$ is called the *functional semisimple system.*

7. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t \neq 1$ is called *complicated system.*

8. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ is called *parametric complicated system.*

9. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t \neq 1$ is called *functional complicated system*.

10. The system with nonconservation the criteria of reciprocity and simplicity and with $\sigma_t \neq 1$ is called *absolute complicated system*.

With taking into account 15 basic types of generalized mathematical transformations we have 150 types of hybrid systems; practically 150 types of the formalization and modeling of knowledge and science [1].

Only first four types of hybrid systems may be considered as mathematical, last four types are not mathematically. Therefore HTS may be describing all possible system of knowledge. Problem of verbal and nonverbal systems of knowledge is controlled with help of types the mathematical transformations and parameter connectedness [1].

HTS allows classifying all possible knowledge by step simplicity – complexity. It may be used for the representation evolution of development of concrete science from complexity to simplicity.

Only first four types of hybrid systems may be considered as mathematical, last four types are not mathematically. Therefore HTS may be describing all possible system of knowledge. Problem of verbal and nonverbal systems of knowledge is controlled with help of types the mathematical transformations and parameter connectedness [1, 5 – 7].

HTS may be used for the classification and creation old and new chapters of all science, including computing science.

HTS may be used for the represented of evolution of systems in two directions: 1) from simple system to complex system (example, from classic to quantum mechanics) and 2) conversely, from complex system to simple system (example, from formal logic to mathematical logic) [1].

Hybrid theory of systems is open theory. Parameters of openness are number of generalizing mathematical transformations and parameter of connectedness. Thereby we have finite number of types of systems, but number of systems may be infinite. Hybrid theory of systems allows considering verbal and nonverbal knowledge with one point of view [[1, 5 – 7].

HTS may be represented as application PA (HTS) to the problem of calculation [1, 4 – 6]. This theory was used for the problem of matrix computation and problem of arrays sorting [1].

PA may be represented as natural concept of foundation of mathematics. It may be represented as formalization Pythagorean phrase "Numbers rules of the World", Plato concept of three types of numbers: arithmetical (pure mathematics); sensitive (applied mathematics) and ideal (numerology), and variant resolution S. Beer problem of complexity in more widely sense [1].

Conclusions

1. We show, that problem of calculation is the central problem of mathematics and therefore must be represented as basis of foundation of mathematics.
2. Short analysis of three basic concepts of foundation of mathematics is represented.
3. Basic requirements to theories of foundation of mathematics and foundation of science are formulated.
4. Necessity transition to natural ("calculated") concept of foundation of mathematics is shown.
5. The polymetric analysis is based on the functional expansion of number and calculations as elements of measure may be represented as universal system of analysis, synthesis and formalization of knowledge and as possible realization of natural concept of foundation of mathematics.
6. It may be interpreted as foundations of mathematics, theoretical foundation of computing science and foundation of all sciences and knowledge.

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