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APPLICATION OF THE ZEROS AND POLES MATCHED METHOD FOR MODELING OF ELECTRICAL SYSTEMS

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The widespread use of mathematical applications, which include differential equations solvers, and the increase in the speed of computing devices have led to a decrease in interest in operator methods, in particular, the z-transform. Nevertheless, the use of the z-transform capabilities allows the implementation of efficient high-speed computing schemes with high numerical stability. The need for this may arise in the case of real-time simulation or the synthesis of digital control systems. Based on the analysis of literary sources, the relevance and advantages of using the z-transform for modeling the dynamics of electrical engineering systems are shown.

The method of computer modeling is considered, the basis of which is the use of the method of matching zeros and poles of an equivalent continuous transfer function to build a computer model. The process of implementing the modeling recurrent formulas obtained by this method is shown for three elementary dynamic blocks, which are obtained as a result of the expansion of the transfer function according to the Heaviside residue theorem: integral (zero pole), first-order inertial (real pole) and second-order blocks with a real zero and by a pair of complex-conjugated poles. In this way, the parallel decomposition of the researched system is implemented, which makes it possible to reduce the negative impact of the limited bit precision of the system and facilitate the execution of parallel calculations. A discrete transfer function and a simulation recurrent equation were obtained for each such block.

The practical use and advantages of this method are shown on two examples: a simple elastic joint mechanical system, which is described by a second-order differential equation, and a nonlinear model of an asynchronous machine based on a single-phase T-shaped equivalent circuit. Both problems are illustrated by examples of solutions in the environment of the Mathcad mathematical application. The effectiveness of the zeros and poles matched method of compared to classical numerical methods for solving ordinary differential equations is shown.

The use of this method of mathematical modeling makes it possible to provide a stable numerical solution with a specified accuracy for a wide range of solution steps.

Key words: computer simulation; electrical systems; integration numerical methods; transfer function; z-transform; zero-poles matched method.

Introduction

The development of modern microelectronics causes both an increase in the computing power of modern computer systems, which makes it possible to expand the range of solved problems in electrical engineering, in particular, in power engineering and the emergence of advanced microcontroller systems, which lets to implement intelligent control laws in almost all technology's fields. The other side of this process is the progress of a wide range of software, which greatly facilitates access even for an untrained user to the capabilities of modern computer systems. It is worth noting that all this together contributes to the set of problems, the solution of which was previously impossible to even dream of. As an example, we can cite the task of analyzing the dynamics of a large electric power system, in particular, studying its stability, in real-time [1], which is necessary for the operational management of such systems, especially with the development of SmartGrid technologies.. Another example of the task of a complex electrotechnical systems' analyzing is the study of the modern steel furnaces modes [2].

The mathematical foundation of computer modeling of electrical engineering systems is based on classical numerical methods, which, given their origin, are designed to work with smooth continuous functions without discontinuities, that is, those for which there is a Taylor series expansion. As the practice of computer modeling shows, the existing traditional methods of studying the dynamics of modern electric power systems, in particular, with impulse elements, are in many cases poorly adapted or just unsuitable for practical use, but currently, the developers of mathematical applications do not offer any alternative [3]. The development of electrical engineering and its emergence to a new level requires a new method that will provide information about the arisen problem and ways to fast solve it with great accuracy in real-time. Therefore, there is a need to create new effective methods for the development of high-speed computer models of electrical systems, which would be expedient to use in modern systems with impulse elements, starting from simple control algorithms for microcontroller systems and ending with advanced models of whole electrical power systems intended for development programming on powerful multiprocessor systems. As the research carried out by the authors showed, integral methods, in particular, z-transformation, turned out to be quite effective.

Formulation of the problem

Thus, the goal of the research, which proposed in the article, is to illustrate the advantages for the digital control systems synthesis and computer simulation of the electrical systems dynamics in real-time the using z-transformation, in particular, the zeros and poles matched method.

The possibility of using zeros and poles matched method for the transient processes analysis of linear and nonlinear electromechanical systems with higher computational efficiency than classical numerical methods is shown.

Analysis of previous research and publications

Historically, the traditional way of solving problems of the technical systems' dynamics is to describe the investigated system by ordinary differential equations followed by their solution [1]. Most often, appropriate numerical methods are used for this as the most universal method [3–5]. As mentioned earlier, classical numerical methods are an approximation of the solution by a limited expansion in the Taylor series [4], which exists in the condition of smooth continuous differentiable functions. At the same time, in most problems of modern electrical engineering, this situation doesn't work in the case of the need to analyze impulse systems, switching in equipment, etc., that is, in the case of real practical problems of electrical engineering, for which the solution function is not smooth and differentiated. To some extent, this problem is solved by new algorithms for adapting the step of solving the numerical method to the behavior of the function, for example, they search for the switching point, choosing it as the initial conditions for the next step [3, 4].

The advantages of integral methods of solving problems of the technical systems' dynamics are shown in the publications of Prof. A. F. Verlan [6–8], prof. R. V. Filts [9, 10] etc. Integral methods also include operator methods, in particular, z-transformation, the effectiveness of which for solving a wide range of problems of the technical systems' dynamics has been shown both by its authors [11, 12] and by more modern authors [13].

The explanation of the traditional method shortcomings of solving dynamics problems is quite simple: during the differentiation operation, some part of the information is lost, for example, we can cite the operation of differentiating a polynomial, which leads to a decrease in its order. In the case of solving differential equations, they try to recover lost information by taking into account the initial conditions (Cauchy problem), however, in the occasion of using numerical methods, there are problems with the appearance of numerical instability and the accumulation of errors in the solution function. By the way, from a technical point of view, there is a simple explanation: the differentiator (or its mathematical analog – the operation of differentiation) is very sensitive to noise and interference, moreover, the higher the frequency of interference, the stronger its impact. Such noises and interferences in the situation of numerical methods are arithmetic rounding errors, numerical method errors etc. At the same time, integral methods are "not afraid" of discontinuities in the solution function, they solve problems of dynamics with impulse elements without any problems [6–10].

Due to the efficiency, one of the main engineering methods of designing and simulation the widest range of discrete systems, in particular, digital regulators of control systems and digital filters, has become an approach using z-forms (*known as the Tustin substitution*) – an approximate method of obtaining a discrete transfer function on the basis of an analog prototype by replacing the continuous integration operation $\frac{1}{s}$ with its discrete approximation $\frac{h}{2}\frac{1+z^{-1}}{1-z^{-1}}$ (*corresponds to the implicit trapezoids formula*). The disadvantage of this method is low accuracy, which decreases with increasing step, due to which the Tustin substitution is used in digital systems and computer models with an operating frequency that is at least 10–15 times lower than the sampling frequency, which is determined for a digital system by the sampling theorem of Claude Shannon [13].

One of the main disadvantages of using the z-transformation for computer models is the significant amount of necessary analytical work, which is quite difficult to automate even with the help of modern mathematical computer programs. As the complexity of the modeling object grows, the complexity of the building process of a discrete model increases disproportionately, this motivate the residue the complete model into simpler components, which usually causes a loss of accuracy due to the appearance of additional discretization nodes [11–13]. The transition to engineering methods that use simplified methods to build the discrete transfer functions, for example, bilinear transformation or z-forms, leads to a decrease in accuracy and dependence on the discretization period [13].

Presenting main material

Theoretical information

The Z-transform arose from the need to define a samples of discrete signal values in the form of a numbers sequence, which is used by communications and control engineers to study the digital control and in telemetry systems. One of the main problems with applying the Z-transform is to obtain a discrete transfer function $W(z)$ that describes the system.

After obtaining the discrete transfer function, the problem of finding the output signal in the time domain $y(t)$ arises. The most appropriate procedure for solving such a problem with the assistance of computer technology is to obtain a recurrent formula, which can be used to calculate sequential values of the desired original function.

The great value of the z-transform is that with its using, as well as with the using Laplace transform, for a known input signal and for a linear system, it is possible to obtain an analytically true solution for the output signal. Unfortunately, this situation rarely happens in modeling, because in complex feedback systems, the input signals of the internal units are often the sum of the signals from the outputs of the previous components and the feedback. In such a case, it is practically impossible to obtain an analytical expression of the Laplace transform for the input signal of a separate block or system, so it is necessary to apply procedures for transforming the input signal – its approximation by simpler functions for which the Laplace transform exists.

It is worth noting that understanding the necessity of the process of transformation (or approximation) of the input signal and the physical sense of such a process in z-transformation is a non-trivial problem. It may help to realize that finding the system output using the z-transform is an analytical process, just like the Laplace transform.

If it is impossible to obtain an analytical expression for the signal at the input of the studied component, devices that are called holders in the theory of automatic control come to the rescue – on a given interval of length h, they approximate (*hold*) an arbitrary input signal by the pieces of low-order polynomials (*usually not higher than the first order*). In this case, the analytical description of the input signal becomes the transfer function of the corresponding holder. However, in this occasion, this description is approximate, because, as is known, polynomial approximation for an arbitrary curve is performed only with definite accuracy (approximation).

The progress of mathematical applications with the symbolic mathematic capabilities, for example, Mathcad, Maple, MATLAB (Symbolic Math Toolbox) etc., allowed to simplify the obtaining procedure of discrete transfer functions for systems described by continuous transfer functions using the Laplace transform. For this, the method of matching zeros and poles of the transfer function is used, a short algorithm of which is given below [11–13].

1. For the continuous transfer function of the studied system model, find all zeros Z_i ($i = 1 \dots m$) and poles P_j ($j = 1 \dots n$), where the order of the numerator polynomial is m, and the order of the denominator polynomial is n , moreover $n \geq m$.

2. Go to the discrete transfer function, which is written in the form of zeros and poles using the well-known relation [13] $z = e^{sh}$, where h – is the sampling period (time discretization step):

- accordingly, discrete zeros will be determined by the expression $Z_{d_i} = e^{z_i h}$, where $i = 1 \dots m$;
- accordingly, the discrete poles will be determined by the expression $P_{d_j} = e^{p_j h}$, where $j = 1 \dots n$.

3. Find the corrected K^* transfer coefficient for the discrete transfer function.

This method can be summarized as follows:

$$K \frac{\sum_{i=1}^{m}(s - Z_i)}{\sum_{j=1}^{n}(s - P_j)} \Rightarrow K^* \frac{\sum_{i=1}^{m}\left(z - e^{Z_i \cdot h}\right)}{\sum_{j=1}^{n}\left(z - e^{P_j \cdot h}\right)} \Rightarrow K^* = \lim_{s \to 0} K \frac{\prod_{i=1}^{m}(s - Z_i)}{\prod_{j=1}^{n}(s - P_j)} \cdot \lim_{z \to 1} \frac{\prod_{j=1}^{n}\left(z - e^{P_j \cdot h}\right)}{\prod_{i=1}^{m}\left(z - e^{Z_i \cdot h}\right)}.$$

Note that for linear and linearized system models and known input signals, this method gives an analytically true result [14, 15].

The application of the transfer function decomposition of the electrical system into elementary dynamic components simplifies the process of using this method – responses (or discrete transfer functions) are found for elementary components, and then summed up to obtain the final system response. In the presence of non-linear elements, the principle of piecewise linear approximation can be applied, which makes it possible to consider the electrical system as linear at each separate interval. In this case, the sampling time can be chosen sufficiently small under the condition of the errors reducing of the piecewise linear approximation. Such a procedure is possible for real electrical systems, which, as mentioned earlier, can be described by fractional-rational transfer functions, for which the order of the numerator polynomial does not exceed the order of the denominator polynomial. This makes it possible to decompose the transfer function of such a system into elementary components according to the basic theorem of algebra (also known as the Heaviside decomposition theorem) – fractions of the form $\frac{b}{s}$, $\frac{b}{s+a}$ i $\frac{c \cdot s+d}{s^2+p \cdot s+q}$ (see Fig. 1 and Fig. 2):

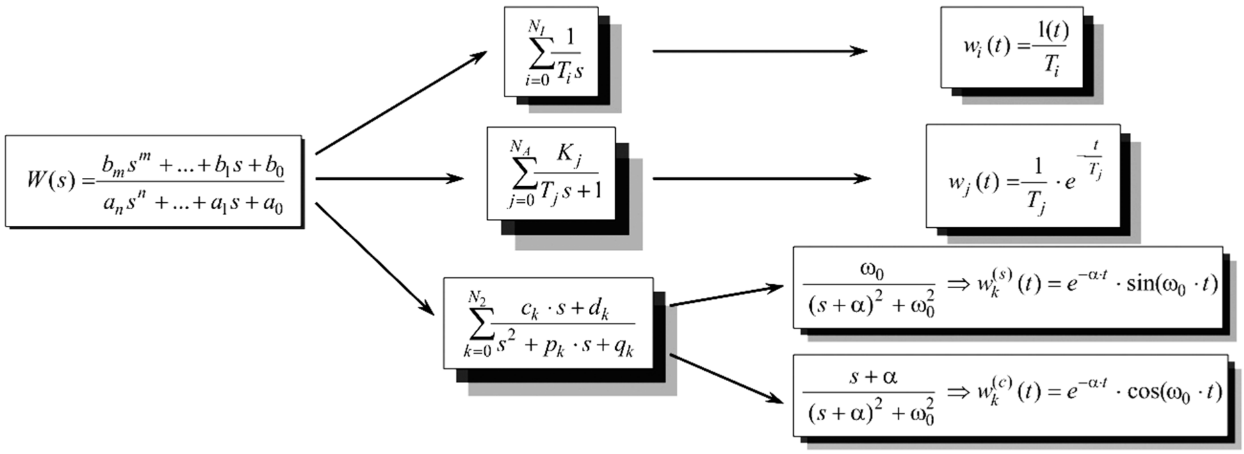

Fig. 1. Decomposition scheme of the transfer function into elementary components

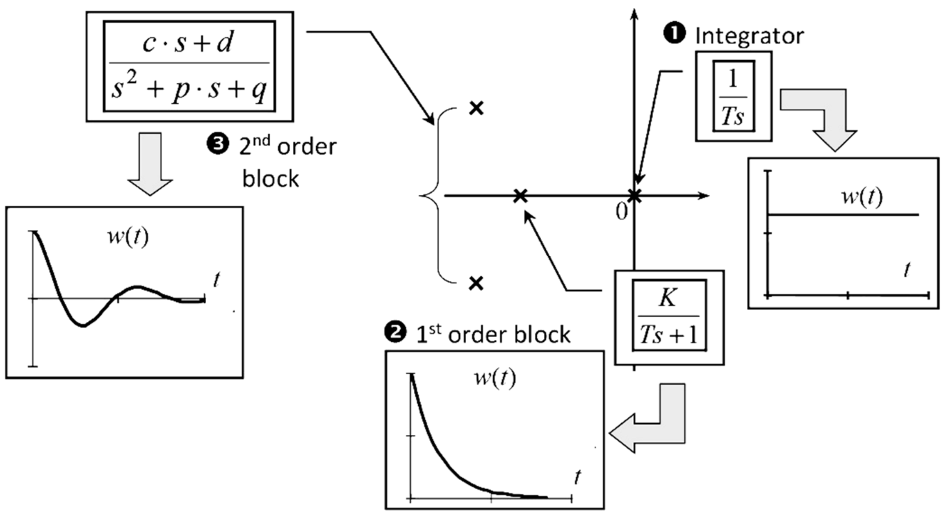

Fig. 2. Location of the poles of elementary dynamic blocks and their corresponding impulse response

- $\frac{b}{s}$ – corresponds to an integrating circuit (or zero pole) having a transfer function $\frac{1}{T \cdot s}$ and an impulse response $w(t)=\frac{1(t)}{T}$;
- $\frac{b}{s+a}$ – corresponds to a first-order block (or real pole) having a transfer function $\frac{K}{T \cdot s+1}$ and an impulse response $w(t)=\frac{K}{T} \cdot e^{-\frac{t}{T}}$;
- $\frac{c \cdot s+d}{s^2+p \cdot s+q}$ – this component takes into account a pair of complex-conjugated poles and is the sum of two components with corresponding weighting coefficients:

– with a transfer function $\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$, that has an impulse response $w(t)=e^{-\alpha t} \cdot \sin(\omega_0 \cdot t)$ (*sine component*);

– with a transfer function $\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$, which has an impulse response $w(t)=e^{-\alpha \cdot t}\cdot\cos(\omega_0\cdot t)$ (*cosine component*).

Using the decomposition of the complete transfer function of the electrical system into elementary components with impulse responses $w_i(t)$ makes it possible to apply its parallel decomposition to further find the response of the entire system as the sum of the responses of simpler (elementary) components (Fig. 3). The use of parallel decomposition has a significant advantage over other methods:

- in this occasion, only one point (node) of discretization of the continuous signal *x(t)* is introduced in order to create its discrete approximation $x^*(t)$, which reduces the total errors due to the loss of information during the transition to a discrete system;
- determination of the roots of the characteristic equation of the continuous transfer function followed by obtaining their discrete mapping in the unit circle (for a discrete system) significantly reduces the impact of possible singularity of the denominator polynomial of the discrete transfer function with a step's decrease. This makes it possible to reduce the sensitivity of the synthesized discrete system (namely, the resulting recurrent equation) to rounding errors and the limited bit rate of the computing device [16].

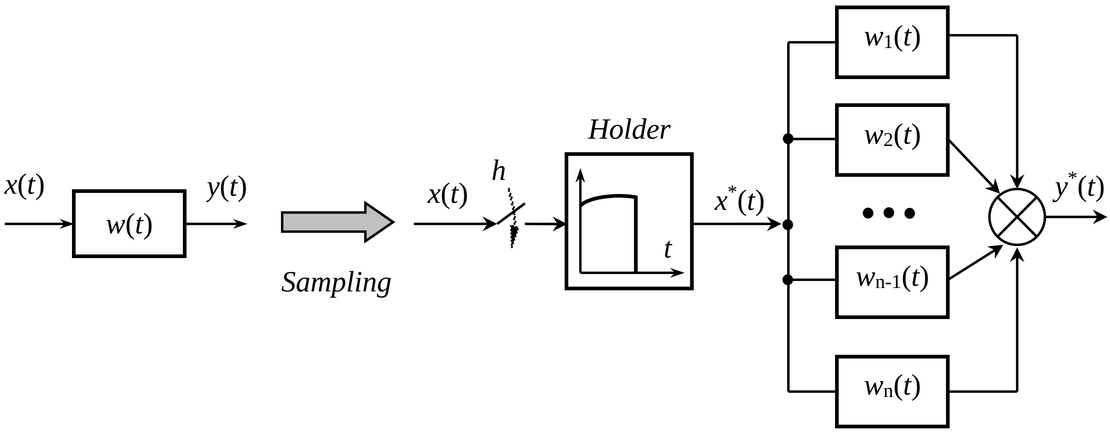

Fig. 3. Parallel decomposition of the continuous transfer function

Recurrent equations for simulation

Reducing all possible options for finding transient processes of dynamic blocks to only a small set makes it possible to develop an acceptable option for building a computing scheme. We note that all recurrent formulas obtained in this way are absolutely stable for any sampling periods (and in the situation of mathematical and computer simulation – the solution step).

In the case of a continuous integrator, we will have a zero pole, which correspond to a single discrete pole. Based on the obtained discrete transfer function, we obtain a recurrent equation for simulation:

$$\frac{1}{Ts} \Rightarrow \frac{h/T}{z-1} \Rightarrow y_{i+1}=y_i+\frac{h}{T}\cdot x_i .$$

The first order block corresponds to a unit real pole and is an analog of the ordinary differential equation of the first order $T\frac{dy}{dt}+y=K\cdot x$ – this is its universality as a basic element in the process of synthesis of digital systems, in particular, during computer simulation. Using the matching of the unit pole, we get its digital model [13, 17]:

$$\frac{K}{Ts+1} \Rightarrow \frac{\left(1-e^{-\frac{h}{T}}\right)K}{z-e^{-\frac{h}{T}}} \Rightarrow y_{i+1}=y_i\cdot e^{-\frac{h}{T}}+\left(1-e^{-\frac{h}{T}}\right)\cdot K\cdot x_i .$$

Applying the matching (correspondence) of zeros and poles to the continuous transfer function of the second-order block $\frac{c\cdot s+d}{s^2+p\cdot s+q}$ leads to such a discrete transfer function and to the corresponding recurrent formula:

$$\frac{c\cdot s+d}{s^2+p\cdot s+q} \Rightarrow \frac{(s-Z_1)}{(s-P_1)(s-P_2)}\begin{cases} Z_1=-d/c; \\ P_{1,2}=\dfrac{-p\pm\sqrt{p^2-4q}}{2} \end{cases} \Rightarrow \begin{cases} Z_1^*=e^{-d\cdot h/c}; \\ P_{1,2}^*=e^{h\cdot\frac{-p\pm\sqrt{p^2-4q}}{2}}; \end{cases} \Rightarrow$$

$$\Rightarrow \frac{K^*\left(z-Z_1^*\right)}{\left(z-P_1^*\right)\left(z-P_2^*\right)} \Rightarrow \frac{K^*\left(z-Z_1^*\right)}{z^2-(P_1^*+P_2^*)z+P_1^*P_2^*} \Rightarrow \frac{\left(z-V_C\right)\cdot K^*}{z^2-V_A\cdot z+V_B},$$

where the auxiliary variables are:

$$V_A=2e^{-\frac{p}{2}h}\cdot\cosh\left(\frac{\sqrt{p^2-4q}}{2}h\right);\ \ V_B=e^{-h\cdot p};\ V_C=e^{-\frac{d}{c}h}.$$

Find the corrected transfer coefficient of the discrete transfer function:

$$\lim_{s\to 0}\frac{c\cdot s+d}{s^2+p\cdot s+q}=\frac{d}{q} \Rightarrow \frac{d}{q}=\lim_{z\to 1}\frac{\left(z-V_C\right)\cdot K^*}{z^2-V_A\cdot z+V_B} \Rightarrow K^*=\frac{d\cdot(1-V_A+V_B)}{q\cdot(1-V_C)}.$$

After simple algebraic transformations, we will have a recurrent modeling formula:

$$y_{i+1}=V_Ay_i-V_By_{i-1}+K^*\cdot\left(x_i-V_Cx_{i-1}\right).$$

The found discrete transfer function has one real discrete zero and two discrete poles, which in the case of a stable continuous system will also fall into the unit circle of the region of discrete systems stability.

Note that these recurrent equations, which are obtained from the corresponding discrete transfer functions, are stable for any simulation step (sampling period).

Practical use

The recurrent equations obtained by the zeros/poles matching method, as already mentioned, are stable **for any** sampling period or simulation step, since they have, as it is usual to say in applied mathematics, the property of strong stability. In this case, the choice of the sampling period for the recurrent equations obtained in this way is no longer determined by the stability conditions of the numerical method, but by the desired level of accuracy and detail of the investigated process. Practically the only negative factors in case of step increase are:

- Phase shift or phase error due to time sampling processes [13, 18]. Such a phase error in a feedback digital control system or in the computer simulation process of feedback automatic control systems can lead to an unstable digital model, although the continuous prototype is stable.
- Loss of information arising as the result of the Shannon sampling theorem, when due to a long sampling period (simulation step) the system no longer passes the high-frequency components of the working signal spectrum. This situation is less critical than the previous one, because:

1) a correctly designed automatic control system is a low-pass filter, so the content of higher components in the operating spectrum is insignificant;
2) the loss of a small part of the higher components of the spectrum usually does not lead to phase shifts and therefore does not affect the system stability.

Experimental results

We will show the use of the zeros and poles matching method for the calculation of dynamic processes on the examples of models of electromechanical systems elements: model of a simple elastic mechanical system and a single-phase model of an asynchronous motor with a squirrel cage.

Elastic mechanical system

An example of the procedure for finding simulation recurrent equations for a mechanical system with elastic connections is illustrated on a simple model (Fig. 4), which is described by a second-order differential equation $m \cdot x'' + \beta \cdot x' + C \cdot x = F$, where *m* is a mass of the moving part of the mechanical system; β is the damping factor; *C* is a coefficient of elasticity; *F* is an external force; *x* is a movement of the moving part of the mechanism [19].

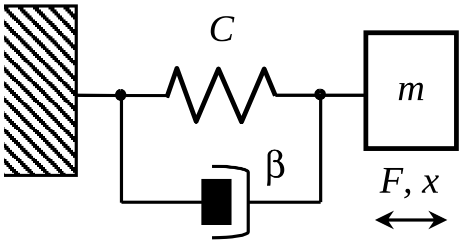

Fig. 4. Functional diagram of a simple elastic mechanical system

Using the zeros and poles matched method directly to a mechanical system with a transfer function $\frac{F}{ms^2 + \beta s + C}$, corresponding to a pair of complex-conjugate poles $P_{1,2}$, begins with finding continuous zeros and poles of this system. This procedure is simple, in particular, with the use of mentioned modern mathematical applications:

$$ms^2 + \beta s + C \;\Rightarrow\; P_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 4Cm}}{2m}.$$

Accordingly, a pair of discrete poles $P_{1,2}^*$ will look like $P_{1,2}^* = e^{\frac{-\beta \pm \sqrt{\beta^2 - 4Cm}}{2m} h}$. The resulting characteristic equation of the discrete transfer function will have the form:

$$(z - P_1^*)(z - P_2^*) = \left(z - e^{\frac{-\beta + j\sqrt{4Cm - \beta^2}}{2m} h}\right)\left(z - e^{\frac{-\beta - j\sqrt{4Cm - \beta^2}}{2m} h}\right) \quad\Rightarrow\quad z^2 - 2\cos\left(\frac{h\sqrt{4Cm - \beta^2}}{2m}\right) e^{-\frac{\beta h}{2m}} z + e^{-\frac{\beta h}{m}}.$$

Let us denote auxiliary variables $A = 2\cos\left(\frac{h\sqrt{4Cm - \beta^2}}{2m}\right) e^{-\frac{\beta h}{2m}}$; $B = e^{-\frac{\beta h}{m}}$. Accordingly, the discrete transfer function of the digital model will have the form [13] $\frac{1 - A + B}{z^2 - A \cdot z + B}$, from which, after simple algebraic transformations, we will have a recurrent simulation formula:

$$x_{i+1} = A \cdot x_i - B \cdot x_{i-1} + (1 - A + B) \cdot F.$$

The calculation results by the proposed method of the behavior of a mechanical system after applying a force *F* = 10 N to its moving part with a zero initial conditions are shown in Fig. 5. For comparison, the same graph shows the graphs of the analytical solution and relative errors for the mechanical system with the following parameters: *m* = 0.05 kg is a mass of the moving part of the mechanical system; *β* = 0.1 N·s/m is a damping factor; *C* = 2 N/m is a coefficient of elasticity.

For a fixed solution step *h* = 0.1 s (50 points for the entire time interval of the transient process calculation), the RMS error for movement relative to the analytical solution was 1.5 %.

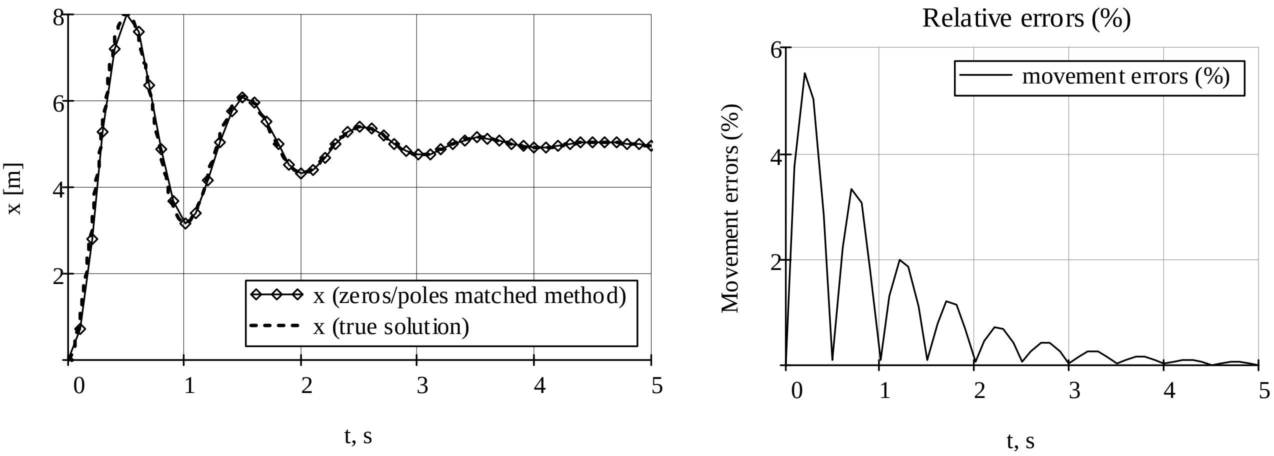

Fig. 5. Movement graphs for a model of a simple mechanical system

Simulation of the asynchronous machine dynamics

One of the main elements of modern AC electric drives is an asynchronous motor, therefore, when simulate it, it is appropriate to evaluate the advantages of the method proposed in the article. A single-phase model of an asynchronous machine (further in the text – AM) is taken as the starting point, which in many cases provides sufficient accuracy of reproduction of main coordinates and is based on an T-shaped equivalent electrical circuit (Fig. 6) in effective values [20, 21], where descriptions are used:

U_{f1} is an effective value of the phase voltage;

I_1 is an effective value of the stator current;

I_2' is a relative to the stator the effective value of the rotor current;

r_1 is an active resistance of the stator;

x_1 is a stator leakage reactive resistance;

r_μ is an active resistance of the magnetization circuit;

x_μ is a reactance of the magnetization circuit;

r_2' is an active resistance of the rotor relative to the stator;

x_2' is a reactive leakage resistance of the rotor relative to the stator;

$s = (\omega_0 - \omega)/\omega_0$ is a sliding AM where

ω_0 is a synchronous angular speed of rotation;

ω is an angular speed of the rotor.

Note: in this example, the variable s does not mean the Laplace operator, but the AM sliding.

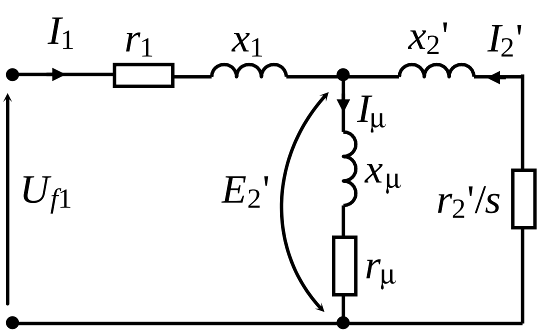

Fig. 6. T-shaped equivalent electric circuit of the AM model

This model is described by the corresponding system of equations [20, 21]:

$$\begin{cases} \dfrac{|x_1 + x_e(s)|}{\omega_e} \cdot \dfrac{dI_1}{dt} + I_1 \cdot |z_1 + z_e(s)| = U_{f1}; \\ \dfrac{|x_2'|}{\omega_e} \cdot \dfrac{dI_2'}{dt} + I_2' \cdot |z_2'(s)| = U_{f1} - I_1 \cdot |z_1|; \\ s = \dfrac{\omega_0 - \omega}{\omega_0}; \\ M_{em}(s) = \dfrac{3I_2'^2\, r_2'/s}{\omega_0}; \\ J_\Sigma \dfrac{d\omega}{dt} = M_{em}(s) - M_{mech}(s), \end{cases}$$

where ω_e is an angular frequency of the power supply voltage; $x_e(s)$ is a relative to the stator, equivalent reactive resistance dependent on slip s; $z_e(s) = \dfrac{z_2'(s) \cdot z_\mu}{z_2'(s) + z_\mu}$ is a relative to the stator, equivalent impedance of the rotor, which depends on the slip s; where $z_2'(s) = \dfrac{r_2'}{s} + j \cdot x_2'$; $z_\mu = r_\mu + j \cdot x_\mu$; $M_{mech}(s) = M_{\mathrm{nom}}(0.1 + 0.9 \cdot (1-s)^2)$ is a loading (mechanical) torque – simulation of a typical "fan" loading torque, where M_{nom} is the nominal torque of an asynchronous machine.

Experimental studies using the proposed calculation method were carried out for the process of direct start with load of fan torque of an asynchronous motor with a squirrel cage rotor type 4A200L6 with parameters:

P_{nom} = 30 kW; n_{nom} = 979 rpm;
r_1 = 0.18 Ohm; r_2' = 0.09 Ohm;
x_1 = 0.47 Ohm; x_2' = 0.5 Ohm;
cos φ_{nom} = 0.9; J_M = 0.45 kg·m^2;
U_{f1} = 220 V; J_Σ = 0.9 kg·m^2;
p_n = 3.

The total model is significantly nonlinear (Fig. 7):

- the loading torque of AM has a fan-like quadratic mechanical characteristic;
- the electromagnetic torque depend to the slipping function of the machine s is described by a non-linear dependence.

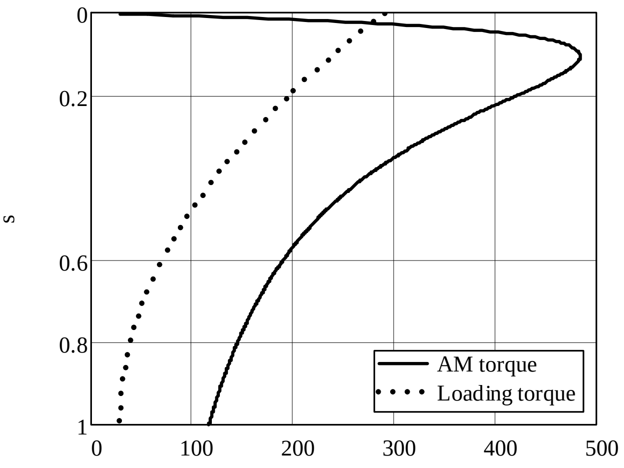

Fig. 7. Dependencies of AM torque and loading from slipping

Let's rewrite the system of equations in a slightly different way, introducing appropriate notations for the equivalent time constants of the model:

$$\begin{cases} \dfrac{|x_1 + x_e(s)|}{\omega_e \cdot |z_1 + z_e(s)|} \cdot \dfrac{dI_1}{dt} + I_1 = \dfrac{U_{f1}}{|z_1 + z_e(s)|}; \\ \dfrac{|x_2'|}{\omega_e \cdot |z_2'(s)|} \cdot \dfrac{dI_2'}{dt} + I_2' = \dfrac{U_{f1} - I_1 \cdot |z_1|}{|z_2'(s)|}; \\ s = \dfrac{\omega_0 - \omega}{\omega_0}; \\ M_{em}(s) = \dfrac{3I_2'^2 \, r_2'/s}{\omega_0}; \\ J_\Sigma \dfrac{d\omega}{dt} = M_{em}(s) - M_{mech}(s), \end{cases}$$

$T_1(s) = \dfrac{x_1 + x_e(s)}{\omega_e \cdot |z_1 + z_e(s)|}$ is an equivalent time constant of the stator circuit; $T_2(s) = \dfrac{x_2'}{\omega_e \cdot |z_2'(s)|}$ is an equivalent time constant of the rotor circle.

Note that these equivalent time constants in the computer model are nonlinear and significantly depend on slipping (angular velocity), which is shown in Fig. 8. During computer studies, the results of which are presented below, during the AM start-up process (slip varied within 0.002–1), the value of the ratio between the simulation step h = 1 ms and the equivalent time constants of the stator and rotor varied within:

$0.33 \le h/T_1 \le 0.72$ is a more than twice;

$0.32 \le h/T_2 \le 28.3$ is an almost a hundred times different.

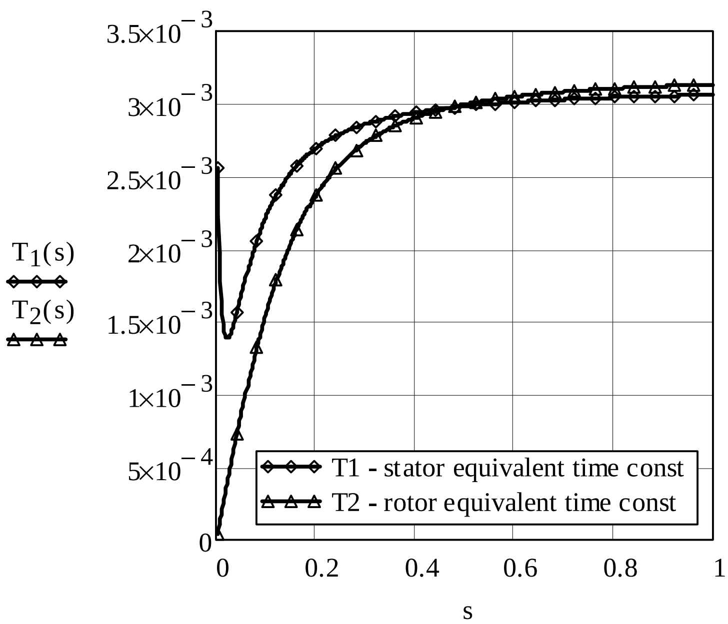

Fig. 8. Values of the equivalent time constants of the AM model depending on the slip

Finally, we write the system of equations in the form:

$$\begin{cases} T_1(s)\cdot\dfrac{dI_1}{dt}+I_1=\dfrac{U_{f1}}{\left|z_1+z_e(s)\right|}; \\ T_2(s)\cdot\dfrac{dI_2'}{dt}+I_2'=\dfrac{U_{f1}-I_1\cdot\left|z_1\right|}{\left|z_2'(s)\right|}; \\ M_{em}(s)=\dfrac{3I_2'^2\,r_2'/s}{\omega_0}; \\ J_\Sigma\dfrac{d\omega}{dt}=M_{em}(s)-M_{mech}(s). \end{cases}$$

Based on this system of equations, we will create a system of recurrent equations for computer simulation. It is worth paying attention to the case that each explored coordinate (stator and rotor currents, angular velocity) is described by the first order differential equations (corresponding to the first-order block or circuit), which simplifies the problem and makes it possible to apply simpler recurrent formulas [13, 17], as shown lower:

$$T\frac{dy}{dt}+y=K\cdot x \quad \overset{\substack{\text{Laplace}\\\text{transform}}}{\Rightarrow} \quad \frac{K}{Ts+1} \quad \overset{\substack{\text{digital}\\\text{transfer}\\\text{function}}}{\Rightarrow} \quad \frac{\left(1-e^{-\frac{h}{T}}\right)K}{z-e^{-\frac{h}{T}}} \quad \overset{\substack{\text{recurrent}\\\text{equation}}}{\Rightarrow} \quad y_{i+1}=y_i\cdot e^{-\frac{h}{T}}+\left(1-e^{-\frac{h}{T}}\right)\cdot K\cdot x_i .$$

$$\begin{cases} s=\dfrac{\omega_0-\omega_i}{\omega_0}; \\ I_{1i+1}=I_{1i}\cdot e^{\frac{-h}{T_1(s)}}+\left(1-e^{\frac{-h}{T_1(s)}}\right)\dfrac{U_{f1}}{\left|z_1+z_e(s)\right|}; \\ I_{2i+1}=I_{2i}\cdot e^{\frac{-h}{T_2(s)}}+\left(1-e^{\frac{-h}{T_2(s)}}\right)\dfrac{U_{f1}-I_{1i}\cdot\left|z_1\right|}{\left|z_2'(s)\right|}; \\ M_{em}=\dfrac{3I_{2i+1}'^{\,2}\,r_2'/s}{\omega_0}; \\ \omega_{i+1}=\omega_i+\dfrac{h}{J_\Sigma}\left(M_{em}-M_{\text{ном}}(0.1+0.9\cdot(1-s)^2)\right). \end{cases}$$

The dynamic characteristics of AM start-up for a fixed simulation step h = 1 ms (500 points per start-up interval) are shown in Fig. 9. The obtained results were compared with reference calculations carried out in the Mathcad environment using standard function for solving ordinary differential equations with integration step automatic selection (combined Adams-BDF method) with an accuracy of 10^{-6}. During the calculations, the relative RMS and maximum errors, respectively, did not exceed:

- 0.4 % and 2/4 % is a for the stator current;
- 0.4 % and 6 % is a for the rotor current;
- 1.02 % and 4.6 % is a for torque;
- 0.42 % and 0.66 % is a for the angular rotor speed.

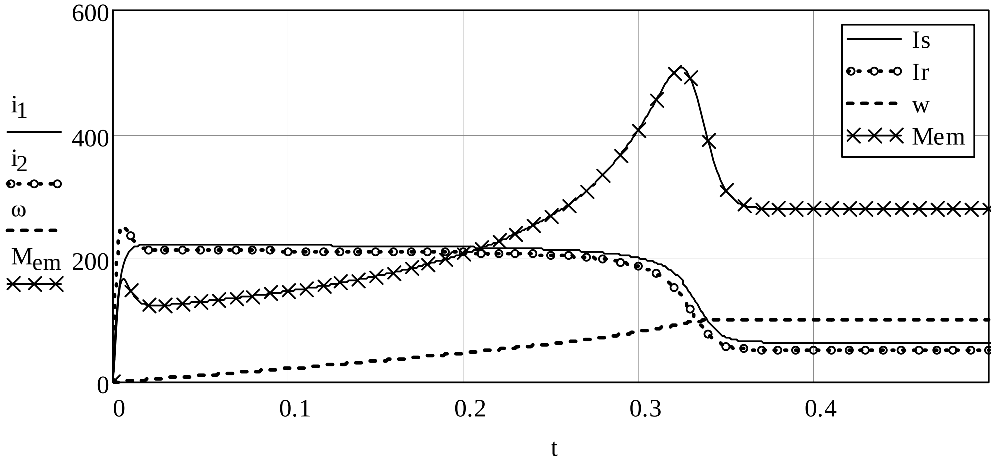

Fig. 9. Mathcad simulation results of the AM start-up process for a step 1 ms

Using zeros/poles matching method makes it possible to perform calculations for the simulation step increased to h = 5 ms (that is, 100 points for the entire startup interval), which is shown in Fig. 10. During these calculations, the relative RMS and maximum errors, respectively, did not exceed:

- 1.5 % and 7.3 % is a for the stator current;
- 1.5 % and 44 % is a for the rotor current;
- 5.8 % and 33.6 % is a for torque;
- 3.1 % and 5.3 % is a for the angular rotor speed.

At the same time, the ratio between the simulation step h and the equivalent time constants of the stator and rotor varied within $1.63 \le h/T_1 \le 3.6$ and $1.6 \le h/T_2 \le 141$ – for such ratios of the solution step and time constants, classical numerical methods are unsuitable due to numerical instability and low accuracy when using a fixed step calculation.

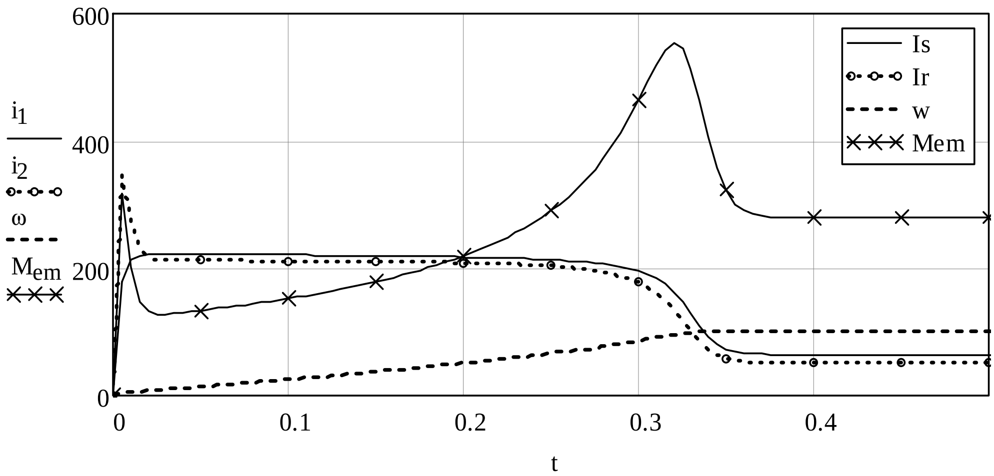

Fig. 10. Simulation results of the AM start-up process for a step 5 ms

Analysis of the obtained results

Using zeros and poles matching method requires the user to understand both the functioning of the technical system and the mathematical description of the processes in the object. Added to this is the need for some analytical work (currently it is simplified by the availability of mathematical applications, for example, Mathcad, MATLAB). All this leads to a certain complication of this method application, which causes certain limitations in the wide use of zeros and poles matching method. Additional competition is created by a large number of computer programs with modern functions for solving problems of the technical systems' dynamics, even with elements of automation of this process – let's just mention the Simulink environment of the mathematical application MATLAB. It is clear that the average user will choose the method that requires less effort.

We note that in the situation of the need to implement a real-time computer model or in another case when the calculation must be performed with a fixed step, there is no other way to obtain a reasonable result in the occasion when the solution step can exceed the time constant more than tenfold.

Conclusions

Applying zeros/poles matching method to obtain simulation recurrence formulas allows you to get a number of advantages, in particular:

- stability of the numerical solution for any simulation step size;
- the possibility of implementing simulation systems in real time due to the simplicity of the formulas and their simple implementation on microcontrollers;
- the possibility of implementing parallel calculations thanks to the application of parallel decomposition of the system model.

It is worth noting that the zeros and poles matching method is also effective for the synthesis of digital control systems, and not only for computer simulation, because:

- makes it possible to use known from the automatic control theory and widely understood methods of analog regulators synthesis;
- provides simple and stable recurrent equations for a wide range of sampling periods for implementation on low-power microcontrollers.

Planned further research

Prospective research on the zeros/poles matching method involves the study of:

- impact on the behavior of a digital system with limited bit computational precision due to the use of microcontrollers;
- effect of the continuous system decomposition method on the properties of the discrete transfer function.

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ЗАСТОСУВАННЯ МЕТОДУ ВІДОБРАЖЕННЯ НУЛІВ І ПОЛЮСІВ ДЛЯ МОДЕЛЮВАННЯ ЕЛЕКТРОТЕХНІЧНИХ СИСТЕМ

Поширення математичних застосунків, які надають засоби розв’язування диференціальних рівнянь, і збільшення швидкодії обчислювальних пристроїв призвели до зменшення зацікавленості операторними методами, зокрема z-перетворенням. Проте використання

можливостей z-перетворення дає змогу реалізувати ефективні швидкодіючі обчислювальні схеми із високою числовою стійкістю. Потреба в цьому може виникнути у випадку моделювання в реальному часі чи під час синтезу цифрових систем керування. На підставі аналізу літературних джерел показано актуальність і переваги використання z-перетворення для моделювання динаміки електротехнічних систем.

Розглянуто спосіб комп’ютерного моделювання, основою якого є використання для побудови комп’ютерної моделі методу відображення (відповідності) нулів і полюсів еквівалентної неперервної передавальної функції. Показано реалізацію отриманих цим методом моделювальних рекурентних формул для трьох елементарних динамічних ланок, які одержують внаслідок розкладу передавальної функції за теоремою розкладу Гевісайда: інтегральної (нульовий полюс), інерційної першого порядку (дійсний полюс) і ланки другого порядку із дійсним нулем і парою комплексно спряжених полюсів. Отже, реалізована паралельна декомпозиція досліджуваної системи, що дає змогу зменшити негативний вплив обмеженої розрядності системи і полегшити виконання паралельних обчислень. Для кожної такої ланки одержано дискретну передавальну функцію та моделювальне рекурентне рівняння.

На двох прикладах продемонстровано практичне використання та переваги цього способу: проста пружна механічна система, яка описана диференціальним рівнянням другого порядку, та нелінійна модель асинхронної машини за однофазною Т-подібною заступною схемою. Обидві задачі проілюстровані прикладами розв’язування у середовищі математичного застосунку Mathcad. Підтверджено ефективність методу відповідності нулів і полюсів порівняно з класичними числовими методами розв’язування звичайних диференціальних рівнянь.

Використання цього способу математичного моделювання дає змогу забезпечити стійкий числовий розв’язок із заданою точністю для широкого діапазону кроків розв’язування.

***Ключові слова:** комп’ютерне моделювання; електрична система; інтегральний числовий метод; передавальна функція; z-перетворення; метод відображення (відповідності) нулів і полюсів.*