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KALMAN FILTER STATIONARITY ANALYSIS IN THE TASK OF INERTIAL NAVIGATION SYSTEM ERRORS ESTIMATION

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Abstract. *Kalman filter main advantage over simpler state estimators is ability to adjust filter's gain accordingly to observable signal instability. It is shown that it is possible to use simple observers if filters gain is stationary regardless of aircraft maneuvers. In this paper model of navigation errors dynamics is considered. This model is used in Kalman filer in integrated system simulation. Frequency domain representation of vehicle dynamics is used in this simulation to investigate filter stationarity. The results show that filter gain is stationary regardless of motion dynamics.*

Keywords: complementary filter; Kalman filter; Integrated Navigation System; filter stationarity.

Introduction

Strapped down Inertial Navigation System (SINS or shortly INS) is main instrument for obtaining navigation parameters in aviation, marine transport, robotics and such applications as biomechanics. This system is fully autonomous in comparison to Satellite Navigation Systems (SNS or GPS) which performance is limited by line of sight and number of satellites. But INS is subjected to errors which due to computing algorithms are accumulating in process. This imposes a constraint on operation time and necessity of periodical calibrations. GPS doesn't have such limitations and can work unlimited time (in ideal conditions). This features makes it possible to calibrate INS with information from GPS which is perfect solution for boosting INS performance on a long period of time. Such systems are called Integrated Navigation Systems or INS/GPS. This system are widely used since 90 s and now are basic navigation systems for aviation. Due to progress in MEMS production this system now can be also used in Mini and Micro UAVs [1–3].

To calibrate INS one's need to estimate sensors errors – gyro and accelerometer biases which causes INS drift. The task is performed by state estimator (or widely used Kalman filter (KF)) which uses differences between INS and GPS data (additionally magnetic and barometric sensors can be used to augment filter performance) for errors estimation. This state estimators often referred as Complementary Filters. The core of estimator's algorithm is an errors dynamics model which represents error generation process with a certain degree of approximation [4; 5].

Errors Dynamics Model

As part of Complementary Filter application research one's need to consider different models of errors dynamics which leads to different estimation algorithms. Each model is characterized by process representation precision which affects quality of estimation, and algorithm's complexity. One's need to compromise between model's precision and complexity because more complex algorithms impose a constraints on hardware [6].

We choose errors model for roll, pitch and yaw angles in which attitude errors are determent from kinematic equations of the aircraft movement. Attitude errors are obtained by adding a perturbation in this equations

$$\Delta\dot{\Theta} = F(\Theta)\Delta\omega + F_d(\Theta,\omega)\Delta\Theta\,, \qquad (1)$$

where $\Delta\omega = \begin{bmatrix} \Delta\omega_x & \Delta\omega_y & \Delta\omega_z \end{bmatrix}^T$ is a vector of gyro biases, Θ is a vector of attitude parameters (roll, pitch and yaw) obtained from kinematic equation [7]

$$\dot{\begin{bmatrix} \gamma \\ \psi \\ \upsilon \end{bmatrix}} = F(\Theta)\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \qquad (2)$$

where $F(\Theta)$ is a transformation matrix

$$F(\Theta)=\begin{bmatrix} 1 & -\mathrm{tg}\upsilon\cos\gamma & \mathrm{tg}\upsilon\sin\gamma \\ 0 & \cos\gamma/\cos\upsilon & -\sin\gamma/\cos\upsilon \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}. \qquad (3)$$

$F_d(\Theta, \omega)$ – derivative matrix

$$F_y(\Theta,\omega)=\begin{bmatrix} tg\upsilon(\omega_y\sin\gamma+\omega_z\cos\gamma) & 0 & (\omega_z\sin\gamma-\omega_y\cos\gamma)/\cos^2\upsilon \\ (-\omega_y\sin\gamma-\omega_z\cos\gamma)/\cos\upsilon & 0 & (\omega_y\cos\gamma-\omega_z\sin\gamma)\sin\upsilon/\cos^2\upsilon \\ \omega_y\cos\gamma-\omega_z\sin\gamma & 0 & 0 \end{bmatrix}.$$

Velocity errors are described by

$$\Delta\dot{V}=W_n\Delta\Theta+M(\Theta)\Delta a \,. \qquad (4)$$

where $\Delta a = \left[\Delta a_x \quad \Delta a_y \quad \Delta a_z\right]^T$ – vector of accelerometers biases, $M(\Theta)$ – direct cosine matrix, W_n – skew-symmetric matrix of accelerations in navigation frame

$$W_n = -\begin{bmatrix} 0 & -a_{nav\ z} & a_{nav\ y} \\ a_{nav\ z} & 0 & -a_{nav\ x} \\ -a_{nav\ y} & a_{nav\ x} & 0 \end{bmatrix}. \qquad (5)$$

Coordinates errors are described by

$$\Delta\dot{P} = \Delta V. \qquad (6)$$

Finally errors model in matrix form is

$$\begin{bmatrix} \Delta\dot{\Theta} \\ \Delta V \\ \Delta P \\ \Delta\omega \\ \Delta a \end{bmatrix} = \begin{bmatrix} F_d & O & O & F & O \\ W_n & O & O & O & M \\ O & I & O & O & O \\ O & O & O & O & O \\ O & O & O & O & O \end{bmatrix} \begin{bmatrix} \Delta\Theta \\ \Delta V \\ \Delta P \\ \Delta\omega \\ \Delta a \end{bmatrix}. \qquad (7)$$

where I is identity matrix of size 3×3 and O – zero matrix of the same size.

Motion Dynamics

Usually in the research of state estimators for navigation parameters of aerial vehicles motion dynamics is specified by some defined types of motion (like takeoff, horizontal flight, maneuvers) or flight data from actual aircraft [8]. However this data is not sufficient for estimator's stationarity analysis. Instead of time domain representation of aircraft motion we choose to implement frequency domain representation of motion dynamics (gyros and accelerometers data or Inertial Measurement Unit (IMU) data). In this case motion dynamics is divided into three types: Low motion (IMU data rate constraints of 0.5 Hz), Middle motion (2 Hz), High motion (5 Hz). Gyro and accelerometer magnitude constrained to ±100 deg/sec and ± 2 m/s^2 respectively.

To generate IMU data we used a colored noise – random signal with determined spectral characteristics obtained by shaping white noise with low pass filter.

Kalman Filter Stationarity

Kalman filter is the most popular instrument for integration INS and GPS. It uses a state space model of the process to estimate it parameters (or states) with a limited observable data. In case of INS/GPS systems GPS velocity and position data is used to augment INS performance and estimate IMU biases. Kalman filter utilizes recurrent algorithm of calculating filter gain (Kalman Gain) on every tact (fig. 1).

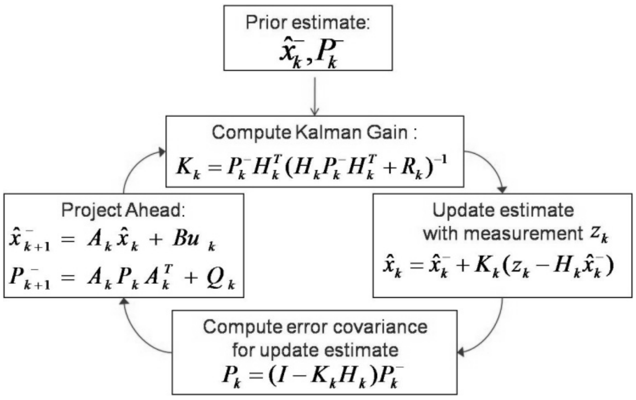

Fig. 1. Kalman Gain computation algorithm.

This part of the KF algorithm consumes most of computing power. If it is possible to get rid of this part of the estimation process it could be used for another tasks. There is a way to accomplish this by implementing simpler algorithm such as Luenberger observer. In this observer gain is a constant and computes using initial conditions of the system. But using stationary observer to estimate non-stationary system may lead to increasing error of estimation [9].

We carried out a simulation of INS/GPS system with a different types of motion to see how KF stationarity behave in different conditions. If it is stable or have a relatively small disturbance it would be possible to implement simpler estimation algorithm without decrease in quality of estimation.

Results

In order to carry out simulation we used two INS blocks from Matlab/AeroSim library – one with IMU biases and the "ideal". Differences between their velocity and position data were processed with KF to obtain estimation of navigation errors and IMU biases. IMU biases were of order 1 m/s^2 for accelerometers and 0.1 rad/s^2 for gyros (figs. 2 – 10).

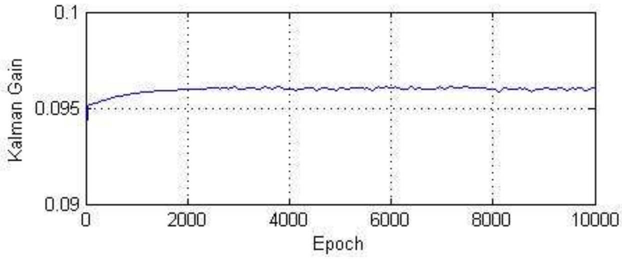

Fig. 2. Kalman Gain of attitude error estimation (Low motion)

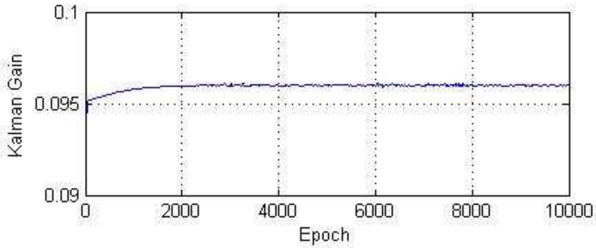

Fig. 3. Kalman Gain of attitude error estimation (Middle motion)

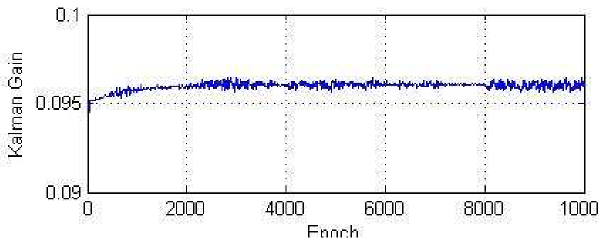

Fig. 4. Kalman Gain of attitude error estimation (High motion)

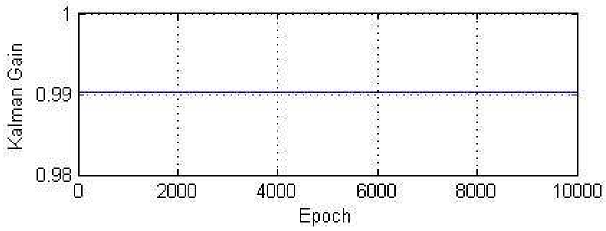

Fig. 5. Kalman Gain of velocity error estimation (Low motion)

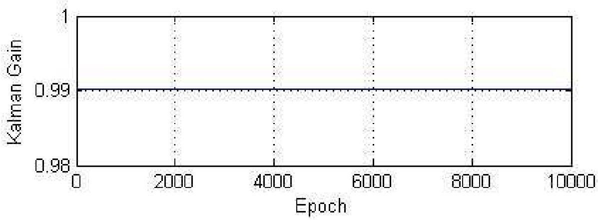

Fig. 6. Kalman Gain of velocity error estimation (Middle motion)

Figs 2 – 4 show Kalman Gain of attitude error estimation for Low, Middle and High motions. It is shown that for Low and Middle motion (figs 2, 3) Kalman Gain in steady state is nearly a constant and have minor instability of less than 1 % in High motion.

Figs 5 – 7 show Kalman Gain of velocity error estimations. It is constant on a whole time scale for all types of motion so even fast maneuvers don't introduce instability in velocity channel.

It is shown on Figs 8 – 10 that Kalman Gain of position estimations is as well as velocity stable on a whole time scale.

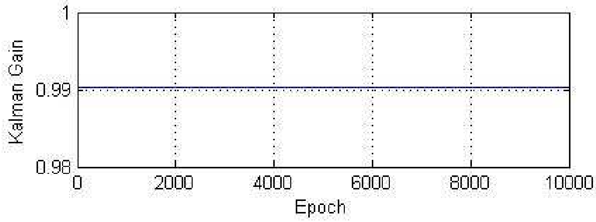

Fig. 7. Kalman Gain of velocity error estimation (High motion)

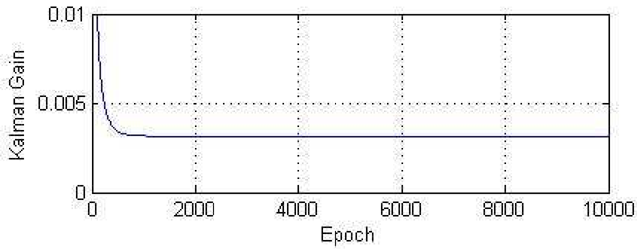

Fig. 8. Kalman Gain of position error estimation (Low motion)

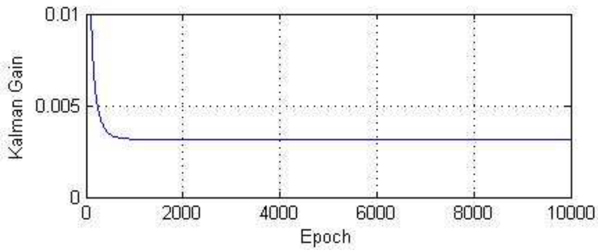

Fig. 9. Kalman Gain of position error estimation (Middle motion)

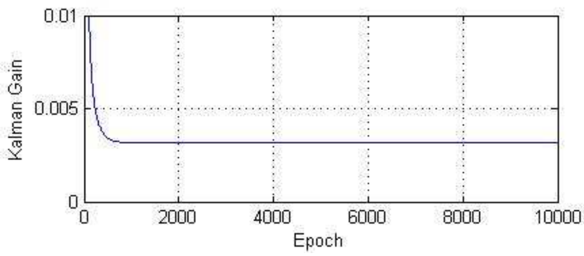

Fig. 10. Kalman Gain of position error estimation (High motion)

Conclusion

As we can see from results of simulation Kalman Gain for all estimated parameters is stable on a whole time scale in steady state for all types of motion. This means that instead of using complex algorithm of computing Kalman Gain on every tact of filtering process it would be possible to use constant value of gain. Thus application of simpler Luenberger observer in integrated systems will make computation of navigation parameters faster without decrease in estimation performance.

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І. О. Ратайчук, В. І. Кортунов. Стаціонарний фільтр Калмана в задачі аналізу оцінювання помилки інерціальної навігаційної системи

Основною перевагою фільтра Калмана над простими оцінками стану є можливість регулювання посилення фільтру відповідно спостережуваної нестабільності сигналу. Показана можливість використовувати прості спостерігачі, якщо посилення фільтрів стаціонарні незалежні від маневрів літального апарату. Розглянуто динаміку моделі навігаційних похибок. Ця модель використовується у фільтрі Калмана в інтегрованій системі моделювання. Частотна область динаміки літака використовується при моделюванні для визначення стаціонарності фільтра. Результати показують, що коефіцієнт посилення фільтра є незмінним незалежно від динаміки руху.
Ключові слова: додаткові фільтри; фільтр Калмана; інтегрована навігаційна система; стаціонарність фільтра.

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И. А. Ратайчук, В. И. Кортунов. Стационарный фильтр Калмана в задаче анализа оценки ошибки инерциальной навигационной системы

Основным преимуществом фильтра Калмана над простыми оценками состояния является возможность регулировки усиления фильтра соответственно наблюдаемой нестабильности сигнала. Показано, что можно использовать простые наблюдатели, если усиления фильтров стационарные независимо от маневров летательного аппарата. Рассмотрена динамика модели навигационных погрешностей. Эта модель используется в фильтре Калмана в интегрированной системе моделирования. Частотная область динамики самолета используется при моделировании для определения стационарности фильтра. Результаты показывают, что коэффициенты усиления фильтра являются неизменными независимо от динамики движения.

Ключевые слова: дополнительные фильтры; фильтр Калмана; интегрированная навигационная система; стационарность фильтра.

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