

## AUTOMATIC CONTROL SYSTEMS

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**Abstract**—Designed the model of the nonlinear stabilization system of the ship course and showed experimental and theoretical research results of the selection an optimal regulator for the system, which excludes, at the same time, appearance of limit cycles in it. As research method, the method of describing function was used. Simulation of the nonlinear stabilization system of the ship course with different types of regulators allowed to perform a comparative assessment of industrial regulators. To improve the properties of the PID-regulator, it is proposed to a nonlinear correction system.

**Index Terms**—Ship; stabilization system; course; nonlinearity; describing function; controller; correction; the frequency transfer function.

## I. INTRODUCTION

Stabilization and control systems take an important place in the automated control systems of inertial objects. Their goal is to achieve a sustainable value of a control magnitude or its change for a given control law.

Currently, there is a whole class of automatic stabilization and control systems of inertial objects such as planes, tanks, ships, submarines. A detailed study of these systems allows to conclude that they are nonlinear.

The tasks of analysis and synthesis of such systems are more complex than similar problems for linear systems. For example, the stability of nonlinear systems unlike linear depends on the size and location of external action application, the nature of surge process changes with changing the magnitude of external action, there are regimes in nonlinear systems that do not exist in linear systems, including the mode of limit cycles. All of this requires applying of special analysis and synthesis methods of nonlinear systems.

In the development of automatic control theory has been created various mathematical methods for analysis and synthesis of nonlinear systems, each of them can only be applied to a class of systems and applications. So, universal analytical methods for the researching of nonlinear systems do not exist.

## II. PROBLEM STATEMENT

The main feature of nonlinear systems is the possibility of limit cycles appearing – undamped oscillations, with amplitude that is independent from external influence and initial conditions, and their fre-

quency is a harmonic or subharmonic of input signal. In this regard, the particular interest is in the development of a stabilization system of the ship course with optimal regulator, which exclude the possibility of limit cycles appearing in it during operation.

## III. PROBLEM SOLUTION

Mathematical model of the stabilization system of the ship course is represented as a block diagram (Fig. 1).

The system has certain features. Firstly, scheme contains sensors of the angular and speed deviation of control object based on the three-stage and two-stage gyroscopes. At the same time, they contain nonlinear feedbacks, which are caused by the presence of dry friction in the bearings of the gyroscope frames. Secondly, in the real system the deviation angle of the executive drive rudder and its rotation speed have natural barriers, so executive drive is nonlinear. Thus, the stabilization and control system of the ship course is nonlinear. There are five nonlinear elements in it.

Considering nonlinearities in the angle and speed sensors and their effect on the sensor is a separate branch of researches. Consider sensors errors caused by the presence of nonlinearities in them, like  $\Delta\psi$  and  $\Delta\beta$ , and pass to linear models of the angle sensor and the speed sensor.

Because of restrictions on the amount of work, it will be presented below research of the system for one nonlinearity of the executive drive. Methods of assessing the impact of another nonlinearity is similar.

The principle of superposition does not apply to nonlinear systems, so, we will consider the system that is only under the action of the control signal.

Because of adopted limitations, the block diagram of a nonlinear system of stabilization takes the form shown in Fig. 2.

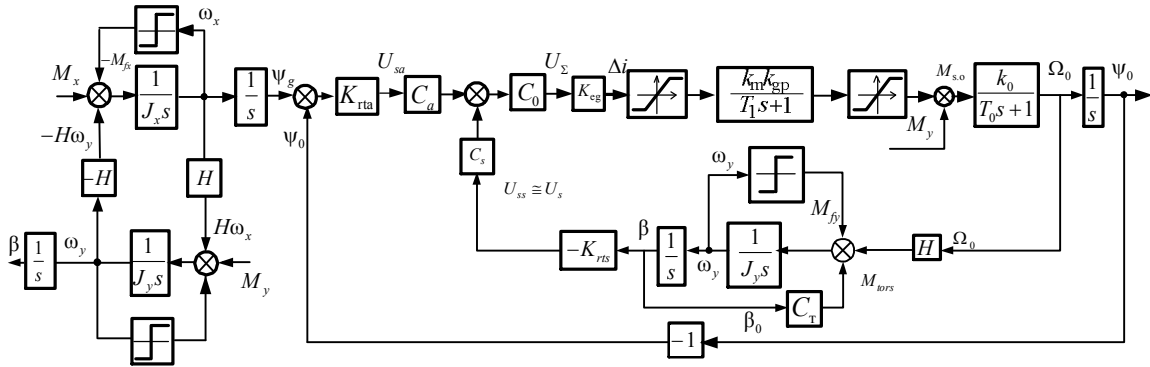


Fig. 1. Nonlinear stabilization and control system of the ship course

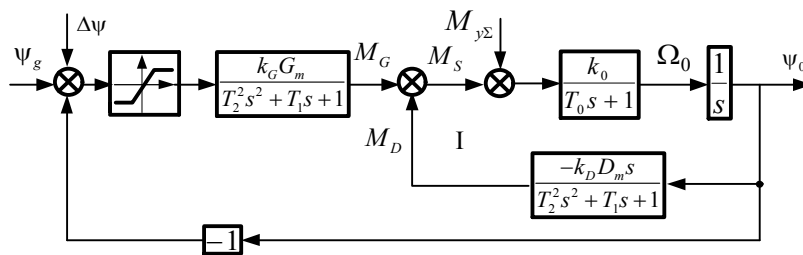


Fig. 2. Calculated model of the nonlinear stabilization system of the ship course

The moment of stabilization is formed through the sensor channel of angular deviation and through the speed sensor of angular deviation of control object:

$$\bar{M}_s = \bar{M}_G + \bar{M}_D \equiv k_G G_m + k_D D_m. \quad (1)$$

Concurrently, the executive drive has a linear characteristic with limitation

$$z = \{-x_{\max} \text{ at } x < -a \quad kx \text{ at } -a \leq x \leq a \quad x_{\max} \text{ at } x > a\}.$$

In order to determine the possibility of limit cycle appearing, we will make calculations according to the method of describing function.

After the convolution of contour I and the transition to the frequency domain, we obtain an equivalent of frequency transfer function of the linear part of the stabilization system

$$W(j\omega) = P(\omega) + jQ(\omega),$$

where

$$P(\omega) = \frac{k_0 k_G G_m [T_0 T_2^2 \omega^2 - (T_0 + T_1)]}{[T_0 T_2^2 \omega^3 - (T_0 + T_1) \omega]^2 + [(1 + k_0 k_D D_m) - (T_0 T_1 + T_2^2) \omega^2]^2}, \quad (2)$$

$$Q(\omega) = -\frac{k_0 k_G G_m [(1 + k_0 k_D D_m) - (T_0 T_1 + T_2^2) \omega^2]}{[T_0 T_2^2 \omega^3 - (T_0 + T_1) \omega]^2 + [(1 + k_0 k_D D_m) - (T_0 T_1 + T_2^2) \omega^2]^2} \frac{1}{\omega}.$$

We will apply to the nonlinear element method of harmonic linearization and will find in the handbook of automation the function which describe it

$$W_f(a_m, \omega) = \frac{N_1 + jC_1}{a_m} = \frac{2k}{\pi} \left( \arcsin \frac{b}{a_m} + \frac{b}{a_m} \sqrt{1 - \frac{b^2}{a_m^2}} \right), \quad (3)$$

where  $b$  is the width zone of linear mode;  $k$  is the gain coefficient in the linear mode.

The condition of the limit cycle appearing in considered system is the implementation of equality

$$W(j\omega) = -W_f^{-1}(a_m, \omega), \quad (4)$$

where  $W_f^{-1}(a_m, \omega)$  is the inverse describing function of nonlinear part of the system;  $W(j\omega)$  is the frequency transfer function of the linear part of the stabilization system.

Combined plotting (Fig. 3) of functions  $W(j\omega)$  and  $-W_f^{-1}(a_m, \omega)$  in the plane  $P(j\omega) \rightarrow jQ(\omega)$ , allows to solve the equation (4).

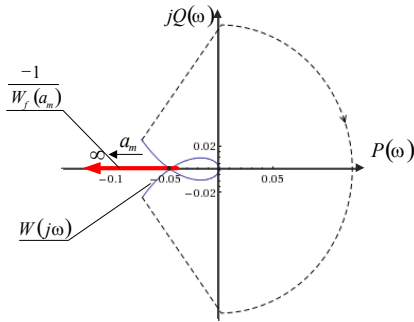


Fig. 3 Inverse describing function and the amplitude-phase frequency characteristic (APFC)

When plotting the inverse describing function, we take into account the particular cases of its calculation:

– for

$$a_m = b \Rightarrow W_f(a_m = b) = \frac{2k}{\pi}(\arcsin 1) = \frac{2k}{\pi} \cdot \frac{\pi}{2} = k;$$

– for

$$a_m \rightarrow \infty \Rightarrow W_f(a_m \rightarrow \infty) = \frac{2k}{\pi} \left( \arcsin 0 + \frac{b}{\infty} \right) = 0.$$

Let's find the coordinate of the point of intersection  $W(j\omega)$  and  $W_f^{-1}(a_m)$ .

First of all we determine from (2) the frequency of possible limit cycle

$$\omega_0|_{Q(\omega)=0} = \sqrt{\frac{1 + k_0 k_D D_m}{T_0 T_1 + T_2^2}}.$$

Substituting the value  $\omega_0$  into the real component of equation (2), we find

$$P(\omega) = \frac{k_0 k_G G_m (T_0 T_1 + T_2^2)}{1 + k_0 k_D D_m}.$$

Taking into account the parameters of the system, determine the frequency of limit cycle and the coordinate point of intersection of inverse describing function and APFC

$$\omega_0 = 0.168 \text{ rad/s}, \quad P(\omega_0) = -0.047.$$

A limit cycle will be possible if  $|1/k| < P(\omega_0)$ , so  $k > 21.27$ .

The amplitude of a possible limit cycle we find, in accordance with equation (4), after the substitution of values in it for  $W(j\omega_0)$  and  $W_f(a_m)$ :

$$-0.047 = \frac{-1}{\frac{2k}{\pi} \left( \arcsin \frac{b}{a_m} + \frac{b}{a_m} \sqrt{1 - \frac{b^2}{a_m^2}} \right)}.$$

On the basis of the tables function  $W_f(a_m)$  at the parameter  $b=1$  we have  $a_m = 1.98 \text{ deg}$ .

Thus, there is possible limit cycle  $\psi_0(t) = -1.98 \sin 0.168 t$  in the stabilization and control system of the ship course. According to the criterion of Goldfarb the limit cycle will be stable.

Oscillogram of the system reaction to the signal with received parameters (Fig. 4) confirmed data calculations about the existence of the limit cycle.

In practice, the possibility of the limit cycle appearing in the stabilization and control system of the ship course should be excluded. Synthesize a regulator that excludes the possibility of the limit cycle appearing.

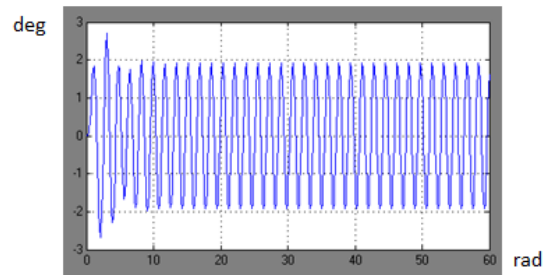


Fig. 4 Oscillogram of the limit cycle

The limit cycle will be excluded if the inverse describing function  $W_f^{-1}(a_m)$  and the APFC of the linear part of the system will not intersect in the complex plane, i.e. if the value  $|k_{reg} W(j\omega)|_{\omega=0.168}$  will be less than the minimum modulo value of inverse describing function

$$|k_{reg} W(j0.168)| < \left| \frac{1}{k} \right|. \quad (5)$$

To provide the necessary modulo supply stability for stabilization systems, take

$$k_{reg} < \left| \frac{1}{2 \cdot W(j0.168)k} \right|. \quad (6)$$

Considering  $k = 22$ , from the equation (6) find transfer coefficient of the P-regulator, which is synthesized  $k_{reg} \approx 0.484$ .

The mutual arrangement of inverse transfer function of the nonlinear element and the APFC of the linear part of the system takes the form, shown on the Fig. 5.

Thus, the possibility of limit cycle appearing in the stabilization and control system of the ship course is excluded.

Schematic decisions of regulators for stabilization systems may be different. The block diagram of the

system, in general, with PID-regulator is shown on the Fig. 6.

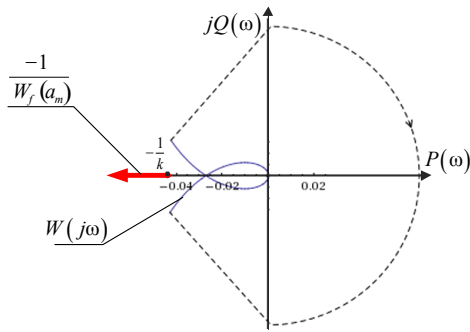


Fig. 5. Effect of the P-regulator to the mutual arrangement of inverse describing function and APFC

Models of the system with PD- and PI-regulators can be obtained by applying restrictions on the PID-model.

Modeling of the stabilization and control system of the ship course with different types of regulators allows to do a comparative assessment of industrial regulators.

A PD-regulator reacts not only to the error signal, but to the speed of its change too. Due to this, when using PD-control law achieved the effect of advanced control. Results of researches of the system with PD-regulator are shown in Fig. 7.

The experiment showed that with selected parameters control occurs by aperiodic law. Fluctuations of the ship are missing. The rapidity of action of the system reaches 120 s. Comparing the step response of the system without the influence of external perturbations and with them, we conclude that the PD-control system is static.

Step response analysis (Fig. 8) for the system with PI-regulator shows that the adding to the control law the error signal and its integral increases the settling time of the system to 200 s and raises its oscillation, thus reducing the stability margin.

At the same, an integral component converts the system into astatic first order one, that ensures elimination of static errors, which is typical for a system with PD-regulator

The step response of the system with PID-regulator is shown in Fig. 9.

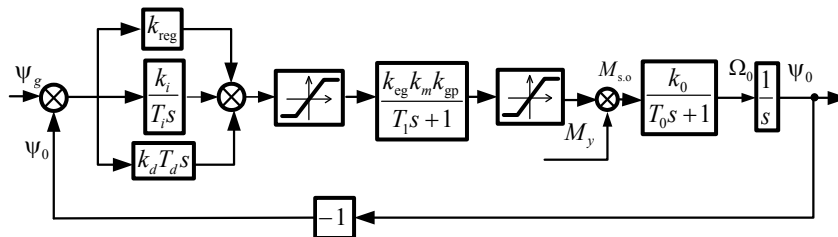


Fig. 6. Block diagram of the system with the PID-regulator

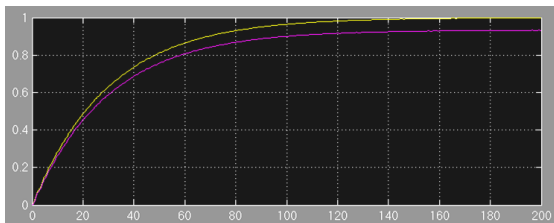


Fig. 7. Step response of the system with the PD-regulator

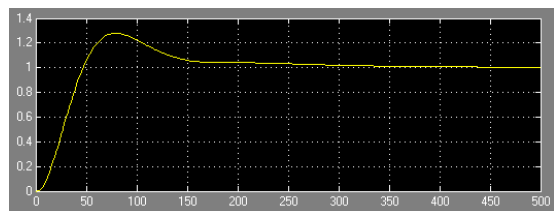


Fig. 8. Step response of the system with the PI-controller

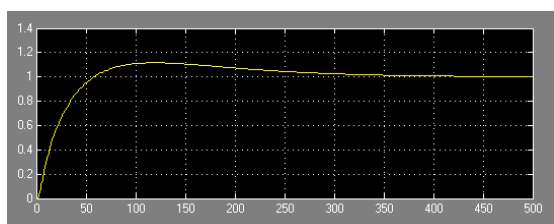


Fig. 9. Step response of the system with the PID-regulator

As in the case with PI-regulator, stabilization and control system with PID-regulator hasn't a static error, but there still is overshoot, albeit much smaller. The system has a larger settling time than in previous systems. This is due to the presence an integrating unit in the regulator. When the control signal reaches the limit, the integrator continues to integrate mismatch error. As a result - the overshoot and rising time of the transition process.

Of course, the developer of stabilization systems can choose which regulator is better from the standpoint of technical requirements to the system. Each option has its advantages and disadvantages.

Since in the stabilization and control system of the ship course there are aforementioned limitations, it is prompted to enter the nonlinear correction system (Fig. 10) into the PID-regulator, which provides, in our opinion, the main advantages combination of PD- and PID-regulators.

The principle of its action is next. Output signal  $u$  of PID-regulator is proportional to the desired helm angle. If there is no saturation, the difference of signals  $\Delta = u - \psi_0$  is zero and the feedback does not

work, so uses above studied law of PID-regulator. If the signal  $u$  exceeds the permissible limit, the difference  $\Delta = u - \psi_0$  is fed with the sign "-" to input of integrating unit through the gain. Thus, at the saturation signal at the input of the integrating unit weakens as strong, as the greater the difference between the desired and acceptable helm angles. So, using of the nonlinear correction system allows to remove the negative impact of integrating component on the transition process.

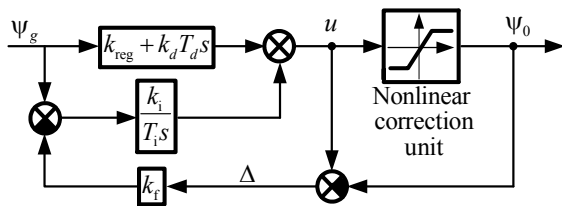


Fig.10 The principle of the nonlinear correction

In order to optimize the regulator with nonlinear correction system it was elected the regulator's feedback coefficient such that the transition process of the system, in the whole, satisfies specified requirements of quality indicators. To select the optimal value of the gain it was applied the numerical optimization procedure of the Simulink Respons Optimization package.

As a result of optimization  $k_f$  with a block Signal Constraint (Fig.11), is received an optimal value of the coefficient  $k_f = 7,2477$ .

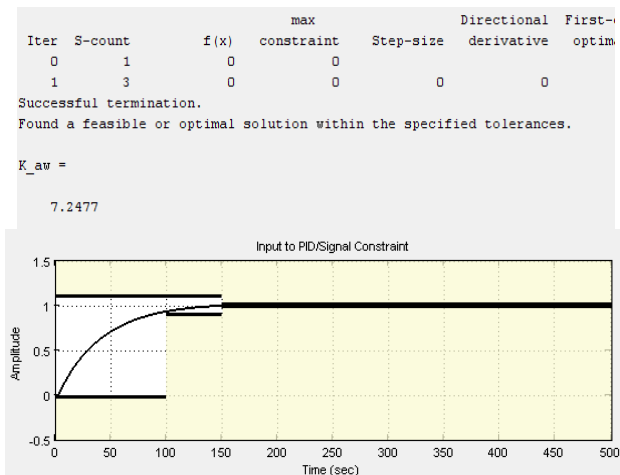


Fig. 11. Results of optimization

Comparative characteristics (Fig. 12) of the system with PID-regulator and optimal (with nonlinear correction of PID-regulator) demonstrate that both systems definitely have the same constant value, but in the optimal system the process is

without overshoot and the rapidity of action of the stabilization system meets the requirements.

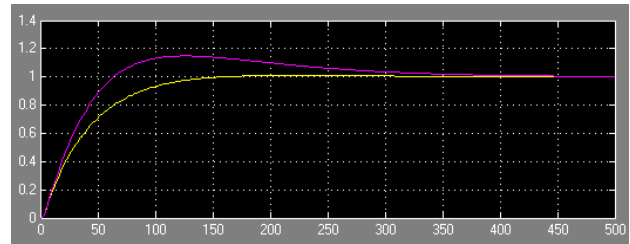


Fig. 12. Comparative characteristics

Summing up research results, we note that for the nonlinear stabilization and control system of the ship course, in terms of quality control and excluding the probability of oscillations, it is optimal the PID-regulator with the proposed system of nonlinear correction.

#### IV. CONCLUSION

The feature of nonlinear systems is the possibility in them of limit cycles appearing – undamped oscillations, which amplitude is independent of external influence and initial conditions, and their frequency is a harmonic or subharmonic of input signal.

Designed model of the nonlinear stabilization and control system of the ship course allowed to set parameters of limit cycle system and synthesize the regulator to exclude the appearing possibility of such cycles.

Theoretical and experimental research showed good convergence of results.

Synthesis of industry regulators allowed to choose the optimal one. Such the nonlinear stabilization and control system of the ship course in terms of quality control and excluding the probability of oscillations is PID-regulator with the proposed system of nonlinear correction.

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**О. К. Аблесімов, М. О. Пилипенко. Оптимальний регулятор для нелінійної системи стабілізації інерційного об'єкта керування**

Розроблено модель нелінійної системи стабілізації курсу корабля і представлено результати теоретичних і експериментальних досліджень з вибору для неї оптимального регулятора, що виключає в той же час виникнення в ній граничних циклів. Як метод досліджень в роботі застосований метод описуючої функції. Моделювання системи стабілізації і керування курсом корабля з різними типами регуляторів дозволило провести порівняльну оцінку промислових регуляторів. Для покращення властивостей ПІД-регулятора, до його складу запропоновано ввести нелінійну систему корекції.

**Ключові слова:** корабель; система стабілізації; курс; нелінійність; описуюча функція; регулятор; корекція; частотна передатна функція; регулятор.

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**А. К. Аблесимов, М. А. Пилипенко. Оптимальный регулятор для нелинейной системы стабилизации инерционного объекта управления**

Разработана модель нелинейной системы стабилизации курса корабля и представлены результаты теоретических и экспериментальных исследований по выбору для системы оптимального регулятора, исключающего в то же время вероятность возникновения в ней предельных циклов. Как метод исследований в работе применен метод описывающей функции. Моделирование системы стабилизации и управления курсом корабля с различными типами регуляторов позволило провести сравнительную оценку промышленных регуляторов. Для улучшения свойств ПИД-регулятора, в его состав предложено ввести нелинейную систему коррекции.

**Ключевые слова:** корабль; система стабилизации; курс; нелинейность; описывающая функция; регулятор; коррекция; частотная передаточная функция; регулятор.

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