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A GATING ALGORITHM WITH REDUCED COMPUTATIONAL COMPLEXITY FOR LINEAR KALMAN FILTERS IN EMBEDDED SYSTEMS

Purpose. Kalman-based filters are the most commonly used algorithms of processing noisy signals from real sensors in different application areas. However, they have no intrinsic means of fighting off outliers, which can deteriorate filter accuracy drastically. Existing outlier rejection and accommodation techniques are computationally expensive for embedded systems. Our goal is to develop a multiple-step gating procedure suitable for implementation in firmware.

Research methods. Primarily the work is based upon statistical and probabilistic methods. For deduction, representation and corroboration of principal ideas it utilizes also linear algebra and analytic geometry. Verification of the developed algorithm has been performed using simulation techniques including the Monte-Carlo method.

Scientific novelty. A new gating algorithm has been developed, verified and presented in detail. In contrast to the well-known combination of the rectangular and elliptic gates, the proposed algorithm assumes an additional step which allows rejection of points not lying between the tangents of tilted confidence ellipses parallel to their major axes. The step helps avoid computing inverse covariance matrices required by the Mahalanobis distance due to the fact that it recognizes most outliers that are able to pass the rectangular gate, which makes invocation of the elliptic gate redundant.

Practical value. The proposed algorithm has been implemented and verified both theoretically and practically upon a variety of the multivariate linear Kalman filters and normally distributed random numbers for simulating outliers. Implementation has been done using the stm32f407vg microcontroller by STMicroelectronics. Simulation results have corroborated that the proposed interim step between application of the rectangular and elliptic gates reduces computational complexity of the whole gating procedure without any losses in accuracy.

Results. The developed algorithm is integrated into a complex hardware-software tool intended for verification, estimation and investigation into micro-electromechanical inertial sensors.

Key words: gating, outlier, outlier rejection, Kalman filter, Gaussian distribution, confidence ellipse, tangent

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Problem and its connection with scientific and practical tasks. The Kalman filter is an algorithm that has been a great success since its first appearance in 1960. It is widely used in a range of applications including navigation. The linear Kalman filter is said to be optimal in the least squares sense if a problem being solved is linear or its non-linearity is insignificant and can be adequately described by process noise, provided that process and measurement uncertainty is modeled as Gaussian [1, 2]. The first Kalman-based filter able to deal with non-linear problems was the extended Kalman filter (EKF) whose main idea is to linearize the problem in discrete points and apply the linear Kalman filter to solve the linearized problem. The EKF assumes forming Jacobian matrices, thus its main difficulty consists in the fact that the closed-form solution to the problem may be non-existent or highly difficult and non-trivial to compute. In this case numerical methods should be used which may be intractable for the available computational resources, especially if the filter is supposed to be executed in an embedded system. Less complex Kalman-based filters include the unscented Kalman filter (UKF) and numerous particle filters. The UKF is based on the linear Kalman filter [3].

The cornerstone of the linear Kalman filter is summation and multiplication of Gaussians. I.e., the uncertainty in the process model and measurements are expected to be Gaussian. If the condition is not fulfilled, the filter quickly diverges. The filter can only provide trustworthy results if it has been fed an adequate process model and measurement noise. If the process noise covariance matrix is large in comparison with the measurement noise covariance matrix, the filter will closely follow measurements. It may be that outliers occasionally happen amongst generally accurate measurements. The Kalman filter has no intrinsic means of distinguishing between inliers and outliers. An outlier is a measurement that disobeys the general pattern and does not follow the statistical distribution of the bulk of the data. Outliers result from some disorder or unexpected conditions in the system, such as gross measurement, sampling, computing or recording errors, transient malfunctioning, noise, missing data, human errors, etc [4]. Another popular definition – an outlier is a measurement that markedly deviates from its expected value. A procedure that enables exclusion of unlikely measurements and/or combination of measurements is called gating.

There exist different gating techniques. Some of them are simple, easy-to-compute but occasionally mistake outliers for normal measurements. Others, on the contrary, are accurate but computationally expensive. Two-step gating is usually taken in order to reject the majority of bad measurements coarsely and then refine the results by a more computationally demanding but accurate filter. The work is aimed at development a gating procedure with reduced resource consumption, primarily for use in embedded systems.

Related work. Two main approaches to dealing with outliers are used concurrently – rejection and accommodation. Rejection assumes that if a measurement is considered suspicious, it should be discarded. In this case a prediction step is not followed by a corresponding update step. Instead, a new prediction should be made. There still remains some probability that several outliers will occur in a row. However, it is supposed that even several measurement innovation steps skipped sequentially would not impede the filter performance noticeably. Accommodation means that any measurement will be taken into account, even if it seems erroneous. However, less trustworthy measurements are assigned smaller weights. Hence, they do not impact the filter in the same way as measurements that seem accurate do. Both approaches have pros and cons. The main idea that backs up rejection is that all measurements fall into two groups: “normal” (fitting the measurement model) and “abnormal” (generated by a completely different model) [5]. However, the measurement model may not be accurate enough, which would lead to rejection of normal measurements. Moreover, [5] provides research results showing that there is a small but dangerous possibility that normal measurements are discarded when spurious ones are accepted due to overlapping outlier and measurement-noise distributions and inaccurate modeling. Outlier identification methods suffer from masking and swamping effects. Masking refers to mistaking an actual outlier for a normal measurement. Swamping is mistaking a legitimate measurement for an outlier. The effects are complimentary, so a trade-off should be found.

One of the ways to enhance the linear Kalman filter immunity to outliers is to model measurement noise by distributions other than Gaussian, and a good deal of research efforts are concentrated in this field. To name a few, heavy-tailed Gaussian mixture and t-distributed noise models are used to modify the Kalman filter for better robustness [6]. In [7], outliers are considered results of measurement noise with variable covariances. Under assumption that the noise covariances obey an inverse Gamma distribution, the authors propose to add a procedure of adaptive identification of the key parameters involved in the inverse Gamma distribution to the Kalman filter. Within the accommodation approach one should mention [8-10] and [11]. Some researchers consider outliers as inputs added to normal measurements and estimate them together with the state [12, 13].

Under assumption of nearly Gaussian distributions, the 3σ rule is widely applied for outlier identification. The probability of observing a measurement that is more than three standard deviations away from its expected value is only about 0.3%. However, in practice, one needs to consider 5σ or 6σ to make for the fact that the actual distribution differs from Gaussian, as it is stated in [14]. The presence of outliers in the dataset biases the calculated mean and standard deviation and thus impairs the outlier identification procedure. Since particle filters, which are gaining popularity recently, are not restricted to Gaussian distributions, many researchers [14] do not use the 3σ rule.

Outlier rejection is often performed using a prior or a median window. Median windows consider a set of measurements and discard those far from the median. Gating with a prior assumes that there is a predicted measurement value, which is compared with a measurement. If the two are not in good agreement, the measurement is considered an outlier. The simplest way of comparing the predicted state with the measured one is to consider each state variable separately. Thus, one may choose to check whether the difference between the predicted and measured variables exceeds 5σ or not. This approach is called rectangular gating. It is easy to implement, however it is rather coarse because it does not take into account the covariance of the state variables. Fig. 1 illustrates this drawback of rectangular gating for the case if the state vector contains two variables (these can be the distance and velocity, for instance). Correlated variables in 2D are represented by ellipses. For highly correlated variables the rectangular gate is remarkably coarse. According to [14] in 5 dimensions it is twice as likely to accept a bad measurement as the hyper-ellipsoid.

In order to take the covariance into account, the predicted and actual measurements are often compared using the Mahalanobis distance. Essentially, the Mahalanobis distance calculates the scalar standard deviation distance from a point to a distribution. If the covariance matrix is the identity matrix, the Mahalanobis distance comes down to the commonly used Euclidean distance.

Accuracy in outlier detection comes at cost of computational load, since one should calculate the inverse covariance matrix for each measurement innovation. Obviously, the more variables are being tracked in the state vector, the more computationally expensive the problem will be. The problem gets intractable for embedded systems with their limited computation resources and poor floating-point logic. There have been research works aimed at reduction of computational complexity of elliptic gates without impairing masking and swamping. An intuitive approach is to filter twice. The first time a new measurement should be tested coarsely with the rectangular filter. If it managed to pass through the gate, the more precise elliptic gate is applied. Thus, the inverse covariance matrix will be computed not for all measurements but only for those that have not been rejected by the rectangular filter, which saves the computational resources drastically.

In [15] sorting algorithms were used to accelerate the gating procedure and to lower computational complexity primarily for multi-hypothesis tracking problems. Besides rectangular and elliptic gates, other shapes are also used, depending on the nature of an object being tracked. For example, a fighter jet would have a maneuver gate whose shape resembles a cone projecting in front of the current travel direction [14]. The literature overview shows that there still exist ways of further improving gating.

Problem statement. The work is aimed at development and verification of a new gating procedure suitable for implementation in embedded systems.

Main statements and results. The underlying idea of the work is best illustrated for a 2D tracking problem, which means that the state vector is comprised of two variables and the gate can be represented by an ellipse. We select a point (x, y) lying on the ellipse and find its tangent in this point, given by its linear equation $ax + by = d$. The tangent divides the plane into two semi-planes. Due to the fact than an ellipse is a convex figure, one semi-plane contains all the points belonging to the ellipse while another incorporates none of the ellipse points. Thus, it is sufficient to take any of the points belonging to the ellipse except (x, y), substitute its coordinates into $ax + by – d$ and check the sign. The same sign holds for any point of the same ellipse (except (x, y)).

By this check all the points confined between the rectangle and the tangent (Fig. 1) can be excluded during gating. It is most expedient to select the center of the ellipse as a reference point, because its coordinates have been already calculated.

It is convenient to analyze multivariate Gaussian distributions in a pair-wise manner, i.e., to represent the iso-contour for any two variables as an ellipse (called error ellipse, or confidence ellipse). If the covariance matrix has non-zero off-diagonal elements, the error ellipse is tilted, i.e., its axes are not parallel to the coordinate axes. The stronger the covariance, the more tilted the error ellipse. The sum of squared Gaussian data points follows Chi-Square distribution. One can use probability tables to find out which scale s of the error ellipse corresponds to 99% confidence.

The axes of the error ellipse have the lengths $2\sqrt{s\lambda_1}$ and $2\sqrt{s\lambda_2}$, where λ_1 and λ_2 are the eigenvalues of the covariance matrix or, more precisely, the sub-matrix that corresponds to the two variables being currently analyzed. The tilt angle θ can be found as $\theta = \arctan(v_1(y)/v_1(x))$, where v_1 is the eigenvector corresponding to the largest eigenvalue (Fig. 1).

As one can see, both tangents going through points A and B are parallel to the major axis of the ellipse. The major axis goes through the center of the ellipse (μ_x, μ_y) and forms angle θ with the horizontal coordinate axis. Thus the line equation can be uniquely defined as

$$y = x \cdot tg\theta - \mu_x tg\theta + \mu_y .$$

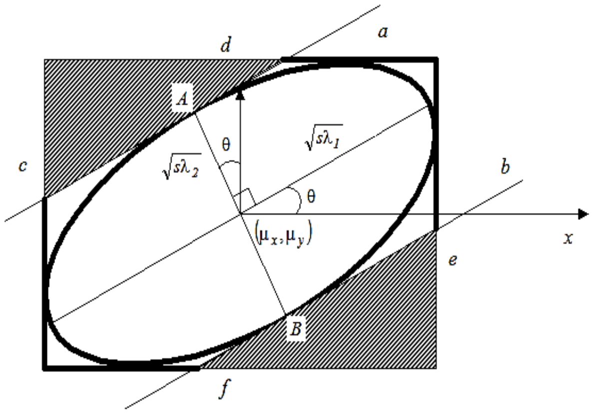

Fig. 1. Error ellipse and illustration of the gating complexity reduction idea

Both tangents can be found as

$$y = x \cdot tg\theta - \mu_x tg\theta + \mu_y + b,$$

$$y = x \cdot tg\theta - \mu_x tg\theta + \mu_y - b, \qquad (1)$$

where b is $\sqrt{s\lambda_2}\,/\cos\theta$.

As shown in Fig. 1, an ellipse can be confined by a hexagon formed by two tangents (*a* and *b*) and the sides of the rectangle (*c*, *d*, *e*, *f*). Thus, we can perform three-step gating, first rejecting all the points outside the rectangle, then points not lying between the two tangents and finally applying an elliptic gate. On the one hand, computation of the eigenvalues and eigenvectors, constructing the equations of the two tangent lines and an additional check puts extra computational load on the gating procedure. Moreover, the stated steps should be repeated for different pairs of state variables. On the other hand, chances are that a measurement will be rejected after performing hexagon gating over only a few pairs of state variables, without the necessity of going through them all. On the contrary, the Mahalanobis distance calculation assumes that the whole inverse covariance matrix should be computed prior to making any conclusions. Our hypothesis is that for multivariate cases, the proposed three-step gating procedure will be more efficient than the classic two-step rectangular-elliptic one.

The ideas represented in the previous section in their geometrical sense are to be applied to the linear Kalman filter problem. The complete algorithm is as follows.

Step 1. Calculate the predicted state for the problem being solved as

$$\hat{x}_k^- = Ax_{k-1} + Bu_{k-1} + w, \qquad (2)$$

where $\hat{x}_k^-$ is the predicted state at step *k*, *A* is the state transition matrix, x_{k-1} is the estimated state at step (k – 1), *u* is the control input to the state, *B* is a matrix that links the control input to the state, and *w* is the process noise. The minus in the superscript of $\hat{x}_k^-$ reflects the fact that $\hat{x}_k^-$ is a prior, i.e., it is to be updated by a real measurement in order to obtain the posterior estimate. The exact value of *w* is unknown, however we can find the prior mean as

$$\hat{x}_k^- = Ax_{k-1} + Bu_{k-1},$$

since *w* is supposed to be zero-mean white noise.

Step 2. Represent the predicted state in the measurement space using

$$\hat{z}_k = H\hat{x}_k + v,$$

where $\hat{z}_k$ is the predicted measurement, *H* is the measurement function, *v* is the measurement noise.

Step 3. Get the real measurement z_k (the vector is composed of the current readings of different sensors) that corresponds to the prior $\hat{x}_k^-$.

Step 4. Calculate the difference $(\hat{z}_k - z_k)$, which is called the innovation, or the residual. Any normal measurement z_k should be scattered around the mean ($\hat{z}_k$) no farther than three standard deviations away. As was stated earlier, in practice one should not be restricted to 3σ because pretty normal measurements can be met, for instance, 4σ away from the mean. Thus we use a more general notation *N*σ instead of 3σ. For 2D problems $\hat{z}_k$ is the center of an ellipse, represented by covariance matrix *P*. For higher dimensions it is the center of a (hyper)ellipsoid, correspondingly. An outlier lies outside an *N*σ ellipse or a (hyper)ellipsoid.

Step 5. Compute the prior's covariance as

$$P_k^- = AP_{k-1}A^T + Q,$$

where P_{k-1} is the prior's covariance matrix at the previous step and *Q* is the process noise covariance noise. Initialization of *Q* is not a trivial task, since not all state variables may be observed directly. One should remember to initialize the prior's covariance matrix with non-zero values. Otherwise one obtains a "smug" filter which assumes the process model to be impeccable and ignores measurements completely. Here *A* is the same state transition matrix that has been already mentioned in (2).

Step 6. Apply the rectangular gate. If z_k does not pass the gate, it is rejected and the update for the current prediction is skipped. Otherwise Step 7 is taken.

Step 7. Chose the largest off-diagonal item σ_{ij} in matrix P_k^-. Should there be several such items, opt for lesser indices. Obviously, $\sigma_{ij} = \sigma_{ji}$ for any $i \neq j$.

Step 8. For the sub-matrix

$$\begin{bmatrix} \sigma_i^2 & \sigma_{ij} \\ \sigma_{ij} & \sigma_j^2 \end{bmatrix}, \qquad (3)$$

calculate the eigenvalues and eigenvectors and find the equations (1) for the two tangents parallel to the major axis of the error ellipse represented by sub-matrix (3) and selected confidence level.

Step 9. Substitute $\hat{z}_{ki}$ and $\hat{z}_{kj}$ into equations (1) found at Step 8 and check the signs of the calculated expressions. Do the same with z_{ki} and z_{kj}. The signs should coincide. If they do not, it means that the whole vector z_k failed to pass the gate. Thus it is rejected and the update for the current prediction is skipped. Otherwise, choose the next largest off-diagonal item in matrix P_k^- and repeat steps 8 – 9. The question is when to stop considering the error ellipse for the next pair of state variables. Obviously, there is no sense in running through all the possible pairs because each new pair increases the risk that one finally will have to apply the elliptic gate after having made all the redundant checks. It is clear that if the measurement had not been rejected when considering highly correlated state variables, it is unlikely to be rejected when checking loosely correlated ones. There are several ways of proceeding. We suggest that one should stop repeating steps 8 – 9 under either of the following conditions: 1) ¼ of the upper off-diagonal items have been already considered; 2) $\sigma_{ij\_prev} / \sigma_{ij\_curr} \geq 2$, where subindices _prev and _curr denote the previous and current steps, correspondingly. If one of these conditions holds true and the measurement still has not been rejected, Step 10 should be taken.

Step 10. The gate using the Mahalanobis distance assumes computation of

$$d = \left(\hat{z}_k - z_k\right)^T \left(P_k^-\right)^{-1} \left(\hat{z}_k - z_k\right).$$

If d exceeds some threshold, z_k is rejected, otherwise it is considered a legitimate measurement.

Verification of the algorithm has been done using theoretical reasoning and simulation. Inversion of an *n*×*n* matrix, when performed by optimized CW-like algorithms, can be reduced to $O(n^{2,373})$ and no more. The proposed algorithm assumes several matrix operations – finding the minimal item in an *n*×*n* matrix and solving several pairs of linear equations. It involves probabilities since there is no telling at which step the gate will be successful. Nevertheless, one can reasonably assume that for high-dimensional problems pair-wise operations are less computationally complex than inverse matrix calculation. In order to deal with probabilities, we performed the following steps. At the first step, we designed four linear Kalman filters with 2, 8, 12 and 16 state variables. An array of measurements, Z_k, were simulated by adding Gaussian noise to the state vectors $\hat{x}_k^-$ obtained at the prediction steps (500 predict-update steps were taken for each linear Kalman problem). Matrix *R* was filled with the numbers, corresponding to the variance and covariance of the artificially added noise terms.

At the second step, we generated a number of outliers as random numbers scattered more than 4σ away from each $\hat{x}_k^-$ value. The number changed from 1 to 10 % of measurements for each Kalman filter (we took a discrete range of 1%, 2%, 4%, 8% and 10%). In this way a new array, $Z_{k\_outlier}$, was constructed for each case (in total, 20 specimens of array $Z_{k\_outlier}$ were obtained).

At the third step, we benchmarked our gating algorithm against the classic rectangular-elliptic one. During this step we evaluated the execution time taken by the two concurrent procedures and the amount of times the elliptic gate has been applied. Additionally, we checked the performance of the Kalman filter for each text case. Since we dealt with simulated data and no real measurements, for evaluation of the Kalman filter performance it is allowed to use the normalized estimated error squared (NEES), which is defined as:

$$\varepsilon = \tilde{x}^T P^{-1} \tilde{x}, \text{ where } \tilde{x} = x - \hat{x}.$$

The filter stores its own estimate of its error in the covariance matrix. However, the latter cannot serve as an indicator of the filter performance. One can quite often observe a diverging filter whose covariance matrix is getting smaller at each step. As the covariance matrix gets smaller, the NEES gets larger for the same error [14]. Since the whole system is simulated, it is trivial to calculate $\tilde{x}$. Obviously, this approach is not applicable for real-data systems. Both gating procedures refer to the elliptic gate, which is considered to be very accurate, each time when a measurement manages to pass the previous gate(s). That is why it is expected that both procedures have the same accuracy. The NEES was used only for an additional control and in practice it has proved the same accuracy for the two procedures, as expected. On the contrary, the execution time of the two gating procedures was completely different. The proposed algorithm proved to be inefficient for 2D problems. However, for higher dimensions it showed good results, as one might expect from theoretical reasoning. Despite the fact that the results were obtained by generating limited amounts of random numbers, they are sufficient to illustrate (but not strictly prove) that our hypothesis about an additional gates was true.

Conclusions and future work. A modified gating algorithm has been presented. Its main difference with the known solutions consists in an interim step, which is taken after applying the rectangular gate and before the elliptic one. The algorithm considers highly correlated variables in a pair-wise fashion, computes the axes and tilt angles for the corresponding confidence ellipses and the tangents parallel to their major axes. Thus it allows rejection of points outside the confidence ellipses that the rectangular gate has failed to detect. Numerical experiments and theoretical considerations have shown that on average the proposed gating algorithm has less computational complexity than the rectangular-elliptic one due to avoidance of calculating the inverted covariance matrix. Several improvements to the algorithm are possible. Firstly, the criterion of selecting pairs of variables can be revised. The ratios between the variances and covariances can be taken into account. Secondly, the condition for stopping selection of new variable pairs can be chosen more thoroughly. Finally, the exact dependencies between the algorithm efficacy and the problem dimensionality and the probability of outlier occurrence are to be discovered yet. These issues comprise the scope of our future work.

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СПОСТЕРІГАЧ СТАНУ ТЯГОВОЇ ЕЛЕКТРОМЕХАНІЧНОЇ СИСТЕМИ З ДВОМА АСИНХРОННИМИ ДВИГУНАМИ, ЩО ЖИВЛЯТЬСЯ ВІД ОДНОГО ІНВЕРТОРА

Мета роботи. Виконати аналіз тягової електромеханічної системи, у якій живлення декількох асинхронних двигунів здійснюється від одного інвертора, при оцінці змінних стану кожного двигуна за допомогою адаптивного спостерігача.

Методи дослідження. Аналіз рівнянь стану асинхронного двигуна здійснювався з використанням математичних методів диференційного числення, аналіз процесів у тяговій електромеханічній системі здійснювався шляхом математичного моделювання, робота напівпровідникового перетворювача (інвертора) моделювалася з використанням методу перемикаючих функцій.

Наукова новизна. У роботі запропоновано структуру та порядок розрахунку коефіцієнтів адаптивного спостерігача стану асинхронного двигуна у складі електромеханічної системи з живленням декількох двигунів від одного інвертора.

Практична значимість полягає у розробці практичних рішень, які можуть стати основою синтезу алгоритмів керування тяговими електромеханічними системи, у яких здійснюється керування декількома двигунами змінного струму від одного інвертора. Така ситуація є типовою для залізничного транспорту, а тому використання розроблених підходів може слугувати для подальшого покращення показників енергоефективності та точності керуванні.

Результати. У роботі розглянуто рівняння стану асинхронного двигуна та на їх основі розроблено адаптивний спостерігач стану та запропоновано підхід до розрахунку коефіцієнтів спостерігача задля забезпечення його стійкості. Оскільки важливою умовою функціонування системи векторного керування є коректна орієнтація системи координат, що обертається, то у випадку живлення декількох двигунів від одного інвертора орієнтація даної системи координат повинна здійснюватися за усередненим потокозчепленням ротора двигунів. Представлено аналітичні залежності, що описують зміну усереднених величин при наявності відхилень у параметрах двигунів, що відповідає реальним умовам функціонування. Розглянуто метод розрахунку сигналів завдання контурів керування струму з урахуванням наявності відхилень у параметрах двигунів, що дозволяє підвищити точність керування та інші показники якості. Розроблену систему досліджено шляхом математичного моделювання, результати якого свідчать про те, що використання адаптивного спостерігача стану дозволяє забезпечити функціонування багатодвигунної системи без використання датчиків магнітного потоку та кутової швидкості.

Ключові слова: асинхронний двигун, інвертор, векторне керування, потокозчеплення ротора, кутова швидкість, спостерігач стану

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Проблема та її зв'язок з науковими і практичними задачами. За останні десятиліття завдяки бурхливому розвитку мікропроцесорної техніки та силової електроніки було розроблено чимало систем регульованого електроприводу змінного струму. Проте, у більшості таких систем виробники не передбачають можливість здійснення керування декількома двигунами від одного інвертора, обмежуючи можливість функціонування такої системи лише у режимі скалярного керування, що суттєво знижує показники якості керування та енергоефективність. В даний час до найбільш популярних випадків, де здійснюється живлення декількох двигунів змінного струму від одного інвертора відносяться тягові електромеханічні системи залізничного транспорту та електроприводи рольгангів у металургійному виробництві. Проте, наявність ефективних підходів до побудови систем керування такими багатодвигунними системи дозволило б розширити сферу їх застосування, оскільки зменшення кількості інверторів, що використовуються, дозволило б досягти меншої вартості та зменшення габаритних розмірів.

Аналіз досліджень і публікацій. Дослідження тягових електромеханічних систем змінного струму [1-5] є актуальною задачею, оскільки відбувається поступове витіснення двигунів пос-