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TRAFFIC FLOWS MODELLING BASED ON PROBABILITY OF COINCIDENCE OF CHAOTIC IMPULSES OF RANDOM DURATION AND RANDOM INTERVALS BETWEEN THEM

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Abstract. *The features of applying the theory of chaotic electrical impulse flows in mathematical modelling of traffic flows have been considered. Both the moments of appearance of vehicles on the traffic lane and the moments of appearance of electrical impulse on the time axis are supposed to obey the Poisson law. It is assumed that the duration of an impulse is equivalent to the length of a vehicle, and the intervals between the impulses are equivalent to the intervals between the vehicles in the spatial-temporal axis.*

Key words: *traffic flow, chaotic electrical impulse flow, coincidence of impulses.*

МОДЕЛЮВАННЯ ТРАНСПОРТНИХ ПОТОКІВ НА ОСНОВІ ЙМОВІРНОСТІ ЗБІГУ ІМПУЛЬСІВ, ЩО ПРЯМУЮТЬ ХАОТИЧНО, ВИПАДКОВОЇ ТРИВАЛОСТІ Й ВИПАДКОВИХ ІНТЕРВАЛІВ МІЖ НИМИ

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Анотація. *У наш час при моделюванні транспортних потоків розглядається, в тому числі, й клас макроскопічних моделей, коли рух транспортних засобів уподібнюється будь-якому фізичному потоку (гідро- і газодинамічні моделі). Як фізичний аналог транспортного потоку запропоновано використовувати теорію хаотичних електричних імпульсних потоків.*

Ключові слова: *транспортний потік, транспортний засіб, хаотичний електричний імпульсний потік, збігання транспортних засобів.*

МОДЕЛИРОВАНИЕ ТРАНСПОРТНЫХ ПОТОКОВ НА ОСНОВЕ ВЕРОЯТНОСТИ СОВПАДЕНИЯ ХАОТИЧНЫХ ИМПУЛЬСОВ СЛУЧАЙНОЙ ДЛИТЕЛЬНОСТИ И СЛУЧАЙНЫХ ИНТЕРВАЛОВ МЕЖДУ НИМИ

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Аннотация. *В настоящее время при моделировании транспортных потоков рассматривается, в том числе, и класс макроскопических моделей, когда движение транспортных средств уподобляется любому физическому потоку (гидро- и газодинамические модели). В качестве физического аналога транспортного потока предложено использовать теорию хаотических электрических импульсных потоков.*

Ключевые слова: *транспортный поток, транспортное средство, хаотичный электрический импульсный поток, совпадение транспортных средств.*

Introduction

Computer simulation of traffic plays an important role in improving the transport situation on the roads. The application of various mathematical methods and modelling systems enables to improve the traffic flow capacity, reduce accident rate on the roads, and preserve the environment with minimal financial expenses.Road traffic is closely connected with the welfare and life of people. Therefore, the major road accidents are often covered in media, on television, on the Internet.

Analysis of Publications

According to the information of State Motor Vehicle Inspectorate of Ukraine, the most frequent reason for traffic accidents is related to the actions of road traffic participants, for which reason more than 90 % of accidents from their total number take place (Fig. 1) [1, 2]. Some of the most frequent violations connected with vehicles (except those involving pedestrians) are: exceeding the safe speed, violation of manoeuvring rules, non-observance of the distance and driving onto the oncoming lane of the road.

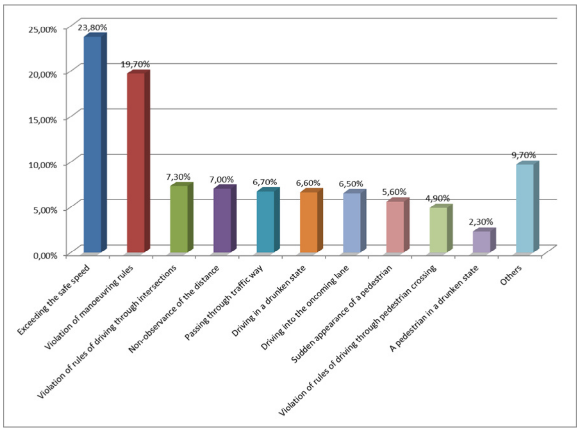

Fig. 1. The diagram of traffic accidents in Ukraine

Currently, in modelling traffic flows, the class of macroscopic models, when the motion of vehicles is likened to any physical flow (hydro- and gas-dynamic models) is also considered [3].

Purpose and Tasks

The aim of the work is mathematical modelling of transport flows on the basis of choosing the theory of chaotic electric impulse flows as a physical analogue of the transport flow.We will proceed from the assumption that violation of the manoeuvring rules, including driving onto the adjacent lane of traffic and onto the oncoming lane, will result in a collision of vehicles when they are in the cross section for a moment before the convergence or at the time of the convergence, that is, in the zone of compatible parallel movement in the cross section of the road (Fig. 2). We will assume that the moments of the appearance of vehicles on the lane of motion obey the Poisson law, that is, they meet the criteria of ordinarinessand not controlling the after-effect [4].

Let there be a transport flow with z random lengths of vehicles and random intervals of d between them and known densities of probability $P(d)$ for the intervals between the vehicles and $Q(z)$ for their lengths.

The distance between successive vehicles may be different. In some cases, it can slightly exceed the length of the vehicle, while in others it can be quite large. Similarly, the time interval between successive vehicles varies. This variability in time and space is the most notable characteristic of the traffic flow [5].

To know the intervals of time and distance between successive vehicles is sometimes more important than knowing the intensity or density of the flow, as it reflects the true nature of the traffic flow more fully. The intervals of time and distance between successive vehicles are those “bricks” on which the entire traffic flow is built [5]. Since vehicles are dynamic objects, which depending on the speed at each moment of time take a new position in the spatial-temporal plane [4] (traffic lane), this process can be represented as a random electric impulse flow on the time axis [6]. Therefore, it can be assumed that the duration of an impulse is equivalent to the length of a vehicle, and the intervals between the impulses are equivalent to the intervals between the vehicles.

To do this, we use the theory of random impulse flows. Let us consider the n probability of coincidence of vehicles of random lengths that go chaotic, and random intervals between them and the distribution of their coincidence [7].

Substantiation of mathematical modelling of traffic flows on the basis of choosing the theory of chaotic electric impulse flows as a physical analogue of the transport flow

Let the lengths of the vehicles x_i $(\mathrm{i}=1,2)$ in two lanes of motion $(n=2)$ be distributed according to independent laws $f_i(x_i)$ in intervals $(0, x_{im})$, where x_{im} is the largest length of a vehicle on the lane. The distribution of the moments of the appearance of vehicles on each lane of motion is subject to the Poisson law, which is characterized by average density $\lambda \ll 1/x_{im}$. We find the three-dimensional distribution law $W_{(3)}(x_1,x_2,z)$, where x_1 and x_2 are the lengths of the vehicles, which make a coincidence of with z duration.

As we know

$$W_{(3)}(x_1,x_2,z)=\varphi_1(x_1)\cdot\varphi_{21}(x_2/x_1)\cdot$$
$$\cdot W_{12}(z/x_1,x_2) \quad (1)$$

where $\varphi_1(x_1)$ is the law of the distribution of the duration of coincidence of vehicles, which are on the first lane of motion; $\varphi_{21}(x_2/x_1)$ is the conditional law of distribution of length x_2 of coincidence of vehicles of the second traffic lane to the length $x_{1\ \text{ы}}$ of vehicles of the first lane; $W_{12}(z/x_1,x_2)$ is the conditional law for the distribution of z duration of coincidence of the vehicles to the lengths of x_1 and x_2. The law of distribution $\varphi_i(x_i)$ differs from the law $f_i(x_i)$, as the probability $p=\lambda\cdot(x_1+x_2)$ of coincidence of vehicles turns out to be higher with longer vehicles [4] and therefore the latter coincide on average more often than shorter vehicles. With the unchanged length x_2 of the vehicles on the second lane, the distribution law is

$$\varphi_1(x_1)=C\cdot f_1(x_1)\cdot\lambda\cdot(x_1+x_2), \quad (2)$$

where on the condition of normalization $C=\left[\lambda\cdot(\bar{X}_1+x_2)\right]^{-1}$ In the case of random magnitude, the law expressed by formula (2) should be considered as a conditional law of distribution

$$\varphi_1(x_1)\ \varphi_{21}(x_2/x_1)=\frac{(x_1+x_2)}{x_1+\bar{X}_2}\cdot f_2(x_2)$$

$\varphi_{12}(x_1/x_2)$. So

$$\varphi_{12}(x_1/x_2)=\frac{(x_1+x_2)}{\bar{X}_1+x_2}\cdot f_1(x_1). \quad (3)$$

An unconditional distribution law is found by averaging in the (2) probability $p=\lambda\cdot(x_1+x_2)$ by length x_2

$$\varphi_1(x_1)=C_1\cdot f_1(x_1)\cdot\int_0^{x_{2m}}\lambda\cdot(x_1+x_2)\cdot f_2(x_2)dx_2=$$
$$=C_1\cdot\lambda\cdot f_1(x_1)$$

where C_1 is the normalizing factor. Thus

$$\varphi_1(x_1)=\frac{(x_1+\bar{X}_2)}{\bar{X}_1+\bar{X}_2}\cdot f_1(x_1);$$
$$\varphi_2(x_2)=\frac{(\bar{X}_1+x_2)}{\bar{X}_1+\bar{X}_2}\cdot f_2(x_2). \quad (4)$$

To determine the law $W_{12}(z/x_1,x_2)$, let us consider two vehicles with the lengths of x_1 and x_2 correspondingly, (Fig. 2), and for certainty we will accept $x_2 \ge x_1$.

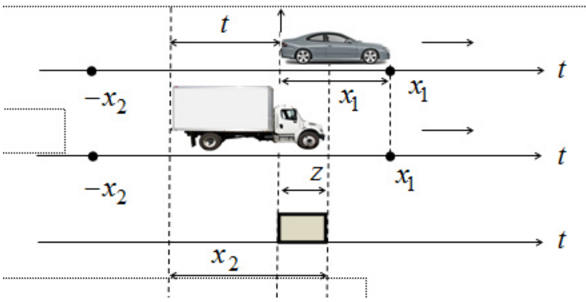

Fig. 2. The diagram of two vehicles coincidence

Let us mark as T a random magnitude which fixes the moment t of appearance of vehicles on the second traffic lane relative to the beginning of vehicles on the first traffic lane. When t changes in the interval $(-x_2,x_1)$, the random magnitude Z of coincidence duration changes according to trapezoid law $z(t)$ taking the value $0\le z\le x_1\le x_2$ (Fig. 3)

$$\begin{array}{lll} & -x_2\le t\le x_1-x_2 & z=x_2+t \\ \text{with} & x_1-x_2\le t\le 0 & z=x_1=\text{const}\ (5) \\ & 0\le t\le x_2 & z=x_1-t \end{array}$$

So, some value of z, where $0\le z\le x_1$, corresponds to two values of $t: t_1$ та t_2.

Section $[-x_2,(x_1-x_2))$ of the graph presented on Fig. 2 characterizes the coincidence of two vehicles, and segment $(0,x_1]$ characterizes their removal from each other. Segment $[(x_1-x_2),0]$ characterizes the complete coincidence of two vehicles and the value of this segment is determined by a longer vehicle, and its duration is determined by the speed of the vehicles convergence. The slope of the characteristics on segments $[-x_2,(x_1-x_2))$ and $(0,x_1]$ depends on the speed of convergence and drifting of vehicles from each other.

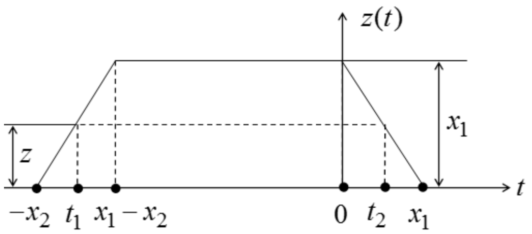

Fig. 3. The graph of changing duration of coincidence of two vehicles

Taking into account that the probability of coincidence $dp=\lambda dt$, shown in Fig. 2, we find an integral distribution law

$$F(z)=P(Z<z)=\frac{1}{p}\int_{-x_2}^{t_1}\lambda dt+\frac{1}{p}\int_{t_2}^{x_1}\lambda dt,$$

where $(0<z<x_1)$.

Integrating and taking into account the fact that according to (5) $t_1=z-x_2$ and $t_1=z-x_2$, and $p=\lambda\cdot(x_1+x_2)$, we find

$$F(z)=\frac{(t_1+x_2)+(x_1-t_2)}{(x_1+x_2)}=\frac{2\cdot z}{(x_1+x_2)},$$
$$(0<z<x_1).$$

At the point $z=x_1$ function $F(z)$ receives a finite increment

$$\Delta F=P(Z=x_1)=\frac{1}{p}\int_{x_1-x_2}^{0}\lambda dt=\frac{(x_2-x_1)}{(x_1+x_2)}.$$

Thus,

$$F(z)=\frac{2\cdot z}{(x_1+x_2)}+\frac{(x_2-x_1)}{(x_1+x_2)}\cdot H(z-x_1)$$
$$(z\geq 0), \qquad (6)$$

where $H(z)$ is a solitary function;
with $z<0$ $F(z)=0$.

Function (6) can be considered as a conditional integral function of distribution Z relative to X_1 and X_2, that is,

$$F(z)=F_{12}\left(\frac{z}{x_1},x_2\right)=P$$
$$(Z<z/X_1=x_1,X_2=x_2).$$

Hence the conditional distribution law is

$$W_{12}(z/x_1,x_2)=\frac{dF}{dz}=$$
$$\begin{Bmatrix}\dfrac{2}{(x_1+x_2)}+\dfrac{(x_2-x_1)}{(x_1+x_2)}\cdot\delta(z-x_1) & (0<z<x_1)\\ 0 & (z\leq 0,z>x_1)\end{Bmatrix}, \qquad (7)$$

where $\delta(z)$ is the delta function. A similar expression is found at $x_1>x_2$.

Substituting expressions (3), (4) and (7) into formula (1), we obtain

$$W_{(3)}(x_1,x_2,z)=$$
$$=\begin{Bmatrix}\dfrac{f_1(x_1)\cdot f_2(x_2)}{\bar{X}_1+\bar{X}_2}\left[2+(x_2-x_1)\cdot\delta(z-x_1)\right]\\ \dfrac{f_1(x_1)\cdot f_2(x_2)}{\bar{X}_1+\bar{X}_2}\left[2+(x_1-x_2)\cdot\delta(z-x_2)\right]\end{Bmatrix}$$

$$\begin{Bmatrix}(0<z<x_1)\\(z\leq 0,z>x_1)\end{Bmatrix}. \qquad (8)$$

Integrating, we define a two-dimensional distribution law

$$W_{(2)}(x_1,z)=\int_z^{x_2}\frac{f_1(x_1)\cdot f_2(x_2)}{\bar{X}_1+\bar{X}_2}\cdot[2+(x_1-x_2)\cdot$$
$$\cdot\delta(z-x_2)dx_2]+$$
$$+\int_{x_1}^{x_{2m}}\frac{f_1(x_1)\cdot f_2(x_2)}{\bar{X}_1+\bar{X}_2}\cdot\left[2+(x_2-x_1)\cdot\delta(z-x_1)dx_2\right]=$$
$$=\frac{f_1(x_1)}{\bar{X}_1+\bar{X}_2}[2\cdot\Psi_2(z)+(x_1-z)\cdot f_2(z)+$$
$$+\delta(z-x_1)\int_{x_1}^{x_{2m}}x_2\cdot f_2(x_2)dx_2-\delta(z-x_1)\cdot x_1\cdot\Psi_2(x_1)]$$

where accepted is

$$\Psi_i(z)=1-\int_0^z f_i(y)dy. \qquad (9)$$

Integrating again, we find a one-dimensional law of distribution of duration of vehicles' coincidence on two $(n = 2)$ lanes of motion

$$W(z) = W_2(z) = \int_z^{x_{1m}} W_{(2)}(x_1, z)\,dx_1 =$$
$$= \frac{1}{\bar{X}_1 + \bar{X}_2}[2 \cdot \Psi_1(z)\Psi_2(z) + f_1(z)\int_z^{x_{2m}} \Psi_2(x)\,dx +$$
$$+ f_2(z)\int_z^{x_{1m}} \Psi_1(x)\,dx]. \qquad (10)$$

When $f_1(x_1) = f_2(x_2) = f(x)$,

$$W(z) = W_2(z) = \frac{1}{\bar{X}}\left\{[\Psi(z)]^2 + f(z)\int_z^{x_m} \Psi(x)\,dx\right\}. \ (11)$$

The task of determining one-dimensional law $W(z)$ of the distribution of the duration of vehicles' coincidence by n lanes $(n > 2)$ is equivalent to the task of determining the law of distribution of the duration of the vehicles' coincidence by two lanes, one of which is the first given lane, and the other one is the equivalent lane, the length of the vehicle on which is equal to the length $z = z_{n-1}$, the coincidence of vehicles on $n-1$ given lanes. So, knowing distribution law $W_{n-1}(z)$ for the $n-1$ lanes, we can find distribution law $W_n(z)$ using formula (10).

Let us find the probability p_2 of coincidence of vehicles for two lanes of motion, the lengths of which are distributed according to laws $f_i(x_i)$ $(i = 1, 2)$, provided that vehicles appear on one of the lanes.Let us assume that a vehicle of a certain length x_1 moves on the first lane of the road. Then the probability of a case A_{x_1} / x_2, consisting in the coincidence of the vehicle with the length x_2 that appeared in the second lane with the vehicle of length x_1, is expressed by the equality [7]

$$P(A_{x_1} / x_2) = \lambda \cdot (x_1 + x_2). \qquad (12)$$

This probability can be considered as the probability of case A_{x_1} associated with the hypothesis x_2, whose probability is equal to $f_2(x_2)dx_2$. Therefore, the average (full) probability of coincidence of the vehicle with the length x_1 with a vehicle of any length (no matter which) moving along the second lane of motion, is found by averaging probability (12) by x_2

$$P(A_{x_1}) = \int_0^{x_{2m}} \lambda \cdot (x_1 + x_2) \cdot f_2(x_2)\,dx_2 =$$
$$= \lambda \cdot (x_1 + \bar{X}_2). \qquad (13)$$

In turn, the probability expressed by the formula (13) can be considered as a conditional probability $P(A / x_1)$ of case A (the coincidence of two vehicles of the same or different length, moving along two adjacent lanes of the road) associated with the hypothesis x_1 (the one of occurrence of the vehicle with length x_1 on the first lane), whose probability is equal to $f_1(x_1)dx$. Thus, the sought probability of coincidence of vehicles of random length on two lanes is

$$p_2 = \int_0^{x_{1m}} \lambda \cdot (x_1 + \bar{X}_2) \cdot f_1(x_1)\,dx_1 =$$
$$= \lambda \int_0^{x_{1m}} \int_0^{x_{2m}} (x_1 + x_2) \cdot f_1(x_1) \cdot f_2(x_2)\,dx_1 dx_2 =$$
$$= \lambda \cdot (\bar{X}_1 + \bar{X}_2). \qquad (14)$$

The given considerations can be extended to any number of n lanes $(i = 1, 2, \ldots, n)$. For unchanged lengths of vehicles for each of the lanes, the probability of their coincidence (subject to the appearance of vehicles on one of the lanes) is expressed by equality [7]

$$p_n^{'} = \lambda^{n-1} \sum_{i=1}^{n} \frac{1}{x_i} \prod_{i=1}^{n} x_i. \qquad (15)$$

The probability of coincidence of vehicles of random length along the n lanes is found by averaging probability $p_n^{'}$ by lengths x_i on all lanes of motion.

$$p_n = \lambda^{n-1} \int_0^{x_{1m}} \ldots \int_0^{x_{nm}} \sum_{i=1}^{n} \frac{1}{x_i} \prod_{i=1}^{n} x_i \cdot f_i(x_i)\,dx_i =$$
$$= \lambda^{n-1} \sum_{i=1}^{n} \frac{1}{\bar{X}_i} \prod_{i=1}^{n} \bar{X}_i. \qquad (16)$$

Thus, as to the probability of coincidence, the lane of motion of a vehicle of a random length is equivalent to a lane of motion of a vehicle of a permanent length, which is equal to the mathematical expectation of the length of the vehicles on the lane of the road.

It can be said that the distribution of moments of occurrence of vehicles involved in the coincidence of vehicles is also subject to the Poisson law with average density

$$\lambda_n = \lambda \cdot p^n . \qquad (17)$$

Mathematical expectation $\bar{Z} = \bar{Z}_2$ of the duration of the coincidence of vehicles on two lanes of motion $(n = 2)$ can be found by formula (10). Let us look into more general considerations. By unchanged lengths of vehicles x_1 and x_2 the average duration of coincidence of vehicles on two lanes of motion is expressed by formula [3] $\bar{Z} = x_1 \cdot x_2 / (x_1 + x_2)$. By random values of X_1 and X_2, the result expressed by the latter equality, must be averaged by lengths x_1 and x_2 of the vehicles involved in the coincidence of the two vehicles, that is

$$\bar{Z}_2 = \int_0^{x_{1m}} \int_0^{x_{2m}} \frac{x_1 \cdot x_2}{(x_1 + x_2)} \cdot \varphi_{(2)}(x_1, x_2)\, dx_1 dx_2 , \qquad (18)$$

where, according to equalities (3) and (4),

$$\varphi_{(2)}(x_1, x_2) = \varphi_1(x_1) \cdot \varphi_{21}(x_2 / x_1) = = \frac{(x_1 + x_2)}{\bar{X}_1 + \bar{X}_2} \cdot f_1(x_1) \cdot f_2(x_2) . \qquad (19)$$

Substituting (19) into (18) and integrating, we find

$$\bar{Z}_2 = \frac{\bar{X}_1 \cdot \bar{X}_2}{\bar{X}_1 + \bar{X}_2} .$$

When $n = 3$, it is possible to replace two lanes with one lane, on which there are vehicles of random length z_2, from where

$$\bar{Z}_3 = \frac{\bar{Z}_2 \cdot \bar{X}_3}{\bar{Z}_2 + \bar{X}_3} = \left(\frac{1}{\bar{X}_1} + \frac{1}{\bar{X}_2} + \frac{1}{\bar{X}_3} \right)^{-1} .$$

Passing from p and $p+1$ lanes of motion, we find the average value of duration of coincidence of vehicles by n lanes

$$\bar{Z}_n = \left(\sum_{i=1}^{n} \frac{1}{\bar{X}_i} \right)^{-1} . \qquad (20)$$

We emphasize that relation (20) does not depend on the type of distribution laws.

Let us consider the case where the length of vehicles on the lanes is distributed according to the limited exponential law

$$f(x) = \frac{A}{x^*} e^{\frac{-x}{x^*}} \qquad (0 < x \le x_m), \qquad (21)$$

where x^* – is the parameter of the distribution lawand A – is the normalizing factor

$$x^* = \int_0^{\infty} \frac{x}{x^*} e^{\frac{-x}{x^*}} dx \, ; A = \frac{1}{1 - e^{-\gamma_m}} \, ; \gamma_m = \frac{x_m}{x^*} . \qquad (22)$$

We note that with limited law (21), the average length is $\bar{X} \ne x^*$

$$\bar{X} = \int_0^{x_m} x \cdot \frac{A}{x^*} \cdot e^{\frac{-x}{x^*}} dx = A \cdot x^* \left[1 - (1 + \gamma_m) \cdot e^{-\gamma_m} \right] = = x^* \cdot \left[1 - \gamma_m \cdot e^{-\gamma_m} - \gamma_m \cdot e^{-2\gamma_m} - \ldots \right] . \qquad (23)$$

According to expressions (9) and (11), for the case $n = 2$ we find

$$W_2(z) = \frac{A^2}{\bar{X}^2} \cdot \left[2 \cdot e^{-2\gamma} + e^{-2\gamma_m} + e^{-\gamma_m - \gamma} \cdot (\gamma - \gamma_m - 3) \right], \qquad (24)$$

$$\text{where } \gamma = \frac{z}{x^*} \qquad \left(0 \le \gamma \le \gamma_m = \frac{x_m}{x^*} = \frac{z_m}{x^*} \right) . \qquad (25)$$

Let us consider the boundary case, when x^* is so small that we can take $x_m \to \infty$ (practically $x_m \gg x^*$). In this case $\bar{X} \to x^*$, $A \to 1$ and

$$W_2(z) = W_2(z)_{x_m \to \infty} = \frac{2}{x^*} \cdot e^{-2\frac{z}{x^*}} = \frac{1}{x_2^*} \cdot e^{-\frac{z}{x_2^*}} , \qquad (26)$$

where $x_2^* = x^* / 2$.

At that, the average duration of the coincidence is $(\bar{Z}_2)_{x_m \to \infty} = x_2^* = x^* / 2$. Actually, when

$x_m \geq 5 \cdot x^*$, one may ignore the restrictions on the duration of the distribution law (22). This is illustrated by the relation of mathematical expectations

$$\frac{\bar{Z}_2}{(\bar{Z}_2)_{x_m \to \infty}} = \frac{\bar{Z}_2}{x^*/2} = \frac{1-(1+\gamma_m)\cdot e^{-\gamma_m}}{1+e^{-\gamma_m}} = \\ = 1-\gamma_m\left(e^{-\gamma_m}+e^{-2\gamma_m}+...\right). \quad (27)$$

Assuming $f(x)=\frac{1}{x^*}\cdot e^{-\frac{x}{x^*}}$ it is not difficult to find the distribution law of the duration of the vehicles coincidence by n lanes.

$$W_n(z)=\frac{n}{x^*}\cdot e^{-\frac{nz}{x^*}}=\frac{1}{x_n^*}\cdot e^{-\frac{z}{x_n^*}},$$

where $x_n^* = x^* / n = \bar{Z}_n$.

As we can see, the exponential law of the distribution of the vehicles' length by the lanes has stability: the duration of the coincidence is also distributed according to the exponential law.

Let us determine the probability of a collision of vehicles for lanes of I–IV categories of roads by violation of the rules of manoeuvring and driving the vehicle onto the adjacent or oncoming lane of motion. When the vehicle moves onto the oncoming traffic lane, the probability of vehicles' collision is denoted by p which depends on a number of parameters, including: flow density, vehicles' motion speed, condition of road surface, mental and physical state of the driver, his behaviour on the road, time of day, day of the week and other random factors.

We will mark the following cases for categoriesII–IV of roads. Let A_1 be the case when a vehicle goes on the roadside with the control lost, and A_2 is the case when a vehicle goes onto the oncoming traffic lane. Then the probability of a collision will be equal to

$$P(A_1)=P(A_2)=0,5. \quad (28)$$

We will mark with F the case of a collision of vehicles at vehicle moving onto the oncoming traffic lane, and G – will be the case of vehicles collision for the II–IV categories of roads. Then

$$P(F)=P(F|A_2)=p. \quad (29)$$

From where

$$P(G)=P(A_1)\cdot P(F|A_1)+P(A_2)\cdot P(F|A_2)= \\ =0,5\cdot 0+0,5\cdot p. \quad (30)$$

Obviously, expression (2) is valid for the adjacent (second) lane of one traffic direction for the road of category I. Therefore, expression (2) will be written in the form of

$$P(F_1)=P(F_1|A_2)=p. \quad (31)$$

Let us determine the probability of vehicles' collision on the third lane of motion in the same direction. Let's mark the case of collision of vehicles on the third lane of traffic with F_2. Then

$$F_2=\left(\bar{F}\cap F\right). \quad (32)$$

From where the probability of the collision will be equal to

$$P(F_2|A_2)=P(\bar{F})\cdot P(F)=(1-p)\cdot p, \quad (33)$$

and the probability of case G will be equal to

$$P(G)=P(A_2)\cdot P(F_2|A_2)= \\ =0,5\cdot\left(p+(1-p)\cdot p\right). \quad (34)$$

Similarly, expressions (32) and (33) at the collision of vehicles on the fourth lane will be rewritten in the form of

$$F_2=\left(\bar{F}\cap\bar{F}\cap F\right), \quad (35)$$

$$P(F_3|A_2)=P(\bar{F})\cdot P(\bar{F})\cdot P(F)= \\ =(1-p)\cdot(1-p)\cdot p. \quad (36)$$

Then the probability of the case G will be equal to

$$P(G)=P(A_2)\cdot P(F_3|A_2)= \\ =0,5\cdot\left(p+(1-p)\cdot(1-p)\cdot p\right). \quad (37)$$

Let us mark the collision of vehicles on any of four lanes for the first category road with V. Then

$$V=\left(F_1\cup F_2\cup F_3\right), \quad (38)$$

From where the probability of the collision will be equal to

$$P(V|A_2) = P(F_1|A_2) + P(F_2 \mid A_2) + P(F_3 \mid A_2) =$$
$$= p + (1-p)\cdot p + (1-p)\cdot(1-p)\cdot p\,. \quad (39)$$

Then the probability of case G for the first category road will be equal to

$$P(G) = P(A_2)\cdot P(V|A_2) =$$
$$= 0{,}5\cdot\left(p + (1-p)\cdot p + (1-p)^2\cdot p\right). \quad (40)$$

Taking into account the traffic in both directions for roads of I–IV categories with different number of lanes in each direction, with the loss of controllability of the vehicle and its collision with another vehicle on one of the adjacent lanes, the probability of case G can be written in general as

$$P(G) = 0{,}5\cdot p\cdot\sum_{k=0}^{n-2}(1-p)^k\,. \quad (41)$$

Conclusion

The probability of n coincidence of vehicles of random length, whose moments of appearance on the lanes of motion are subject to the Poisson law, is determined only by the average value of the lengths of the vehicles on the lanes and the average frequency of their repetition.

The average value of duration of n vehicles coincidence under the specified conditions is determined only by the average value of the lengths of the vehicles on the lanes, regardless of the laws of distribution of the vehicles' lengths by the lanes.

The law of the distribution $W_n(z)$ of duration of n vehicles' coincidence, generally speaking, significantly depends on the laws of distribution $f_i(x_i)$ of lengths x_i of vehicles on the lanes. However, with the increase in the number of n lanes, the law $W_n(z)$ approaches the exponential one. At the exponential law of the distribution of vehicles lengths on the lanes, the duration of n vehicles coincidence is also distributed according to the exponential law.

Taking into account the traffic in both directions for roads of categoriesI–IV with different number of lanes in each direction, with the loss of controllability of the vehicle on one of the lanes, the probability of case G (collision of vehicles on one of the lanes) is determined by expression (41).

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