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DESIGN OF L_1-OPTIMAL LATERAL AUTOPILOT FOR UAV

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Abstract—*The paper deals with the L_1-optimization concept advanced in the modern control theory with an application to the design of the L_1-optimal lateral autopilot. The control aim is to maintain the desired roll orientation of an UAV in the presence of external unmeasurable disturbances, in particular, of a gust. To achieve a high performance index, the two separate control algorithms are proposed. The first algorithm is designed in order to implement the L_1-optimal PI control for the stabilization of a given roll velocity. The second algorithm ensures the L_1-optimal P control necessary to stabilize the roll of the aircraft. Results of a simulation example are given to illustrate the properties of the control method.*

Index Terms—Feedback control system; lateral autopilot; optimization; PI controller; UAV

I. INTRODUCTION

The problem of efficiently controlling the motion of an unmanned aerial vehicle (UAV) in a nonstationary environment is important enough from the practical point of view [1], [2]. On the other hand, the problem of synthesizing the perfect feedback control system which decouples its output with respect to an unmeasured disturbance is a relevant one in control theory. Different methods in this area based on the achievement of the modern control theory have been reported by many scientists. See, for example, [3]–[6]. Recently, new results in this direction have been presented, in particular, by the Canadian and Brazilian researches [7]. Nevertheless, most of available works dealt with an ideal case when there are no disturbances.

One of the efficient methods devised in the modern control theory for rejecting any unmeasured disturbance is based on the l_1-optimization concept [8] applicable to discrete-time control systems. This concept has been utilized in [9] to design the digital lateral autopilot for UAV capable to cope with a gust.

This paper extends the approach of [9] to synthesizing a lateral autopilot system. Namely, the L_1-optimization method whose discrete-time counterpart is the l_1-optimization method, is proposed to reject an external disturbance. But, in contrast with [9], the lateral autopilot is here designed as a two-circuit continuous-time control system containing the L_1-optimal PI controller in the inner loop and the L_1-optimal controller in the external loop.

The remainder of this paper is organized as follows. In Section II, the assumptions are made and the control problem is stated. The main theoretical results including the choice of the control strategy, the background on the L_1-optimization concept and the synthesis of L_1-optimal PI and P controllers are presented in Section III. The simulation results are given in Section IV. Section V concludes this paper.

II. PROBLEM STATEMENT

A. Basic Assumptions

Let γ and ξ denote the roll angle and the aileron deflection of an UAV, respectively. As shown in [10], the transfer function from ξ to γ derived from the linearized lateral equation of the aircraft motion can be approximated by

$$W_\xi(s) = \frac{K_\xi}{s(T_\xi s + 1)}, \qquad (1)$$

where K_ξ and T_ξ represent a gain and a time constant (more certainly, T_ξ is the damping derivative in the roll channel and K_ξ is the roll moment). s is the Laplace variable.

As in [10, chap. 4], it is assumed that the transfer function from the controller output u to be formed by an autopilot system to the aileron input, which describes the aileron servo dynamics, is

$$W_1(s) = \frac{K_1}{T_1 s + 1}, \qquad (2)$$

where K_1 and T_1 are its gain and time constant, respectively.

Let d be an external signal (in particular, a gust) disturbing the angular velocity $\dot{\gamma}$. Using the fact that the transfer function from ξ to $\dot{\gamma}$ is $sW_\xi(s)$, we suppose that the equation

$$\dot{\Gamma}_d(s) = sW_\xi(s)\Xi(s) + \underbrace{W_d(s)D(s)}_{V(s)} \quad (3)$$

holds to describe the lateral motion completely. In this equation, $\dot{\Gamma}(s) := L\{\dot{\gamma}(t)\}, \Xi(s) := L\{\xi(t)\}$ and $D(s) := L\{d(t)\}$ define the Laplace transforms of $\dot{\gamma}(t)\}$, $\xi(t)$ and $d(t)$, respectively (t denotes the continuous time). $W_d(s)$ plays the role of a transfer function of some shaping filter for the additive disturbance $v(t)$ whose Laplace transform, $V(s)$, is here emphasized. As in (9), the main assumption with respect to $d(t)$ is that

$$|\dot{d}(t)| \le C_{\dot{d}} < \infty. \quad (4)$$

B. Control Objective

Denote by $\gamma^0(t)$ a desired roll orientation at the time t. Now, define the output error, $e(t)$, as

$$e(t) = \gamma^0(t) - \gamma(t). \quad (5)$$

Further, introduce the performance index of the control system to be designed in the form

$$J := \lim_{t\to\infty} \sup |\gamma^0(t) - \gamma(t)|. \quad (6)$$

The problem to be stated is formulated as follows. Devise the controller which is able to minimize J assuming that the variables $\gamma(t)$ and $\dot{\gamma}(t)$ can be measured and the constraint of the form (4) takes place. Hence, the aim of the controller design may be written as the requirement

$$\lim_{t\to\infty} \sup |e(t)| \to \inf_u, \quad (7)$$

where the expressions (5) and (6) were utilized.

III. Lateral Autopilot Design

A. Control Strategy

To implement the controller design concept proposed in this paper, two feedback loops similar to that in [9] are incorporated in the autopilot system, as shown in Fig. 1. The aim of the inner feedback loop containing the controller 1 is to stabilize the angular velocity, $\dot{\gamma}(t)$, around a given value, $\dot{\gamma}^0(t)$, formed by the controller 2. The external feedback loop which contains this controller 2 is used to stabilize the roll angle, $\gamma(t)$, around the desired value, $\gamma^0(t)$.

In order to choose the parameters of the controllers 1 and 2, the so-called L_1-optimization approach is employed.

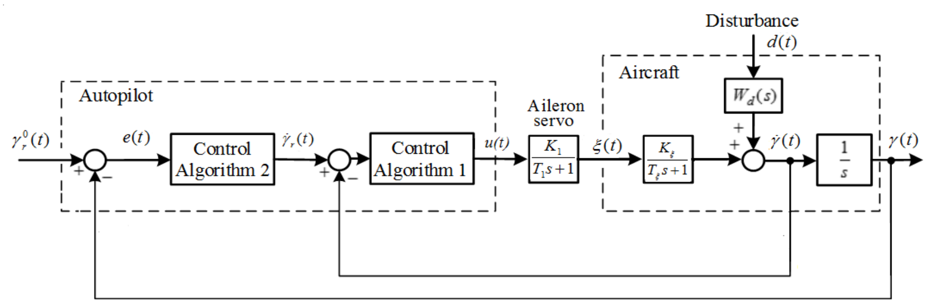

Fig. 1. Structure of lateral autopilot system

B. Background on L_1-Optimization Concept

The L_1-optimization of linear continuous-time control systems deals with the minimization of the value of the H_1-norm of certain closed-loop transfer functions $H(s)$ defined as

$$\|H\|_1 := \int_0^\infty |k(t)|\,dt, \quad (8)$$

where $k(t) = L^{-1}\{H(s)\}$ denotes its impulse response determined by the inverse Laplace transform of $H(s)$. (The lower index "1" in (8) was taken from the notation L_1 of the space of all absolutely integrable functions $f(t)$ of a continue t for which the 1-norm given by $\|f\|_1 = \int_0^\infty |f(t)|\,dt$ is upper bounded)

To clarity the L_1-optimization approach, consider the typical feedback system depicted in Fig. 2. A plant has some transfer function $P(s) = P'(s)P''(s)$ and the controller has a transfer function $C(s)$. The signal $v(t)$ represents an external unmeasurable disturbance acting on this plant and $y(t)$ is the control system output. Then from the signal balance equation

$$Y(s) = P''(s)V(s) - C(s)P'(s)P''(s)Y(s)$$

written for the Laplace transforms $Y(s) = L\{y(t)\}$ and $V(s) = L\{v(t)\}$, it follows that

$$Y(s) = H(s)V(s) \tag{9}$$

with the closed-loop transfer function

$$H(s) = \frac{P''(s)}{1 + C(s)P'(s)P''(s)}. \tag{10}$$

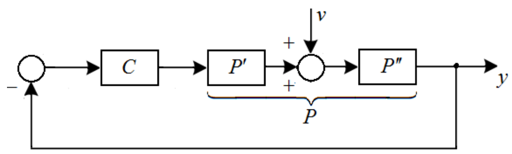

Fig. 2. Basic feedback loop configuration

Now, suppose that the controller is the PI-type controller. This gives that $C(s) = \tilde{C}(s)/s$. Then, the expression (10) leads to

$$H(s) = \tilde{H}(s)s, \tag{11}$$

where

$$\tilde{H}(s) = \frac{P''(s)}{s + \tilde{C}(s)P'(s)P''(s)}. \tag{12}$$

Substituting (11) into (9) finally yields

$$\limsup_{t\to\infty} |y(t)| \le \|\tilde{H}\|_1 \sup_{0\le t<\infty} |\dot{v}(t)| \tag{13}$$

in which $\|\tilde{H}\|_1$ is the L_1-norm of $\tilde{H}(s)$ specified by

$$\|\tilde{H}\|_1 := \int_0^\infty |\tilde{k}(t)|\, dt, \tag{14}$$

where $\tilde{k}(t) = L^{-1}\{\tilde{H}(s)\}$ is the impulse response from $\dot{v}$ to y.

From the Theorem 10 given in [11, sect. 6.7] we can concluded that the asymptotic stability of $\tilde{H}(s)$ is the necessary and sufficient condition to guarantee $\|\tilde{H}\|_1 < \infty$.

Denote (as in [12]) the space of the all asymptotic stable $H(s)$s by RH_∞. Then, assuming that $\dot{v} \in L_\infty$, in which L_∞ denotes the space of all upper bounded in modulus functions, from (13) we further get

$$\limsup_{t\to\infty} |y(t)| \le \|\tilde{H}\|_1 \|\dot{v}\|_\infty < \infty, \tag{15}$$

if and only if $H(s) \in RH_\infty$, where $\|\dot{v}\|_\infty := \sup_{0\le t<\infty} |\dot{v}(t)| < \infty$ is the ∞-norm of $\dot{v}$.

Let k be a vector of controller parameters meaning $\tilde{C}(s) = \tilde{C}(k, s)$. Then, due to (12), the expression (15) can be rewritten as

$$\limsup_{t\to\infty} |y(t)| \le \|\tilde{H}(k)\|_1 \|\dot{v}\|_\infty, \tag{16}$$

where $\|\tilde{H}(k)\|_1$ is the L_1-norm of $\tilde{H}(k, s) \in RH_\infty$ depending on k according to

$$\tilde{H}(k, s) = \frac{P''(s)}{s + \tilde{C}(k, s)P(s)}. \tag{17}$$

(Recall that $P(s) := P'(s)P''(s)$.)

The inequality (16) shows that the minimization of the upper bound on $J := \lim_{t\to\infty} \sup |y(t)|$ implies

$$\|\tilde{H}(k)\|_1 \to \inf_k .$$

The L_1-optimization problem can now be formulated in the following form: find

$$k^* = \arg\inf_k \|\tilde{H}(k)\|_1 \tag{18}$$

provided that $\tilde{H}(k, s)$ belongs to RH_∞.

It turns out that there is the explicit expression for the L_1-norm of $\tilde{H}(k, s)$ as a function of k if only $\deg Q(k, s) \le 2$, $\deg P''(s) = 0$, where $Q(k, s) = s + \tilde{C}(k, s)P(s)$ represents the denominator of $\tilde{H}(k, s)$ in (17). In general case, numerical techniques are required to estimate $\|H(k))\|_1$. To exploit one of these techniques, we first note that the expression (14) produces

$$\|\tilde{H}\|_1 = |\tilde{h}_1 - \tilde{h}_0| + |\tilde{h}_2 - \tilde{h}_1| + \ldots = \sum_{\nu=0}^{\infty} |\tilde{h}_{\nu+1} - \tilde{h}_\nu|, \tag{19}$$

where $\tilde{h}_0 = 0$ and $\tilde{h}_1, \tilde{h}_2, \ldots$ are the sequential extreme values of the step response

$$\tilde{h}(t) = \int_0^t \tilde{k}(\tau)\, d\tau \tag{20}$$

corresponding to $\tilde{H}(s)$ at time instants $t_1, t_2, \ldots$, respectively. Note that if $\tilde{H}(s) \in RH_\infty$ then $\|\tilde{H}\|_1 \ge \tilde{H}(0)$. Due to (20), the expression (14) yields $\|\tilde{H}\|_1 = \tilde{H}(0)$ if $\tilde{k}(t)$ has no extreme values.

To gain a better understanding of how the expressions similar to (14) and (19) may be used for estimating the norm $\|H\|_\infty$ of any $H(s) \in RH_\infty$, the plots of $|k(t)|$ and $h(t)$ are depicted in Fig. 3.

C. Design of Control Algorithm 1

The synthesis of the lateral autopilot system starts with the design of the control algorithm 1. This algorithm is chosen to implement the continuous-time PI control law

$$u(t) = k_{\mathrm{P}}^{(1)} e_1(t) + k_{\mathrm{I}}^{(1)} \int_0^t e_1(\tau)\, d\tau, \qquad (21)$$

where $e_1(t) := \dot{\gamma}^0(t) - \dot{\gamma}(t)$ denotes the deflection of the true angular velocity, $\dot{\gamma}(t)$, from a given angular velocity, $\dot{\gamma}^0(t)$. $k_{\mathrm{P}}^{(1)}$ and $k_{\mathrm{I}}^{(1)}$ are the parameters of PI controller to be optimized. Then, the transfer function from e_1 to u produced by (21) will be described as

$$C_1(s) = \frac{k_{\mathrm{P}}^{(1)} s + k_{\mathrm{I}}^{(1)}}{s}. \qquad (22)$$

To find the transfer function $H(s)$ from v to $\dot{\gamma}$, we first inspect Fig. 1 to establish the signal balance equation

$$\dot{\Gamma}(s) = -\frac{K K_{\xi}(k_{\mathrm{P}}^{(1)} s + k_{\mathrm{I}}^{(1)})}{s(T_1 s + 1)(T_{\xi} s + 1)} \dot{\Gamma}(s) + V(s),$$

utilizing (1), (2) and (22) together with (3). This equation gives

$$H_1(s) = \tilde{H}_1(s) s \qquad (23)$$

with

$$\tilde{H}_1(s) = \frac{(T_1 s + 1)(T_{\xi} s + 1)}{T_1 T_{\xi} s^3 + (T_1 + T_{\xi}) s^2 + (1 + K_1 K_{\xi} k_{\mathrm{P}}^{(1)}) s + K_1 K_{\xi} k_{\mathrm{I}}^{(1)}}. \qquad (24)$$

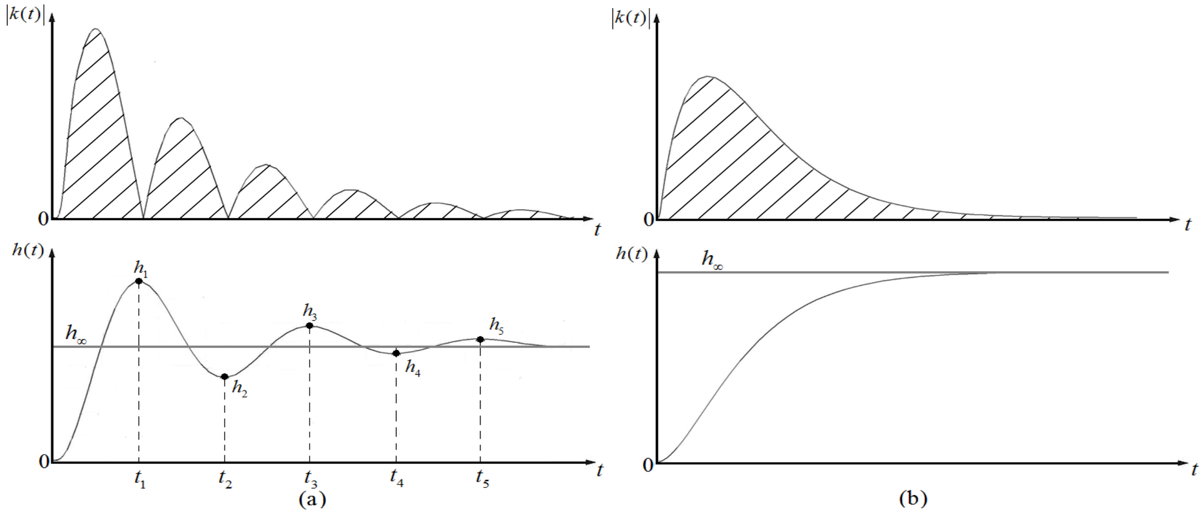

Fig. 3. Variables $|k(t)|$ and $h(t)$ used to estimate $\| H \|_1$: (a) $h(t)$ has extreme values $h_1, h_2, \ldots$ at $t_1, t_2, \ldots$; (b) there is no extreme values of $h(t)$

Due to (4), $\dot{v}$ will be upper bounded if and only if $W_d(s) \in RH_{\infty}$. In this case, from (23) it follows that the asymptotic stability of the denominator of (24) given by

$$Q_1(s) = T_1 T_{\xi} s^3 + (T_1 + T_{\xi}) s^2 + (1 + K_1 K_{\xi} k_{\mathrm{P}}^{(1)}) s + K_1 K_{\xi} k_{\mathrm{I}}^{(1)}$$

guarantees the boundedness of $\dot{\gamma}$.

Applying the well-known Routh–Hurwitz stability criterion [13, subsect. 8.3] to the polynomial $Q_1(s)$, we derive the necessary and sufficient conditions

$$\left.\begin{aligned} & k_{\mathrm{P}}^{(1)} > -\frac{1}{K_1 K_{\xi}}, \\ & k_{\mathrm{I}}^{(1)} > 0, \\ & k_{\mathrm{I}}^{(1)} < \frac{T_1 T_{\xi}}{T_1 T_{\xi} K_1 K_{\xi}} + \frac{T_1 + T_{\xi}}{T_1 T_{\xi}} k_{\mathrm{P}}^{(1)} \end{aligned}\right\} \qquad (25)$$

to achieve $\tilde{H}_1(s) \in RH_{\infty}$.

Within the restrictions on $k_{\mathrm{P}}^{(1)}$ and $k_{\mathrm{I}}^{(1)}$ given by (25), the optimal vector $k_1^* = [k_{\mathrm{P}}^{*(1)}, k_{\mathrm{I}}^{*(1)}]^{\mathrm{T}}$ is determined according to (18) as

$$k_1^* = \arg\min_{k_1} \| \tilde{H}_1(k_{\mathrm{P}}^{(1)}, k_{\mathrm{I}}^{(1)}) \|_1, \qquad (26)$$

where the notation $\tilde{H}_1(k_P^{(1)}, k_I^{(1)})$ has been introduced to indicate that $\tilde{H}_1(s) \equiv \tilde{H}_1(k_P^{(1)}, k_I^{(1)}, s)$ depends on $k_P^{(1)}$ and $k_I^{(1)}$.

To compute k_I^*, we propose to employ the simplest version of the Powell's optimization method [14] to (26) after calculating the norm $\| \tilde{H}_1(k_P^{(1)}, k_I^{(1)}) \|_1$ via the formula (19).

D. Design of Control Algorithm 2

The control algorithm 2 is chosen to implement the simple P control law

$$\dot{\gamma}^0(t) = k_P^{(2)} e(t) \tag{27}$$

in which $e(t)$ is given by (5). Its transfer function is

$$C_2(s) = k_P^{(2)}. \tag{28}$$

Inspecting Fig. 1 and taking into account the expressions (22) for $k_P^{(1)} = k_P^{*(1)}$ and $k_I^{(1)} = k_I^{*(1)}$ together with (28) we derive the transfer function from v to γ as $H_2(s) = \tilde{H}_2(s)s$ in which

$$\tilde{H}_2(s) = P_1(s) / Q_2(s), \tag{29}$$

where $$P_1(s) = (T_1 s + 1)(T_\xi s + 1), \tag{30}$$

$$Q_2(s) = T_1 T_\xi s^4 + (T_1 + T_\xi) s^3 + (1 + K_1 K_\xi k_P^{*(1)}) s^2$$
$$+ (K_1 K_\xi k_P^{*(1)} k_P^{(2)} + K_1 K_\xi k_I^{*(1)}) s + K_1 K_\xi k_I^{*(1)} k_P^{(2)}. \tag{31}$$

It can be established that the transfer function of inner circuit from $\dot{\gamma}^0$ to γ with no external feedback loop will be described by

$$G(s) = \frac{K_1 K_\xi (k_p^{*(1)} s + k_I^{*(1)})}{[T_1 T_\xi s^3 + (T_1 + T_\xi) s^2 + (K_1 K_\xi k_p^{*(1)} + 1) s + K_1 K_\xi k_I^{*(1)}] s} \tag{32}$$

assuming the parameters $k_p^{*(1)}$ and $k_I^{*(1)}$ of the control algorithm 1 are already optimal.

To find the upper bound $\bar{k}_p^{(2)}$ on $k_p^{(2)}$ in the control algorithm (27) under which the external closed-loop circuit will be stable, we utilize the well-known Nyquist stability criterion [13, p. 4–10]. More certainty, we use the Theorem 4.1 of [12] according to which

$$\bar{k}_p^{(2)} = \frac{1}{m} \tag{33}$$

with

$$m = \min \{ \mathrm{Re}\, G(j\omega) : \ \mathrm{Im}\, G(j\omega) = 0 \}, \tag{34}$$

where $G(j\omega)$ is the frequency response obtained by substituting $s = j\omega$ into (32). This implies that if $k_P^{(2)}$ satisfies $0 < k_P^{(2)} < \bar{k}_P^{(2)}$ then $\tilde{H}_2(s) \in RH_\infty$.

The optimal value $k_P^{*(2)} = k_P^{(2)}$ in (27) is now determined as follows:

$$k_P^{*(2)} = \arg \min_{k_P^{(2)} \in (0, \bar{k}_P^{(2)})} \| \tilde{H}_2(k_P^{(2)}) \|_1 . \tag{35}$$

Based on the representation (35), where (29) to (31) have to be used, we propose to exploit the golden section technique [15] to compute $k_P^{*(2)}$ numerically.

IV. Numerical Example and Simulations

To illustrate the features of the L_1-optimal control algorithms developed in this paper, two simulation experiments were done. In both experiments, the parameters of $W_1(s)$ and $W_\xi(s)$ were given as in [10, p. 135]: $K_1 = 1$, $T_1 = 0.1$ s, $K_\xi = 10.84$, $T_\xi = 0.493$ s.

With these parameters, the stability region of the inner closed-loop circuit defined by (25) is depicted in Fig. 4.

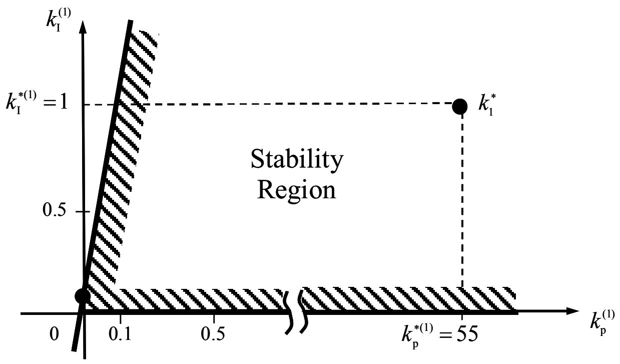

Fig. 4. Set of PI controller parameters for $K_1 = 1$, $T_1 = 0.1$ s, $K_\xi = 10.84$, $T_\xi = 0.493$ s under which the inner circuit is stable

In this numerical example, according to (26), the optimal control parameters become $k_P^{*(1)} = 55$, $k_I^{*(1)} = 1\,\mathrm{s}^{-1}$, see Fig. 4, in which $k_I^* = [55, 1]^{\mathrm{T}}$ is pointed out.

Fig. 5 demonstrates the shape of the surface $\| \tilde{H}_1(k_p^{(1)}, k_I^{(1)}) \|_1$ to clarify of how this variable depends on the parameters $k_p^{(1)}$ and $k_I^{(1)}$. We observe that $\| \tilde{H}_1(k_p^{(1)}, k_I^{(1)}) \|_1$ as a surface is not convex.

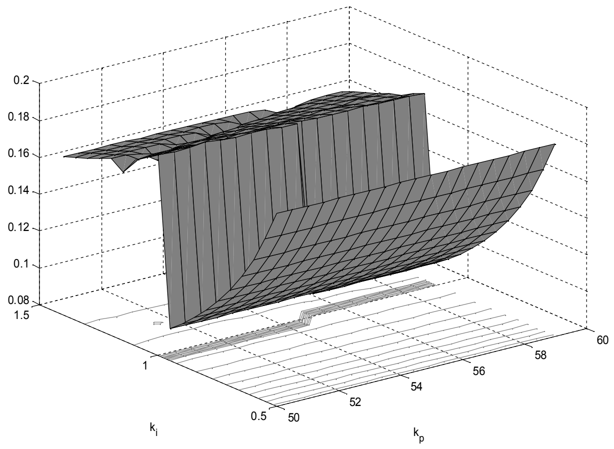

Fig. 5. Surface $\| \widetilde{H}_1(k_{\mathrm{p}}^{(1)}, k_{\mathrm{I}}^{(1)}) \|_1$

In order to calculate $\overline{k}_{\mathrm{p}}^{(2)}$ by the formulas (33), (34), the Nyquist plot $G(j\omega)$ presented in Fig. 6 was designed. This gave $\overline{k}_{\mathrm{p}}^{(2)} \approx 12$. Thus, $\widetilde{H}_2(s) \in RH_\infty$ if $0 < k_{\mathrm{p}}^{(2)} < 12$; see Fig. 7.

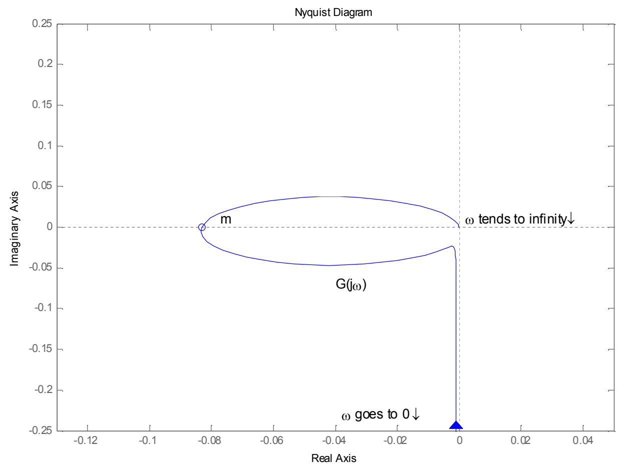

Fig. 6. The Nyquist plot of $G(j\omega)$ derived from (32)

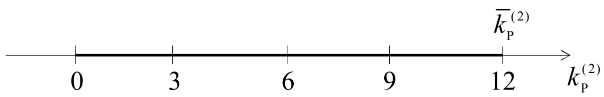

Fig. 7. Stability domain of external closed-loop circuit

The optimal value of $k_{\mathrm{p}}^{(2)}$ corresponding to (35) is $k_{\mathrm{p}}^{*(2)} = 11.8$.

With the system variables, pointed out above, it was taken: $\gamma^0(t) \equiv 0$. $d(t)$ was simulated as a random variable via the Simulink Software.

Results of two simulation experiments are presented in Figs. 8 and 9. We can observe that the performance of the control system is successful enough.

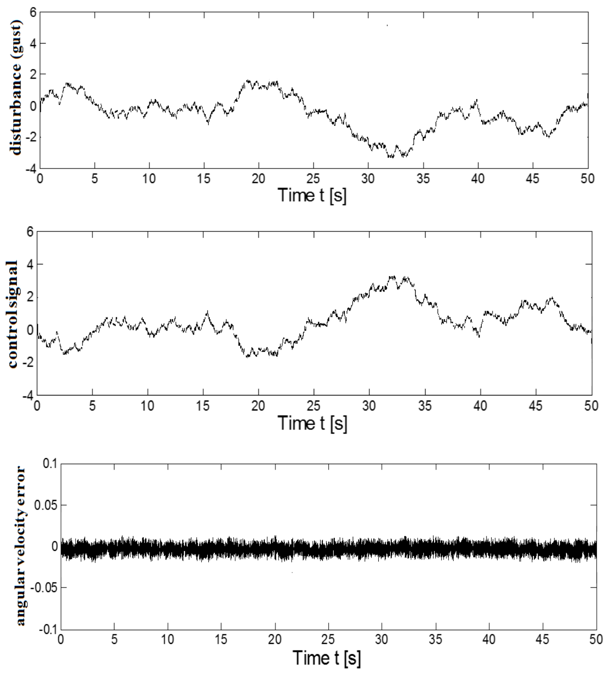

Fig. 8. Stabilization of angular velocity (simulation results 1)

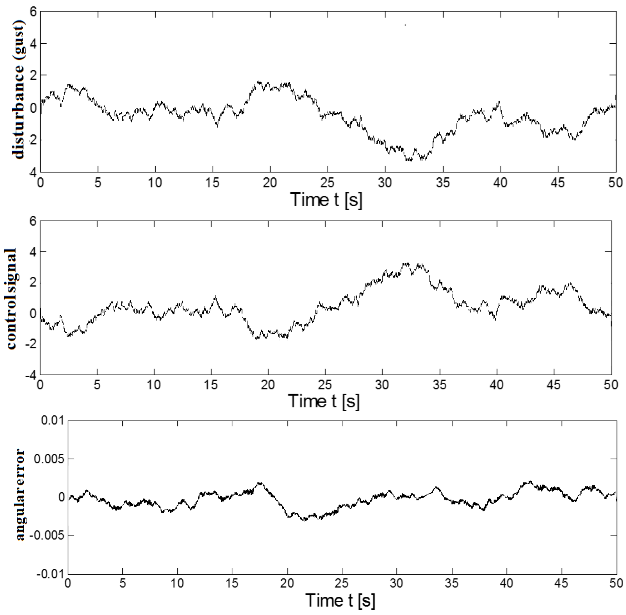

Fig. 9. Stabilization of roll angle (simulation results 2)

V. Conclusion

This paper addressed the L_1-optimization approach applied for synthesizing the continuous-time lateral autopilot system of UAV. It was established that the two-circuit L_1-optimal PI and P control laws can cope with the wind gust and ensure the desired roll orientation. This makes it possible to achieve the control objective given in (7).

A distinguishing feature of the control algorithms is that they are sufficiently simple. This is ther important advantage from the practical point of view.

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Л. С. Житецький, А. Ю. Пільчевський, А. О. Кравченко, Б. В. Биков. Проектування L_1-оптимального автопілоту бічного руху БПЛА

Стаття стосується L_1-оптимізаційної концепції, висунутої в сучасній теорії керування, з застосуванням її до побудови L_1-оптимального автопілоту бічного руху. Мета керування полягає у підтриманні бажаної орієнтації по крену деякого БПЛА за наявності зовнішнього невимірюваного збурення, зокрема, вітру. Для досягнення високого показника якості функціонування запропоновано два окремих алгоритми керування. Перший алгоритм будується для здійснення L_1-оптимального ПІ-закону керування, аби стабілізувати задану швидкість крену. Другий алгоритм забезпечує L_1-оптимальне керування, необхідне для стабілізації крену літального апарату. Для ілюстрації властивостей запропонованого методу керування наведено результати одного модельного прикладу.

Ключові слова: система керування зі зворотним зв’язком; автопілот бічного руху; оптимізація; ПІ-регулятор; БПЛА.

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Л. С. Житецкий, А. Ю. Пильчевский, А. О. Кравченко, Б. В. Быков. Проектирование L_1-оптимального автопилота бокового движения БПЛА

Статья касается L_1-оптимизационной концепции, выдвинутой в современной теории управления, с применением ее к построению L_1-оптимального автопилота бокового движения. Цель управления состоит в поддержании заданной ориентации по крену некоторого БПЛА при наличии внешнего неизмеряемого возмущения, в частности, ветра. Для достижения высокого показателя качества функционирования предложено два отдельных алгоритма управления. Первый алгоритм строится для обеспечения L_1-оптимального ПИ-закона управления, чтобы стабилизировать заданную скорость крена. Второй алгоритм обеспечивает L_1-оптимальное управление, необходимое для стабилизации крена летательного аппарата. Для иллюстрации свойств предложенного метода управления приведены результаты одного модельного примера.

Ключові слова: Система управления с обратной связью; автопилот бокового движения; оптимизация; ПИ-регулятор; БПЛА.

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