ISSN 2078–5097. Вісн. Львів. ун-ту. Сер. прикл. матем. та інф. 2017. Вип. 25. С. 82–92
Visnyk of the Lviv University. Series Appl. Math. and Informatics. Issue 25. P. 82–92

UDC 532.543

ABOUT MATHEMATICAL WATER FLOW MODELS IN PSEUDOPRIZMATIC CHANNEL

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The mathematical model of unsteady fluid flow in the open pseudoprizmatic channel was considered. It was derived from the general Navier-Stokes equations. For this model formulated stability criterion of flow which depends on the emergence of waves with increasing amplitude. Variational formulation of the problem was built and solved by finite element method. The influence of the order approximations and selecting basic functions for convergence to a solution of the problem was analyzed. Investigated the movement of water in the river network, where the network branches are united and form a tree structure of river basin, where in each section of the channel can be changed its trajectory and angle of the midline of bottom. Results approved on test examples with complex relief of bottom and it shows the influence of the choice of basis functions on accuracy of the solutions and calculated orders of convergence for time and spatial variables.

Key words: unstable motion, Navier-Stokes equations, equation of conservation mass, channel fluid flow, criterion of stability, cross-sectional area of the channel, flow velocity, variation formulation of the problem, finite element method, basis functions, the order of convergence, bottom relief, river network.

1. INTRODUCTION

Today humanity is faced with the problem of rational use of natural resources in general and water in particular. Hydrological systems, which include watersheds, rivers, lakes, subjected to strong anthropogenic influence. That economic activity human during the use water resources, regional and global climate change could have a significant impact on the hydrological cycle and processes of formation of river flow. So, today is an issue evaluation of these changes. Sometimes they can be made based on experimental data by comparing the hydrological characteristics before and after human impact. However, the possibility such estimates are very limited because the meteorological conditions change quickly. [1, 2, 4, 5, 10, 11, 14, 17] One of the main research perspectives behavior of natural water systems at present is the use of mathematical modeling of water flows, estimation of influence on them physical and natural factors.

This work relates to the development of mathematical models to describe the flow channel, construction of initial boundary value problems and variational formulations, creating projection schemes discretization them by spatial and time variables, estimates of convergence developed recurrent schemes, their stability and testing on test examples.

2. REVIEW OF EXISTING MODEL RUNOFF CHANNEL

In order to construct a mathematical model of reservoir filling streams falling into rivers [1] used ratio based on one-dimensional equations St. Wenan [2]. So in famous packages MIKE11 [16], as described in [2] used finite difference method and built a 6-point implicit scheme Abbott. Of unknown quantities are separation flow and cross-sectional area and for coefficient Shezi following formula is used:

$$C = R^{0.5} / n$$

where n – coefficient roughness of the bottom (in Maning n); R – hydraulic radius, which is calculated by the formula, where χ – wetted perimeter of the channel section.

$$R = \omega / \chi$$

For program MOSRIV, developed in [3] were used one-dimensional Saint Wenan equations, written in divergent form. These equations are solved using difference schemes [1], the unknown values were depth and cross-sectional area, and they took into account the coefficient of hydraulic friction:

$$\lambda = 2gn^2 R^{-1/3}$$

where g – acceleration of free fall.

Often to describe the flow channel using the theory of solitons [11], which is determined by the distribution of the leading edge waves [10], but in this method not fully taking into account the viscosity of the fluid.

In [13] solved the two-dimensional problem and received a velocity distribution of fluid during its movement established in the tray, the width of which is much greater height. Based on these results, it is impossible to correctly describe the movement of fluid in the channels because these models do not take into account the initial velocity of the water and the influence of geometrical dimensions and forms of section channel fluid flow. So, even of the literature [1-12] can be considered that the review model of channel flow without listed above deficiencies is an important problem.

Therefore, quite accurate calculation of of fluid flow in open channels can be made in models that take into account the shape of the river and presence of inflow and flow turbulence at high speeds and inequalities in the bottom of the beds and change its trajectory.

3. EQUATION OF WATER FLOW IN OPEN PSEUDOPRIZMATIC CHANNEL

In this paper the model of the fluid in the pseudoprizmatic riverbed with the vertical plane of symmetry is considered. These channels are formed during movement some curve along flat bottom line, while it is assumed that the depth of the stream is very small compared to the radius of curvature of the bottom line. One-dimensional model of unidentified slow-alternating movement described by equations [1]:

$$\frac{\partial (UF)}{\partial x} + \frac{\partial F}{\partial t} = q\,;$$

$$\frac{1}{g}\frac{\partial U}{\partial t} + \frac{\alpha}{g}U\frac{\partial U}{\partial x} - \frac{\alpha - 1}{g}\frac{U}{F}\frac{\partial F}{\partial t} + \frac{1}{B}\frac{\partial F}{\partial x} + \frac{U^2}{C^2 R} = i, \qquad (1)$$

where U – flow velocity and F – cross-sectional area; g = *9,8* м/с² – acceleration of gravity; C = const – coefficient of Chezy; $i = \sin\delta$ – the angle of the midline of the channel bottom to the x-axis; $B = b_- + b_+$ – width of the channel; R = const – hydraulic radius; α – parameter adjustments of movement; $q = q(x; t)$ – a side inflow.

Complement these equations by initial $U\big|_{t=0} = u_0(x)$, $F\big|_{t=0} = f_0(x)$ on [0, L] and boundary conditions *U (t, 0) = 0, F (t, 0) = 0* and obtain initial-boundary problem of the unknown – the flow velocity *U* and cross-sectional area *F*.

4. THE STABILITY CRITERION FOR THE ESTABLISHED CHANNEL FLOW

This mathematical model depends on many factors that can be changed fast enough, so this model should be resistant to external and internal influences that significantly modify the solution of the problem.

In the flow stability may affect such natural phenomenon: the emergence of cross circulation; aeration flow; emergence of standing waves; emergence of waves with increasing amplitude. Given the characteristics of constructed models of these natural phenomena is essential impact waves with increasing amplitude. To investigate the stability conditions of the model (1) rewrite the flow through separation and depth:

$$\begin{aligned} &\frac{\partial Q}{\partial x} + \frac{\partial (HB)}{\partial t} = 0 \\ &\frac{1}{g}\frac{1}{F}\frac{\partial Q}{\partial t} - \frac{\alpha}{g}\frac{QB}{F^2}\frac{\partial H}{\partial t} + (1-\alpha\frac{Q^2B}{F^3g})\frac{\partial H}{\partial x} + \alpha\frac{Q}{F^2g}\frac{\partial Q}{\partial x} = \frac{iK^2 - Q^2}{K^2} \end{aligned} \quad (2)$$

where *Q* – flow separation, *F* – cross sectional area, *B* – width of the channel, *H* – depth, $K = CF\sqrt{R}$ – bandwidth channel. Consider the consequences of small changes even flow in pseudoprizmatic channel. Under these conditions, it is assumed that

$$H = H_0 + \Delta H\ ;\ Q = Q_0 + \Delta Q\ ;\ K = K_0 + K'\Delta H\ . \quad (3)$$

where H_0, Q_0, K_0 – constant values that retain H, Q, K in conditions of evenly flow, $K' = \frac{dK}{dH}$.

Substituting the values of *H* and *Q* (3) in (2):

$$\begin{aligned} &\frac{\partial \Delta Q}{\partial x} + \frac{\partial (B\Delta H)}{\partial t} = 0; \\ &\frac{1}{g}\frac{1}{F}\frac{\partial \Delta Q}{\partial t} - \frac{\alpha}{g}\frac{QB}{F^2}\frac{\partial \Delta H}{\partial t} + (1-\alpha\frac{Q^2B}{F^3g})\frac{\partial \Delta H}{\partial x} + \alpha\frac{Q}{F^2g}\frac{\partial \Delta Q}{\partial x} = \\ &\qquad = \frac{Q_0^2(K_0 + K_0'\Delta H)^2 - (Q_0 + \Delta Q)^2 K_0^2}{K_0^2(K_0 + K_0'\Delta H)^2}. \end{aligned} \quad (4)$$

Introduce replacement $s = \frac{B}{F}x;\ \ \tau = t\sqrt{\frac{gB}{F}}$.

In the first equation of (4) the first term multiply by *Q*, and the second term – by *F*. In the second equation of (4) the first and fourth terms multiply by *Q*, and denote

$$u = \frac{\Delta Q}{Q};\ h = \frac{B\Delta H}{F}.$$

The system then takes the form

$$\sqrt{\frac{Q^2B}{F^3g}}\frac{\partial u}{\partial s}+\frac{\partial h}{\partial \tau}=0;$$

$$\sqrt{\frac{Q^2B}{F^3g}}\frac{\partial u}{\partial \tau}-\alpha\sqrt{\frac{Q^2B}{F^3g}}\frac{\partial h}{\partial \tau}+(1-\alpha\frac{Q^2B}{F^3g})\frac{\partial h}{\partial s}+\alpha\frac{Q^2B}{F^3g}\frac{\partial u}{\partial s}=\frac{2Q^2KK'\Delta H}{K^2K^2}\frac{B}{F}\frac{F}{B}-\frac{2Q\Delta QK^2}{K^2K^2}\frac{Q}{Q}.$$

Let $\lambda=\sqrt{\frac{Q^2B}{F^3g}}$; $\mu=\frac{K'F}{BK}$, then

$$\begin{aligned}&\lambda\frac{\partial u}{\partial s}+\frac{\partial h}{\partial \tau}=0\\&\lambda\frac{\partial u}{\partial \tau}-\alpha\lambda\frac{\partial h}{\partial \tau}+(1-\alpha\lambda^2)\frac{\partial h}{\partial s}+\alpha\lambda^2\frac{\partial u}{\partial s}=2i(\mu h-u)\end{aligned}\tag{5}$$

In [17] it is proved that the system (5) is stable if the following condition is performed:

$$\frac{1}{\lambda^2}>\mu^2-2\alpha\mu+\alpha.\tag{6}$$

In criterion (6) becomes the criterion Vedernikov – Ivasa[6]:

$$\frac{1}{\lambda}=\mu-1.\tag{7}$$

5. NUMERICAL SOLUTION PROBLEM OF CHANNEL FLOW

5.1. CONSTRUCTION VARIATIONAL PROBLEMS

Formulate variational formulation of the problem (1). Choose spaces of allowable functions $H:=L^2(\Omega),\ V:=\{v\in H^1(\Omega)\,|\,v(0)=0\}$. To construct the variational problem multiply first equation of (1) an arbitrary function $\phi\in V$, the second – $\psi\in V$ and integrate the results by region Ω .

Then variational formulation of initial-boundary problem (1) can be written as:

Asked: $u_0, f_0\in H$

Find a pair, such that: $(u,f)\in L^2(0,T;V\times V)$ such, that

$$\begin{cases}a(u,f,\phi)+a(f,u,\phi)+b(f',\phi)=0;\\ \frac{1}{g}b(u',\psi)+\frac{\alpha}{g}a(u,u,\psi)+\frac{1}{B}c(f,\psi)+\frac{1}{C^2R}d(u,u,\psi)-\frac{\alpha-1}{g}d(w,f',\psi)=\langle l,\psi\rangle;\\ b(u(0)-u_0,\phi)=0, b(f(0)-f_0,\psi)=0.\end{cases}\tag{8}$$

where $a(u,f,\phi)=\int_\Omega u\frac{\partial f}{\partial x}\phi dx$; $b(u,\phi)=\int_\Omega u\phi dx$; $c(u,\phi)=\int_\Omega\frac{\partial u}{\partial x}\phi dx$; $d(u,f,\phi)=\int_\Omega uf\phi dx$ – bilinear form and $l(\phi)=\int_\Omega i\phi dx$ – linear functional.

This variation problem was solved by finite element method [11,13]. For it was performed discretization time and space variables. In semi-discretization of space variables were selected basis functions as piecewise linear and piecewise quadratic polynomials [7].

5.2. DISCRETIZATION VARIATION PROBLEM IN TIME VARIABLE

Divide the length of time [0,T] in N_T+1 equal parts $\left[t_j, t_{j+1}\right]$ with length $\Delta t = t_{j+1} - t_j$, $j = 0,...,N_T$. On each interval $\left[t_j, t_{j+1}\right]$ looking solutions of (8). Solutions $u(x,t),\ f(x,t) \in L^2(0,T;V)$ to this problem approximate by polynomials form

$$\begin{cases} u_{\Delta t}(x,t) = \{1-\omega(t)\} u^j(x) + \omega(t) u^{j+1}(x); \\ f_{\Delta t}(x,t) = \{1-\omega(t)\} f^j(x) + \omega(t) f^{j+1}(x); \\ t \in [t_{j,} t_{i+1}],\ j = 0,1,...,N_T - 1, \omega(t_j, t) = \dfrac{t-t_j}{\Delta t} \end{cases} \tag{9}$$

with unknown functions $u^j(x), f^j(x) \in V_h$.

For functional $l(x,t) \in V_h^1$ in problem (8) will use the following approximation

$$l_{\Delta t}(x,t) = l_{j+1/2} = l(t_{j+1/2}, x). \tag{10}$$

Then the problem (8) can be written as:

$$\begin{cases} b\left(f^{j+1/2}, \phi\right) + \\ +\Delta t \gamma \left[a\left(u^j, f^{j+1/2}, \phi\right) + a\left(u^{j+1/2}, f^j, \phi\right) + a\left(f^{j+1/2}, u^j, \phi\right) + a\left(f^j, u^{j+1/2}, \phi\right)\right] = \\ = -a\left(u^j, f^j, \phi\right) - a\left(f^j, u^j, \phi\right); \\ \dfrac{1}{g} b\left(u^{j+1/2}, \psi\right) + \dfrac{\alpha}{g} \Delta t \beta \left[a\left(u^j, u^{j+1/2}, \psi\right) + a\left(u^{j+1/2}, u^j, \psi\right)\right] + \\ + \dfrac{1}{B} \Delta t \beta c\left(f^{j+1/2}, \psi\right) + \dfrac{2}{C^2 R} \Delta t \beta d\left(u^j, u^{j+1/2}, \psi\right) - \dfrac{\alpha - 1}{g} d(w^j, f^{j+1/2}, \psi) = \\ = \left\langle l_{j+1/2}, \psi \right\rangle - \dfrac{\alpha}{g} a\left(u^j, u^j, \psi\right) - \dfrac{1}{B} c\left(f^j, \psi\right) - \dfrac{1}{C^2 R} d\left(u^j, u^j, \psi\right), \end{cases} \tag{11}$$

Denote: $u^j = u^j(x),\ f^j = f^j(x);\ u^{j+1/2} = u^{j+1/2}(x) = \dfrac{u^{j+1}(x) - u^j(x)}{\Delta t};$

$$f^{j+1/2} = f^{j+1/2}(x) = \frac{f^{j+1}(x) - f^j(x)}{\Delta t}.$$

The coefficients recurrent scheme defined by formulas $\gamma = \dfrac{(\omega^2, \xi)}{(\xi, 1)}, \beta = \dfrac{(\omega^2, \eta)}{(\eta, 1)}$, where

$$(\xi,1) \int_{t_j}^{t_{j+1}} \xi(\tau) d\tau \neq 0, (\eta,1) \int_{t_j}^{t_{j+1}} \eta(\tau) d\tau \neq 0$$

Then recurrent scheme can be written as

Given:

$$\Delta t, \omega(t) = const > 0,\ u^j, f^j \in V \times V.$$

Find:

$$u^{j+1}, f^{j+1} \in V \times V.$$

such that:

$$
\begin{cases}
b\left(f^{j+1/2},\phi\right)+ \\
+\Delta t\gamma\left[a\left(u^j, f^{j+1/2},\phi\right)+a\left(u^{j+1/2}, f^j,\phi\right)+a\left(f^{j+1/2},u^j,\phi\right)+a\left(f^j,u^{j+1/2},\phi\right)\right]= \\
=-a\left(u^j, f^j,\phi\right)-a\left(f^j,u^j,\phi\right); \\
\frac{1}{g}b\left(u^{j+1/2},\psi\right)+\frac{\alpha}{g}\Delta t\beta\left[a\left(u^j,u^{j+1/2},\psi\right)+a\left(u^{j+1/2},u^j,\psi\right)\right]+ \\
+\frac{1}{B}\Delta t\beta c\left(f^{j+1/2},\psi\right)+\frac{2}{C^2R}\Delta t\beta d\left(u^j,u^{j+1/2},\psi\right)-\frac{\alpha-1}{g}d(w^j, f^{j+1/2},\psi)= \\
=\left\langle l_{j+1/2},\psi\right\rangle-\frac{\alpha}{g}a\left(u^j,u^j,\psi\right)-\frac{1}{B}c\left(f^j,\psi\right)-\frac{1}{C^2R}d\left(u^j,u^j,\psi\right); \\
u^{j+1}=u^j+\Delta t u^{j+1/2},\ f^{j+1}=f^j+\Delta t f^{j+1/2}.
\end{cases}
\tag{12}
$$

The scheme provides that the initial solution (u^0, f^0) defined by initial conditions.

5.3. DISCRETIZATION OF VARIATION PROBLEM FOR SPATIAL VARIABLES

Choose a sequence of finite spaces approximations V_h of the space V with properties $\dim V_h \xrightarrow[h\to 0]{} \infty$. Then $\left(u_h, v_h\right)$ – semi discrete approximation of solution *(u,f)*.

The interval [0, L] divide using sequence equally spaced units: $x_i = i\cdot h, i=0,...,N, h=\frac{L}{N}$ on N finite segments $\left[x_i, x_{i+1}\right], i=0,1,...,N-1$.

Choose a base $\left\{\phi_j\right\}_{j=1}^N$, $\left\{\psi_i\right\}_{i=1}^M$ space approximations V_h. Continuous piecewise defined basis functions $\left\{\phi_i(x)\right\}_{i=1}^N$ of the space V_h chosen as linear polynomials, and $\left\{\psi_i(x)\right\}_{i=1}^M$ – in the form of quadratic functions.

Define functions

$$
u_h^j(x)=\sum_{i=1}^M U_i^j\psi_i\left(x\right), \quad f_h^j(x)=\sum_{i=1}^N F_i^j\phi_i\left(x\right)
\tag{13}
$$

a schedule of for the basis functions $\left\{\phi_i\right\}_{i=1}^N$, $\left\{\psi_i\right\}_{i=1}^M$ and unknown coefficients $U=\left\{U_i\right\}_{i=1}^M, F=\left\{F_i\right\}_{i=1}^N$.

Overlaid matrices we obtain recurrent scheme as follows:

Given:

$$\Delta t,\gamma,\beta = const > 0;$$
$$u^j, f^j \in R^n.$$

Find:

$$u^{j+1}, f^{j+1}\in R^n,$$

such that:

$$\begin{cases}\begin{pmatrix} B1+\Delta t\gamma A1\left(u^j\right)+\Delta t\gamma A2\left(u^j\right) & \Delta t\gamma A3\left(f^j\right)+\Delta t\gamma A4\left(f^j\right) \\ \frac{1}{B}\Delta t\beta C+\frac{\alpha-1}{g}D2(w^j) & \frac{B2}{g}+\frac{\alpha}{g}\Delta t\beta\left(A5\left(u^j\right)+A6\left(u^j\right)\right)+\frac{2\Delta t\beta D1(u^j)}{C^2R}\end{pmatrix}\times \\ \times\begin{pmatrix} f^{j+1/2} \\ u^{j+1/2}\end{pmatrix}=\begin{pmatrix} -AP1\left(u^j,f^j\right)-AP2\left(f^j,u^j\right) \\ L_{j+1/2}-\frac{\alpha}{g}AP3(u^j,u^j)-\frac{1}{B}CP(f^j)-\frac{1}{C^2R}DP(u^j,u^j)\end{pmatrix}; \\ u^{j+1}=u^j+\Delta tu^{j+1/2},\ f^{j+1}=f^j+\Delta tf^{j+1/2}.\end{cases} \tag{14}$$

where $A_{ij}=b1_{ij}+\Delta t\gamma\, a1_{ij}(u^k)+\Delta t\gamma\, a2_{ij}(u^k)$; $B_{ij}=\Delta t\gamma\, a3_{ij}(f^k)+\Delta t\gamma\, a4_{ij}(f^k)$;

$$C_{ij}=\frac{1}{B}\Delta t\beta c_{ij}+\frac{\alpha-1}{g}d2_{ij}(w^k);$$

$$D_{ij}=\frac{1}{g}b2_{ij}+\frac{\alpha}{g}\Delta t\beta(a5_{ij}(u^k)+a6_{ij}(u^k))+\frac{1}{C^2R}2\Delta t\beta\, d1_{ij}(u^k).$$

6. ANALYSIS OF PROBLEM SOLVING APPROXIMATIONS

Since the non-linear problem, the solution becomes large positive and negative values, especially when sharp difference of bottom relief flow. Therefore, increased order approximation the unknown solution and shows the advisability of such approach in different test examples.

Example. Given bottom relief by curve graph of which is shown in Fig. 1. Given the following input data: $\alpha = 1$, $0 \le x \le 1$, $0 \le t \le 2$, $\Delta t = 0.007$, $B = 8$, $g = 9.8$, $C = 60$, $R = 1$

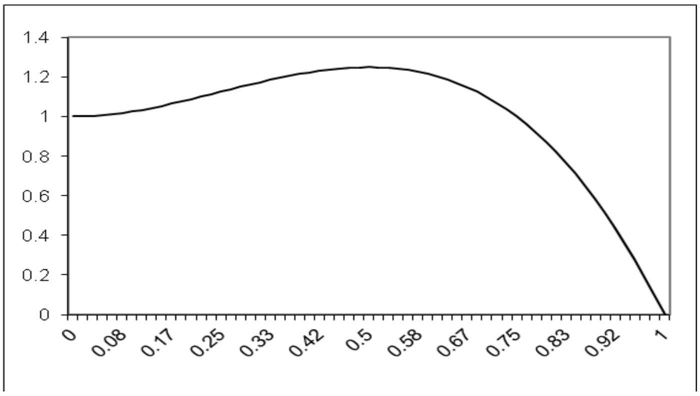
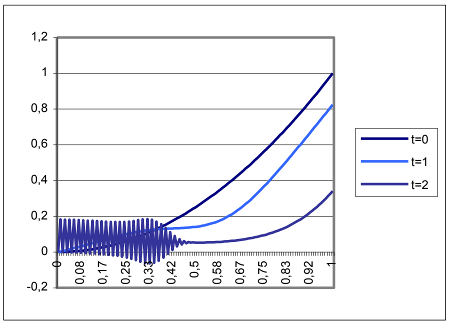


Fig. 1. Bottom surface of flow

Grafics of changes cross-sectional area and flow velocity in time are shown in Fig. 2-5. As in example using linear approximation in solving the system of equations on complex relief oscillations there are, which is clearly seen in Fig. 2.3.



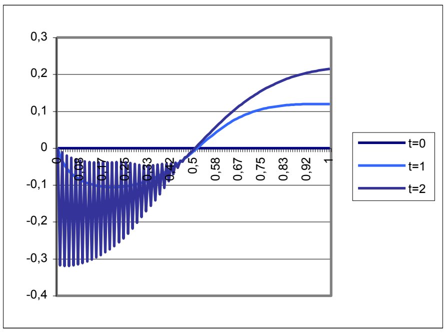
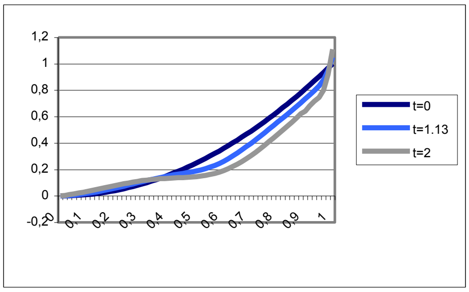


Fig. 2. Cross-sectional area of flow (linear approximation)

Fig. 3. Speed of flow (linear approximation)

Changes in cross-sectional area flow and flow velocity using piecewise quadratic approximations for solving equations (1) are shown in the graphs below.



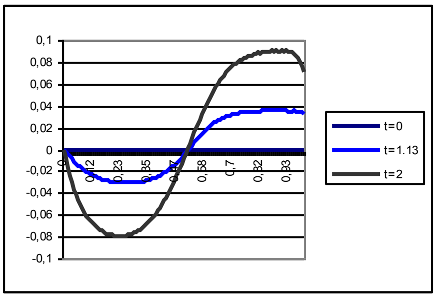


Fig. 4. Cross-sectional area of flow (quadratic approximation)

Fig. 5. Speed of flow (quadratic approximation)

Therefore, on example illustrated fluid motion in channel with the complex relief of bottom and problems that arise in solving nonlinear equations (1) using a linear approximation. To avoid oscillations in the solutions shown in Fig. 2-3 was increased order approximation scheme to the second then received smooth graphics changing cross-sectional area and flow velocity (Fig. 4-5).

7. CALCULATION CHANNEL FLOW FOR RIVER NETWORK

In modeling of water movement in curvilinear line used the fact that the channel is divided into straight parts (Fig. 6, 7), which introduced a local coordinate system with the axis OX directed in the direction of flow. Each rectilinear segment uses a system of equations (1). The transition from one segment to another by means of uniform changes, rotate coordinate system following form

$$(x, y, 1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_0 & -y_0 & 1 \end{pmatrix}\begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \left((x - x_0)\cos\phi - (y - y_0)\sin\phi,\ (x - x_0)\sin\phi - (y - y_0)\cos\phi,\ 1 \right)$$

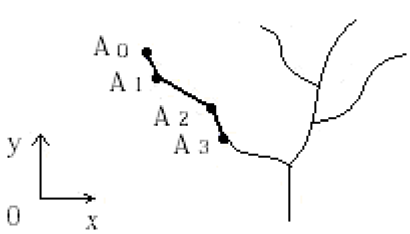


Fig. 6. Structure of river channel

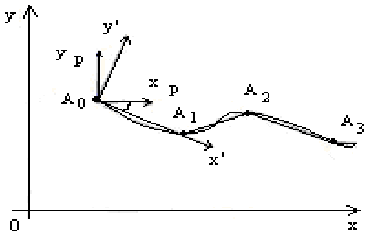


Рис. 7. Changing the local system coordinates

We show conditions for network connections rectilinear parts of the river.

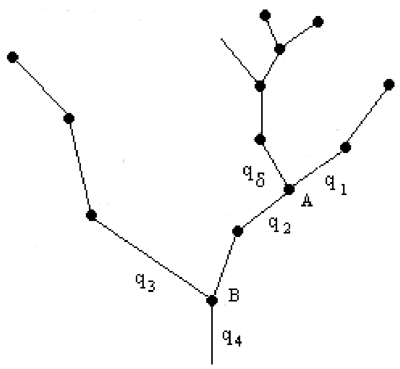


Fig. 8. River network scheme (basin)

Connections of channel flows
in point A $q_2 = q_1 + q_\delta$, $h_1 = h_2$; q_δ – lateral inflow
in point B $q_4 = q_3 + q_2$, $h_4 = h_2 = h_3$.

Thus, studied the movement of water in the river network and output conditions to replenish water from the inflow side and at the connection of channels with different characteristics. These conditions were tested on test examples.

8. CONCLUSIONS

In this paper the model of fluid motion in open pseudo prismatic channel was build. It was described by the system of equations with unknown variables speed and cross-sectional area of flow. Was formulated initial-boundary problem and its variational formulation. Solving the variation problem was conducted using the finite element method.

Selected one-step sampling scheme for recurrent problems in time. Investigated choice of different order approximations for diskretization variation problem for the space variables. We derive stability criterion of unsteady channel flow for waves with increasing amplitude. Consider moving flows of water in the channels with changeable relief of bottom and trajectory. Analysis fluid motion in the river network of selected watershed with different lateral inflow and connection channels with different characteristics.

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Стаття: надійшла до редколегії 25.01.2017
доопрацьована 17.05.2017
прийнята до друку 14.06.2017

ПРО МАТЕМАТИЧНІ МОДЕЛІ СТОКУ ВОДИ У ПСЕВДОПРИЗМАТИЧНОМУ РУСЛІ

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Розглянуто математичну модель неусталеного руху рідини у відкритому псевдопризматичному руслі, яку вивели з загальних рівнянь Нав’є-Стокса. Для цієї моделі сформульовано критерій стійкості потоку, який залежить від виникнення хвиль з наростаючою амплітудою. Побудовано варіаційне формулювання задачі та розв’язано її методом скінченних елементів. Проаналізовано вплив порядку апроксимацій і вибору базисних функцій на збіжність до розв’язку задачі. Досліджено рух води у річковій мережі, де гілки мережі об’єднуються і формують деревовидну структуру басейну річки, в якому на кожній ділянці русла може змінюватися його траєкторія і кут нахилу середньої лінії дна. Результати апробовано на тестових прикладах зі складним рельєфом дна, доведено вплив вибору базисних функцій на точність отриманих розв’язків, обчислено порядки збіжності за часовою та просторовою змінними.

Ключові слова: неусталений рух, рівняння Нав’є-Стокса, рівняння збереження маси, русловий стік рідини, критерій стійкості, площа поперечного перерізу русла, швидкість потоку, варіаційне формулювання задачі, метод скінченних елементів, базисні функції, порядок збіжності, рельєф дна, річкова мережа.