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Mathematical modelling of soil massifs strained-deformed state under soil water level decreasing

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The analytical solutions for the determination of vertical displacements at any point of single-layer and multilayer soil compositions under filtration water flow influence, saline solutions presence and filtration considering soil changing filtration and deformation characteristics have been obtained. The mathematical models of soil filtration and the stress-deformed state from water-saturated ground massifs and bases deformations forecast under internal volumetric forces influence (hydrodynamic forces of the filtration flow, changes in the soils own weight) have been developed and substantiated. Numerical solutions of the corresponding boundary filtration problems and SDS of soil in regions with time-varying curvilinear boundary have been obtained for these mathematical models. They have enabled to perform water-saturated soils and bases deformations forecast under the change of hydrogeological conditions and man-made factors effect.

Keywords: pressure, displacement, conformal mapping, hydrodynamic net

Математичне моделювання напружено-деформованого стану ґрунтових масивів при зниженні рівня ґрунтових вод

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Досліджено вплив зміни гідрогеологічних умов та дії техногенних факторів на деформації водонасичених ґрунтових масивів і основ. Наведено результати експериментальних досліджень впливу концентрації сольових розчинів на фільтраційні та деформаційні властивості ґрунту. Отримано емпіричні залежності у вигляді поліномів коефіцієнта фільтрації, модуля деформації й коефіцієнтів Ламе від концентрації сольових розчинів, які дозволили вдосконалити математичні моделі фільтрації та напружено-деформованого стану ґрунту з урахуванням нелінійних фільтраційних і деформаційних процесів, що відбуваються у ґрунтових масивах за наявності та фільтрації сольових розчинів. Отримано аналітичні розв'язки з визначення вертикальних зміщень у будь-якій точці одно-і багатошарових ґрунтових масивів при дії фільтраційного потоку води, наявності та фільтрації сольових розчинів з урахуванням змінних фільтраційних і деформаційних характеристик ґрунту. Розроблено й удосконалено математичні моделі фільтрації та напружено-деформованого стану ґрунту з прогнозування деформацій водонасичених ґрунтових масивів і основ при дії внутрішніх об'ємних сил (гідродинамічних сил фільтраційного потоку, зміни власної ваги ґрунту). Отримано чисельні розв'язки відповідних крайових задач фільтрації та напружено-деформованого стану ґрунту для плоских областей зі змінною в часі криволінійною межею. Наведено результати числового моделювання вертикальних зміщень ґрунтових водонасичених масивів і основ у процесі їх осушення, наявності водозабірних свердловин та водонапірних споруд.

Ключові слова: напір, переміщення, конформне відображення, гідродинамічна сітка

Introduction. The change in the hydrogeological conditions on Earth is taking place more and more rapidly. The reason for such changes is the development of the last thousand years and the excessive increase in the beginning of the millennium of a new, previously unknown geological agent. This new geological agent is human activity and E.M. Sergeyev [1] considers this to be driving geological force, which not only changes earth surface, but also makes significant changes in the upper part of earth crust, which for scales and consequences coincide with geological processes.

Hydro geological conditions change in soils and presence of man-made factors lead to the formation of hydrodynamic forces of the filtration flow, changes in the soil own gravity, etc.

These factors growth magnitude and intensity can be significantly changed, which leads to earth surface subsidence occurrence. These deformations complicate normal operation, and in some cases lead to accidents in buildings and structures and can bring significant economic damage.

An analysis of the latest sources of research publications. In the paper [2] the frequent earth surface surveying of was performed for the direct study of vertical relief displacements. These studies require a lot of time and considerable material costs. An alternative to this may be the mathematical modeling of soil surface vertical displacement in its drain process [3 – 5].

Existing methods for soil massifs deformations and bases working assessing in the conditions of varying groundwater levels and man-made factors effects have been developed for the case of stabilized groundwater level still do not fully reflect their state [6 – 10].

So, the study of this problem is relevant.

The problem solution is greatly simplified with the development of mathematical methods for water-saturated granular massifs stress-strained state modeling.

Setting objectives. The problem of lowering the groundwater level in the soil mass as a result of water pumping from horizontal drains should is considered (Fig. 1).

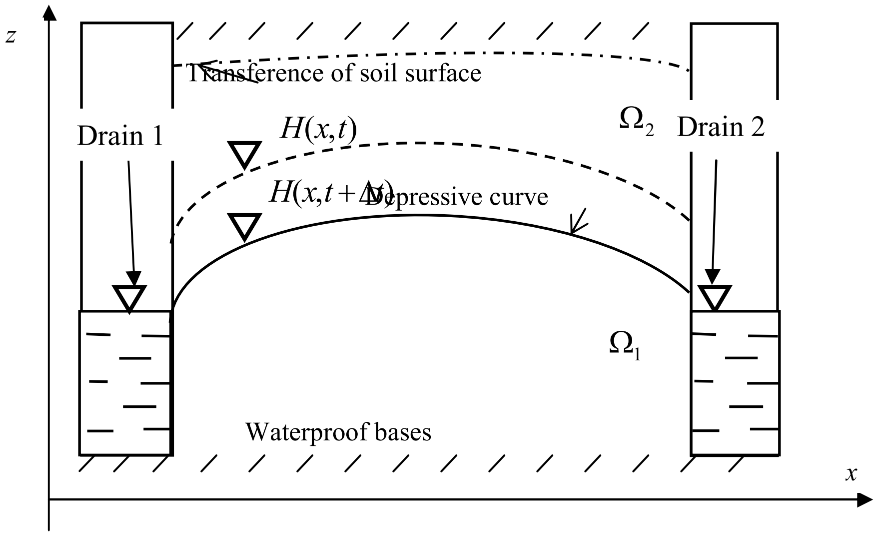

Figure 1 – Scheme of soil massif during its drainage

Due to the drainage of soil upper part and intensive filtration forces, the stress-deformed state of the soil massif changes and it leads to soil surface transference.

In order to find the transference of the soil surface, the pressure at all points of the soil massif at each time moment should be known.

For head determination in the changeable area $\Omega_1 = \{(x,z,t) \mid x \in (0,l), z \in (0,h(x,t)), t > 0\}$ solution of differentiated equation should be found:

$$\frac{\partial H}{\partial t} = \frac{kh_{col}}{\mu}\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2}\right) \quad (1)$$

under such boundary conditions:

$$H(x,z,0) = H_1(x,z), \quad (2)$$

$$H(0,z,t) = H_0(0) - V_0 t, \quad (3)$$

$$H(l,z,t) = H_0(l) - V_l t, \quad (4)$$

$$\frac{\partial H(x,0,t)}{\partial z} = 0, \quad (5)$$

$$H(x,h(x,t),t) = h(x,t), \quad (6)$$

where *H (x, z, t)* – water head at the time moment *t* in the point *(x, z)* of soil massif; *k* –filtration coefficient; h_{col} – capacity of filtration flow; μ –water drainage coefficient; $H_1(x, z)$ – division of water heads at the time moment; $H_0(x)$ – the height of soil water impending at the primary time moment (known function); *h (x, t)* – the height of soil water placement at the time moment *t*.

Symbols are introduced:

$$a^2 = \frac{kh_{col}}{\mu}. \qquad (7)$$

Then the equation (1) will look in the following way:

$$\frac{\partial H}{\partial t} = a^2\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2}\right). \qquad (8)$$

For the solution of this problem it is necessary to find $h\ (x,\ t)$ та $H_1(x,\ z)$. That is why let's consider two additional problems.

Problem 1. In the area $\Omega = \{(x,t) \mid x \in (0,l), t > 0\}$ to find the solution of differential equation

$$\frac{\partial h}{\partial t} = \frac{kh_{col}}{\mu}\frac{\partial^2 h}{\partial x^2} \qquad (9)$$

under such boundary conditions:

$$h(x,0) = H_0(x), \qquad (10)$$

$$h(0,t) = H_0(0) - V_0 t, \qquad (11)$$

$$h(l,t) = H_0(l) - V_l t. \qquad (12)$$

Problem 2. In the area $\Omega = \{(x,z) \mid x \in (0,l), z \in (0, H_0(x))\}$ to find the solution of differential equation

$$\frac{\partial^2 H_1}{\partial x^2} + \frac{\partial^2 H_1}{\partial z^2} = 0 \qquad (13)$$

under such boundary conditions:

$$\frac{\partial H_1(x,0)}{\partial z} = 0, \qquad (14)$$

$$H_1(0,z) = H_0(0), \qquad (15)$$

$$H_1(l,z) = H_0(l), \qquad (16)$$

$$H_1(x, H_0(x)) = H_0(x). \qquad (17)$$

The solution of problem (9) – (12) is the following.

$$h(x,t) = \sum_{n=1}^{\infty}\left(A_n e^{-\left(\frac{\pi n a}{l}\right)^2 t} + \frac{2}{\pi n}(V_0 - V_l(-1)^n)\left(\frac{l}{\pi n a}\right)^2\left(1 - e^{-\left(\frac{\pi n a}{l}\right)^2 t}\right)\right)\sin\frac{\pi n x}{l} +$$
$$+ H_0(0) - V_0 t + \frac{H_0(l) - V_l t - (H_0(0) - V_0 t)}{l}x, \qquad (18)$$

where

$$A_n = \frac{2}{l}\int_0^l H_0(x)\sin\left(\frac{\pi n}{l}x\right)dx + \frac{2}{\pi n}(-H_0(0) + H_0(l)(-1)^n). \qquad (19)$$

Let's (13)–(17) fulfill the numerical conformal reflection of the area $\Omega = \{(x,z) \mid x \in (0,l), z \in (0, H_0(x))\}$ on the parametric rectangle for the solution of the problem. The built net of conformal reflection is the hydrodynamic net, that is the solution of problem. The meanings of heads on the lines of equal heads are equal with the meanings in the upper points of these lines, which in their turn, are equal to the vertical coordinate of soil water surface.

For the solution of problems (1) – (8) it is transfered to variables $\xi,\ \eta$ [11], under this

$$\xi = \xi(x,z),\ \eta = \eta(x,z). \qquad (20)$$

Under the conditions of Koshi-Riman it is:

$$\frac{\partial \xi}{\partial x} = \frac{\partial \eta}{\partial z},\ \frac{\partial \xi}{\partial z} = -\frac{\partial \eta}{\partial x}. \qquad (21)$$

Substituting (20) in (8), we have:

$$\frac{\partial H}{\partial t} = a^2\left(\frac{\partial^2 H}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial H}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 H}{\partial \eta^2}\left(\frac{\partial \eta}{\partial x}\right)^2 + \frac{\partial H}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 H}{\partial \xi^2}\left(\frac{\partial \xi}{\partial z}\right)^2 + \frac{\partial H}{\partial \xi}\frac{\partial^2 \xi}{\partial z^2} + \frac{\partial^2 H}{\partial \eta^2}\left(\frac{\partial \eta}{\partial z}\right)^2 + \frac{\partial H}{\partial \eta}\frac{\partial^2 \eta}{\partial z^2}\right). \qquad (22)$$

Taking into consideration, that $\xi(x,z)$ and $\eta(x,z)$ – harmonic functions, we have:

$$\frac{\partial H}{\partial t} = a^2\left(\frac{\partial^2 H}{\partial \xi^2}\left(\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial z}\right)^2\right) + \frac{\partial^2 H}{\partial \eta^2}\left(\left(\frac{\partial \eta}{\partial x}\right)^2 + \left(\frac{\partial \eta}{\partial z}\right)^2\right)\right) \qquad (23)$$

Taking consideration (21), (23) will be in the following way

$$\frac{\partial H}{\partial t} = a^2\left(\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial z}\right)^2\right)\left(\frac{\partial^2 H}{\partial \xi^2} + \frac{\partial^2 H}{\partial \eta^2}\right). \qquad (24)$$

In this case, a grid constructed with a numerical conformally represented net will be a hydrodynamic grid, that is, the solution of the problem (1) – (8). The solution is the same as solving the problem (13) – (17). The obtained exact formulae are used for ($h(x, t)$ (for calculating the replacement of a series sum by a finite sum, the integration with a parabola method is used. The advantage of this approach is that pressure and movement at any given time can be found without finding them at previous moments of time (only at the initial moment of time).

For finding transferences in the areas below soil water level Ω_1 and above Ω_2, we have the system of differential equations:

$$k_1 \frac{d^2 u_1}{dz^2} = \gamma_{sb} + \gamma_w \left(\frac{dh}{dz} + 1 \right), x \in (0, l_1), \tag{25}$$

$$k_2 \frac{d^2 u_2}{dz^2} = \gamma_n, x \in (l_1, l), \tag{26}$$

$$u_1(0) = 0, \tag{27}$$

$$\frac{du_2(l)}{dz} = 0, \tag{28}$$

$$u_1(l_1) = u_2(l_1), \tag{29}$$

$$k_1 \frac{du_1(l_1)}{dz} = k_2 \frac{du_2(l_1)}{dz}, \tag{30}$$

where γ_{sb}, γ_w, γ_n – specific soil weigh, that is located in weighted state, water and soil, which is in natural state relatively;

$h(z)$ – head (x_1, z) at the time moment t_1;

$k_1 = \lambda_1 + 2\mu_1$, $k_2 = \lambda_2 + 2\mu_2$ – elastic steels;

$u_1(z)$, $u_2(z)$ – transference of point, which is in the moment t_1 located in the point (x_1, z);

l_1 – soil water level, and l – determined vertical coordinate of the upper point of soil's massif in the point $x = x_1$ at the time moment $t = t_1$;

indexes 1, 2 near k, λ, μ, u mean the location of points (x_1, z) below or above soil water level relatively.

Integrating equations, it is got:

$$\frac{du_1}{dz} = \frac{\gamma_{sb} z}{k_1} + \frac{\gamma_w}{k_1}\left(h(z) + z\right) + c_1,$$

$$u_1 = \frac{\gamma_{sb} z^2}{2k_1} + \frac{\gamma_w}{k_1}\left(\int_0^z h(\xi) d\xi + \frac{z^2}{2} \right) + c_1 z + c_2,$$

$$\frac{du_2}{dz} = \frac{\gamma_n z}{k_2} + c_3,$$

$$u_2 = \frac{\gamma_n z^2}{2k_2} + c_3 z + c_4,$$

$$u_1(0) = 0 \Rightarrow c_2 = 0,$$

$$\frac{du_2(l)}{dz} = 0 \Rightarrow \frac{\gamma_n l}{k_2} + c_3 = 0 \Rightarrow c3 = -\frac{\gamma_n l}{k_2},$$

$$k_1 \frac{du_1(l_1)}{dz} = k_2 \frac{du_2(l_1)}{dz} \Rightarrow \gamma_{sb} l_1 + \gamma_w \left(h(l_1) + l_1\right) + c_1 k_1 =$$

$$= \gamma_n l_1 - \gamma_n l.$$

Considering $h(l_1) = l_1$ it is got:

$$c_1 = \frac{1}{k_1}\left(\gamma_n (l_1 - l) - 2\gamma_w l_1 - \gamma_{sb} l_1\right),$$

$$u_1(l_1) = u_2(l_1) \Rightarrow \frac{\gamma_{sb} l_1^2}{2k_1} + \frac{\gamma_w}{k_1}\left(\int_0^{l_1} h(\xi) d\xi + \frac{l_1^2}{2} \right) + c_1 l_1 = \frac{\gamma_n l_1^2}{2k_2} - \frac{\gamma_n l_1 l}{k_2} + c_4 \Rightarrow$$

$$c_4 = \frac{\gamma_{sb} l_1^2}{2k_1} + \frac{\gamma_w}{k_1}\left(\int_0^{l_1} h(\xi) d\xi + \frac{l_1^2}{2} \right) + \frac{l_1}{k_1}\left(\gamma_n (l_1 - l) - 2\gamma_w l_1 - \gamma_{sb} l_1\right) - \frac{\gamma_n l_1^2}{2k_2} + \frac{\gamma_n l_1 l}{k_2} =$$

$$= \frac{1}{k_1}\left(\gamma_w \int_0^{l_1} h(\xi) d\xi - \frac{3\gamma_w l_1^2}{2} - \frac{\gamma_{sb} l_1^2}{2} + \gamma_n \left(l_1^2 - l_1 l\right) \right) + \frac{\gamma_n l_1}{k_2}\left(l - \frac{l_1}{2} \right),$$

So far as we take into consideration all risk factors on soil, transference is located relatively some primary level. This starting point, which we mark l_0, is necessary to be found, using known at the primary moment of meaning l.

$$u_2(l) = l_0 - l, \tag{31}$$

$$u_2(l) = \frac{\gamma_n l^2}{2k_2} - \frac{\gamma_n l^2}{k_2} + c_4 = -\frac{\gamma_n l^2}{2k_2} + c_4.$$

Substituting $u_2(l) = l_0 - l$, found meaning c_4 it is got:

$$l_0 = l - \frac{\gamma_n l^2}{2k_2} + \frac{1}{k_1}\left(\gamma_w \int_0^{l_1} h(\xi) d\xi - \frac{3\gamma_w l_1^2}{2} - \frac{\gamma_{sb} l_1^2}{2} + \gamma_n \left(l_1^2 - l_1 l\right) \right) + \frac{\gamma_n l_1 l}{k_2} - \frac{\gamma_n l_1^2}{2k_2}. \tag{32}$$

Substituting known values in the obtained formula at the primary time moment, we will find l_0.

For finding l at any time moment from (32) we will get quadratic equation:

$$\left(\frac{\gamma_n}{2k_2}\right)l^2+\left(\frac{\gamma_n l_1}{k_1}-1-\frac{\gamma_n l_1}{k_2}\right)l+\left(l_0+\frac{\gamma_n l_1^{\,2}}{2k_2}+\frac{1}{k_1}\left(-\gamma_w\int_0^{l_1}h(\xi)d\xi+\frac{3\gamma_w l_1^{\,2}}{2}+\frac{\gamma_{sb} l_1^{\,2}}{2}-\gamma_n l_1^{\,2}\right)\right)=0. \quad (33)$$

Having solved the equation (33), we will find the coordinate of the soil upper boundary for arbitrary x and t:

$$l=\frac{\left(1+\gamma_n l_1\left(\frac{1}{k_2}-\frac{1}{k_1}\right)+\sqrt{\left(1+\gamma_n l_1\left(\frac{1}{k_2}-\frac{1}{k_1}\right)\right)^2-\left(\frac{2\gamma_n}{k_2}\right)\left(l_0+\frac{\gamma_n l_1^{\,2}}{2k_2}+\frac{1}{k_1}\left(-\gamma_w\int_0^{l_1}h(\xi)d\xi+\frac{3\gamma_w l_1^{\,2}}{2}+\frac{\gamma_{sb} l_1^{\,2}}{2}-\gamma_n l_1^{\,2}\right)\right)}\right)k_2}{\gamma_n}. \quad (34)$$

It has been chosen the sign «+» before square root taking into consideration, that $l > 0$

For finding integral from the function of head

$$\int_0^{l_1}h(\xi)d\xi$$

built hydrodynamic net is used. Geometric algorithm of integral finding in the point $x = x_1$ at time moment $t = t_1$: on the hydrodynamic net for time next. It is determine the straight $x = x_1$. At points of this line intersection with lines of hydrodynamic grid currents, head meaning is found with the help of linear interpolation between adjacent nodes. Integral is found using the formulae of the central rectangles for unevenly located nodes. This algorithm is simply implemented in the programming language using analytical geometry.

In accordance with described algorithm, numerical experiments have been carried out with the determination of vertical displacements under next output data:

$x = 100m$;

$H_0(z)\big|_{t=0} = 35m$; $H_r(z)\big|_{t=t_i} = 23m$;

$\lambda_1 = 18493kPa$; $\mu_1 = 15748kPa$;

$\lambda_2 = 18000kPa$; $\mu_2 = 18000kPa$;

$\gamma_w = 10kN/m^3$; $\gamma_n = 16{,}48kN/m^3$;

$\gamma_{sb} = 9{,}63kN/m^3$; $\gamma_{dr} = 16{,}48\,kN/m^3$.

Given net step $\approx 3m$.

Here λ_1, μ_1 – Lame coefficients for soil in water saturated state coefficients, but λ_2, μ_2 – for drained soil.

Numerical modeling is performed under partial drainage of the soil massif. The results of the vertical displacements at given points of the soil massif for the case during drainage process studies are partly presented in table 1.

Table 1 – Meanings of soil massif vertical displacements

z, *m* \ x, *m*	12,5	35	53,5	66,0	81,5	935	100,0
39,9	-58,56	-58,66	-58,67	-58,64	-58,62	-58,63	-58,74
29,925	-55,53	-55,60	-55,59	-55,57	-55,54	-55,54	-55,61
19,95	-41,44	-41,44	-41,41	-41,36	-41,31	-41,28	-41,28
9,975	-26,28	-26,25	-26,21	-26,17	-26,13	-26,09	-26,06

Numerical meanings of displacements in given nodes, which are presented in the given tables, are measured in millimeters.

For visualization, the results of research in graphic form are presented (fig. 2). The upper part of the figure shows soil surface at the initial and at the final moment of time, in the lower part – a hydrodynamic net.

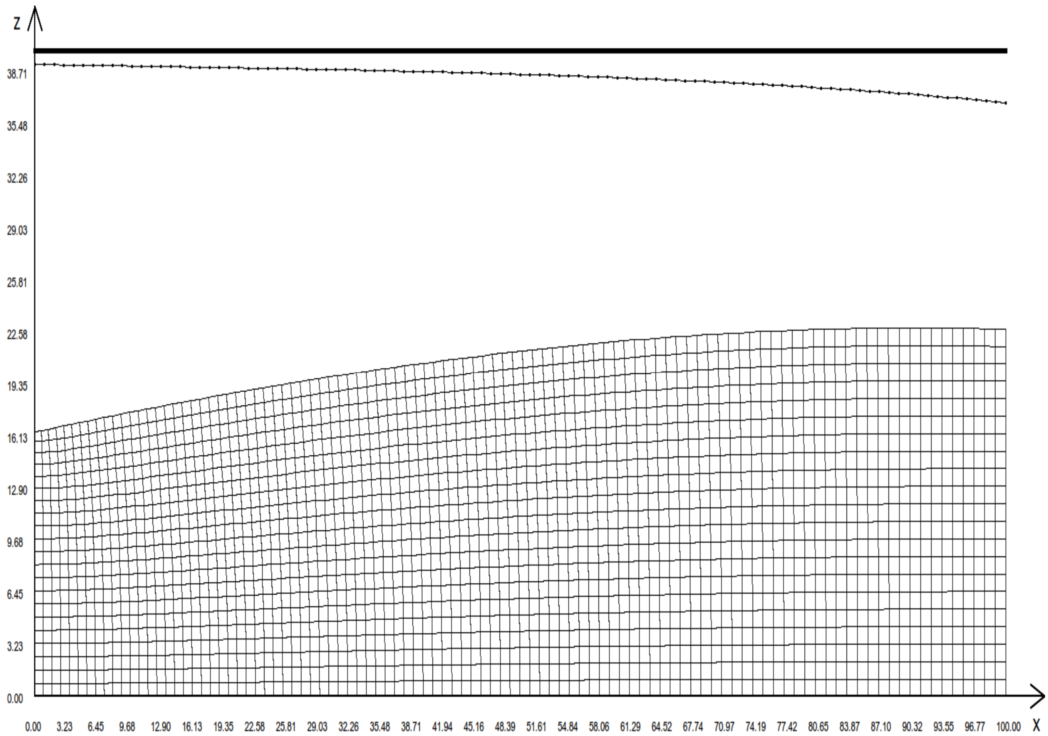

Figure 2 – Graphic representation of research results

Conclusions. In scientific articles only one of the many engineering problems in predicting the subsidence of the earth surface during the groundwater level lowering by pumping water from horizontal drains has been considered. One-dimensional mathematical model has been studied, which may only correspond to some practical problems. The simple solution obtained in this paper has the advantage of its speed and the possibility of finding the soil massif state at any given time, without finding at previous time moments.

The obtained mathematical solutions enables to fulfill the forecast of water-saturated ground masses subsidence during their drainage. Further areas of researches can be mathematical solutions obtaining of the set tasks for a two-dimensional case.

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