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**ОПЕРАТОРНІ РІЗНИЦЕВІ РІВНЯННЯ, ЯКІ ЗАЛЕЖАТЬ НЕЛІНІЙНО  
ВІД СПЕКТРАЛЬНОГО ПАРАМЕТРА**

*V.I. Khrabustovskyi*

**OPERATOR DIFFERENCE EQUATIONS DEPENDING ON SPECTRAL  
PARAMETER NONLINEARLY**

We consider either in regular or singular case operator difference equations containing spectral parameter in Nevanlinna manner.

We obtain analogs of many statements for differential relation from [1] [2]. For example we obtain the eigenfunction expansions in various cases.

**References**

[1] V.I. Khrabustovskyi Analog of generalized resolvents for relations generated

by pair of differential expressions one of which depends on spectral parameter in nonlinear manner J. Math. Phys. Anal. Geom. 9 (2013), no 4, 496-535.

[2] V.I. Khrabustovskyi, Eigenfunction expansions associated with an operator differential equation nonlinearly depending on a spectral parameter, Methods Funct. Anal. Topol. 20 (2014) no 1, 68-91.

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**ФУНКЦІЇ ГРІНА УЗАГАЛЬНЕНИХ РІВНЯНЬ КЛЕЙНА-ГОРДОНА**

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**GREEN FUNCTIONS OF GENERALISED KLEIN-GORDON EQUATIONS**

In [1, 2] it is shown that the four-fold integral corresponding to the Green function of the Klein-Gordon equation diverges. Fourier-components in this integral are the fraction with the  $-q^2 + m^2$ -denominator, where  $m$  is a particle mass, the components of the 4-vector  $q$  are the integration variables. To avoid a divergence of this integral it is proposed to change the  $-q^2 + m^2$ -factor in the denominator on the  $(-q^2 + m_1^2)(-q^2 + m_2^2) \dots (-q^2 + m_N^2)$ -product, where  $N \geq 3$ , The proposed Green functions correspond to differential equations with the a product of the Klein-Gordon differential operators at different masses. The classical solution of corresponding homogeneous equation is sum of N terms. Such terms correspond to the solutions for the positive and the negative frequencies. The

proper fraction in the Fourier-components of new Green functions can be expanded as a sum of common fractions with  $A_k$  residues ( $A_k = (-1)^{k+1} |A_k|, k = 1, 2, \dots, N$ ). The quantized free field is a sum of N terms. The number k term is proportional to  $\sqrt{A_k}$  and includes the creation and the annihilation operators for the particle with the  $m_k$ -mass in usual manner. At such method of a quantization the T-product of free fields is not a Green function of the generalized Klein-Gordon equation.

1. Kulish Yu.V., Rybachuk E.V. The Journal of Kharkiv National University, 2011, n. 955, Is. 2(50), p. 4.

2. Kulish Yu.V., Rybachuk E.V. Problems of atomic science and technology, 2012, n. 1(77), p.16.