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ONE OPTION OF CONSTRUCTING THREE-DIMENSIONAL DISTRIBUTION OF THE MASS AND ITS DERIVATIVES FOR A SPHERICAL PLANET EARTH

Purpose. To build a three-dimensional function of the mass distribution of the Earth's interior according to the parameters (Stokes constant to the second order inclusive) of the external gravitational field of the Earth without considering the minimum deviation from its known density models in geophysics. **Methodology.** The classic methods of constructing mass distribution use only the Stoke's constants zero and second orders. In iterative methods of determining the distribution models the reference model of density is taken for zero approximation which is agreed upon by Stoke's constants up to the second order inclusive. Further, the coefficients of potential expansion to a certain order are taken into account, but their contribution to the function of mass density does not investigate. This research provides an attempt to obtain such an estimation. The proposed method is approximate, but in the iterative process a function of the density is not only used, but also its derivatives. Bringing the order moments of density toward the controlled values (values that are defined on the surface of a sphere) makes it possible to analyze the process of successive approximations. **Results.** In contrast to the second-order model, which describes the global gross irregularities, the obtained distribution function gives a detailed picture of the placement density anomalies (deviation of three-dimensional functions from the average on the sphere – "izoden"). Analysis of maps at different depths (2891 km core-mantle, 5,150 km of the inner-outer core) allow making preliminary conclusions about global redistribution of mass due to the rotating component of gravity across the radius: its dilution along the axis of rotation and accumulation of rejecting it. This is particularly evident for the equatorial regions. On the contrary, there is minimum deviation in the polar regions of the Earth, which also have their own justification since the value of the rotation force decreases when moving away from the equator. The function of mass distribution, which is constructed using the proposed method, describes the mass distribution better. **Originality**. This research is in contrast to the classical results which have been obtained from the Adams-William's equations for the derivatives of the density of one variable (depth), and make attempted to obtain derivatives using Cartesian coordinates. Using the gravitational field parameters up to second order increases the order of approximation of the distribution function of the masses of three variables from two to four through the possibility of restoring the planet's mass distribution by its derivatives. At the same time, in contrast to previous research, geophysical information accumulated in the reference PREM model is used, therefore, features of the internal structure are taken into account. **Practical significance.** The received function of mass distribution of the Earth can be used as a zero-order approximation when used in the presented algorithm Stokes constant of higher order. Their applications give the possibility to interpretate of the global anomalies of the gravitational field, and explore the geodynamic processes deep inside the Earth.

Key words: potential, harmonic function, gravitational field of the Earth, mass distribution model, Stoke's constants.

Introduction

A powerful source for studying the Earth, including its internal structure [H. Moritz, 1994], [Fys M., 2014] is information about its gravitational field [G. Meshcheryakov, 1994, 1991], [Tserklevych A., 2012]. This fact, in turn, requires the development of methods for using the parameters of the gravitational field. It is known [Bullen M., 1978], [Dzewonski A., 1981] that construction of a spherically symmetric mass distribution of the Earth uses mass and Stokes constant second order (polar inertia moment). Applying expansion coefficients of potential using spherical functions is described in articles [Martinenc Z., 1986], [Scherbakov A., 1978], [Meshcheryakov G., 1975], and the functional approach to this problem is proposed in [Korbunov A. I., 2015]. In [Moritz G., 1973], it is proposed to represent the three-dimensional distribution by harmonic function, consistent with the coefficients of expansion to a certain order. G. A. Meshcheryakov proposed an approximate method for constructing three-dimensional density models of the Earth, which takes into account the internal structure and is accountable to Stokes constant given order [Meshcheryakov G., Fys M., 1981]. The degree of reliability of founded distribution model can't be estimated. In this regard, there is need to develop a technique that make it possible to analyze the calculation process and to objectively assess the reliability of density function. Such an attempt was made in articles [Fys M., 2008], [Chernyaha P., Fys M., 2012], where the problem is reduced to controlled variables (surface integrals) and is based on founded derivatives of the density function. Their use is not new in geophysics. Indeed, Adams-William's equation is widely used in geophysics [Williams E., 1923], which describe the change in mass distribution in the mantle with depth. Later this approach was continued in works [Molodenskyy M. C., 1955], [Pankov V. L., 1967]. Resulting density mass models reflect our understanding of this type and are consistent with studies of other

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authors [Byzov A. G., 2015], [Molodenskyy M. C., 2015], and relative value (derivatives of three-dimensional functions) are close to realness.

Purpose

To build a three-dimensional function of the mass distribution of the Earth's interior without considering conditions about the minimum deviation from its known density models in geophysics according to the parameters (Stoke's constant to the second order inclusive) of the external gravitational field of the Earth. To establish a contribution of the paremeters in mass distribution function.

Methodology

The classic methods of constructing mass distribution use only the Stoke's constants zero and second orders. In iterative methods of determining the distribution models the reference model of density is taken for zero approximation which is agreed upon by Stoke's constants up to the second order inclusive. Further, the coefficients of potential expansion to a certain order are taken into account, but did not investigate their contribution to the mass density function. This research provides an attempt to obtain such an estimation. The proposed method is approximate, but also, as a function of the density is used, as also its derivatives. Bringing the order moments of density toward the controlled values (values that are defined on the surface of a sphere) make it possible to analyze the process of successive approximations.

Results

The density distribution of the planet interior d to the second order inclusive considering geophysical information [G. Meshcheryakov, M. Fys, 1986]:

$$d(x_1,x_2,x_3)=d^0(r)+ + \sum_{m+n+k=0}^{2} b_{mnk}W_{mnk}(x_1,x_2,x_3), \quad (1)$$

where $d^0(r)$ – one-dimensional spherical model of mass distribution (for Earth – it is a reference model PREM [Dzewonski A., 1981]) .

The expansion coefficients b_{mnk} in (1) are defined as follows [Meshcheryakov G., 1991]:

$$b_{000}=d_c\left(1-\frac{3}{d_c}\int_0^1 d^0(r)r^2dr\right),$$

$$b_{110}=35d_cS_{21}, \qquad b_{101}=35d_cC_{21},$$

$$b_{011}=35d_cS_{21},$$

$$b_{200}=\frac{7}{2}d_c\left[5\left(\frac{-C_{20}}{2H}+2C_{22}\right)-1-\right.$$

$$\left.-\frac{5}{d_c}\int_0^1 d^0(r)r^4dr+\frac{3}{d_c}\int_0^1 d^0(r)r^2dr\right], \quad (2)$$

$$b_{020}=\frac{7}{2}d_c\left[5\left(\frac{-C_{20}}{2H}-2C_{22}\right)-1-\right.$$

$$\left.-\frac{5}{d_c}\int_0^1 d^0(r)r^4dr+\frac{3}{d_c}\int_0^1 d^0(r)r^2dr\right], \quad (3)$$

$$b_{002}=\frac{7}{2}d_c\left[5\left(1-\frac{1}{2H}\right)C_{20}-1-\right.$$

$$\left.-\frac{5}{d_c}\int_0^1 d^0(r)r^4dr+3\int_0^1 d^0(r)r^2dr\right], \quad (4)$$

where $d_c=5.514\ г/см^3$ – average density of the Earth, C_{nk}, S_{nk} – Stoke's constants.

The function of the mass density of the internal planet, given in formula (1), defines the features of its three-dimensional internal structure. Therefore in paper [G. Meshcheryakov, 1986] a distribution model (1) is proposed to be accepted as a base.

This method of finding the derivatives of density function based on experimental data opens up new possibilities for investigation of the internal structure of planets. For this it is presented in the following form [Fys M., 2013]:

$$\frac{\partial d}{\partial x_i}=\frac{1}{a_i}\sum_{m+n+k=0}^{N} a^i_{mnk}W_{mnk}(x_1,x_2,x_3),\ i=1,2,3. \quad (5)$$

The expansion coefficients a^i_{mnk} in (5) are defined by degree moments of derivatives of the function of mass distribution density within the Earth and on the surface of the ellipsoid, which have the next form, in accordance:

$$I^i_{pqs}=\frac{1}{MR^t}\int_t x_1^p x_2^q x_3^s\frac{\partial d}{\partial x_i}dt, \quad t=p+q+s, \quad (6)$$

$$s_{p_1q_1s_1}=\frac{1}{MR^{t+1}}\iint_s x_1^{p_1}x_2^{q_1}x_3^{s_1}d(x_1,x_2,x_3)ds \quad (7)$$

and are defined using degree density moments of the mass inner planet of lower orders:

$$I^i_{pqs}=pe(i-1)I_{p-1qs}+qe(i-2)I_{pq-1s}+$$

$$+se(i-3)I_{p-1qs-1}+s_{p_1q_1s_1}, \quad (8)$$

where $p_1=p+e(i-1)$,

$s_1=s+e(i-3)$,

$q_1=q+e(i-2)$, $e(n)=\begin{cases}1, & n=0,\\ 0, & n\neq 0,\end{cases}$

s – a surface is described by the equation $x_3=\pm a_3\sqrt{R^2-x_1^2-x_2^2}$. For quantities (8) takes place in the next evaluation:

$$\frac{3d_{\min}(2p-1)!!(2q-1)!!(2s-1)!!}{d_cR(2(p+q+s)+1)!!}\le s_{2p2q2s}\le\frac{3d_{\max}(2p-1)!!(2q-1)!!(2s-1)!!}{d_cR(2(p+q+s)+1)!!}, \quad (9)$$

here, $d_{\min}$, $d_{\max}$ – the minimum and maximum densities on the surface of the Earth, respectively, which help to control the computing process.

To determine the value (8) you must know the function d on the planet's surface. It can be considered known, and thus calculate (8). The function d, however, is defined as unreliable on the surface s, so the degree moments (8) we will try to find the approximate degree moments, by using integral characteristics [Mashymov M., 1991]

$$C_{nk} = \frac{1}{Ma^n}\int_t dU_{nk}dt\ ,$$

$$S_{nk} = \frac{1}{Ma^n}\int_t dV_{nk}dt\ , \qquad (10)$$

where U_{nk}, V_{nk} – inside spherical function. For this we use identity:

$$\int_t j\frac{\partial d}{\partial x_i}dt = -\int_t d\frac{\partial j}{\partial x_i}dt + \iint_s dj\cos a_i ds\ , \qquad (11)$$

here $(\cos a_1, \cos a_2, \cos a_3)$ – normal vector to the surface, where

$$\cos a_i = \pm\frac{x_i}{R}, \qquad i = 1, 2, 3\ . \qquad (12)$$

Using the identity (11) in the formulas (10), we obtain the following:

$$C_{nk} = \frac{1}{Ma_e^n}\left(-\int_t \frac{\partial d}{\partial x_i}U_{nk}^i dt + \iint_s dV_{nk}^i ds\right)$$

$$S_{nk} = \frac{1}{Ma_e^n}\left(-\int_t \frac{\partial d}{\partial x_i}V_{nk}^i dt + \iint_s dV_{nk}^i ds\right) \qquad (13)$$

where

$$U_{nk}^i = \int_0^{x_i} U_{nk}dx_i\ , \qquad V_{nk}^i = \int_0^{x_i} V_{nk}dx_i \qquad (14)$$

Equations (13) contain unknowns s_{pqs}, which can still be added to the identities in the form

$$s_{pqs} = s_{p+2qs} + s_{pq+2s} + s_{pqs+2}\ ,$$
$$p+q+s \geq 0 \qquad (15)$$

According to the calculated values using reference PREM model see Table. 1, unnormalized Stokes constants to the second order, inclusive, and dynamic compression H [Yatskyv J., 1980] as given in Table 2, we determine the degree moments $I_{pqs}(p+q+s \leq 2)$ of the spherical planet (tab. 3) and expansion coefficients b_{mnk} (tab. 4).

Таблиця 1

$\int_0^1 d^0(r)r^2dr$	$\int_0^1 d^0(r)r^4dr$
1.00006261857919E+0	1.65410100398967E-1

Table 2

Unnormalized Stoke's constant and dynamic compression

C_{00}^*	C_{20}^*	C_{22}^*	C_{21}^*	S_{21}^*	S_{22}^*	H
-0.626E-4	1.957E-4	2.793E-6	-2.582E-11	2.582E-11	-9.018E-7	0.327E-2

Table 3

Moments of inertia density considering model PREM

$I_{200}\times10^{-4}$	$I_{020}\times10^{-4}$	$I_{002}\times10^{-4}$	$I_{110}\times10^{-5}$	$I_{101}\times10^{-8}$	$I_{011}\times10^{-10}$
-0.3316	-0.5443	-0.1122	-0.4509	0.1252	-0.2582

Table 4

The expansion coefficients b_{mnk} considering PREM model

$b_{000}\times10^{-4}$	$b_{200}\times10^{-3}$	$b_{020}\times10^{-3}$	$b_{002}\times10^{-1}$	$b_{110}\times10^{-4}$	$b_{101}\times10^{-7}$	$b_{011}\times10^{-9}$
-0.3612	-0.5567	-0.6061	-0.1941	-0.1578	0.2191	-0.4518

The value $s_{pqs}(p+q+s \leq 4)$ is determined approximately using equation (13)–(14). Since the density function of the planet's interior is represented by formula (1), and considering Stoke's constant C_{00}, leads us to the equality:

$$C_{00} = \frac{1}{M}\left(-\int_t \frac{\partial d}{\partial x_1}x_1 dt + \iint_s dx_1\cos a_1 ds\right) =$$

$$= \frac{1}{M}\int_t\left(\frac{3x_1^2}{a_1^2}b_{200} + \frac{x_1^2}{a_1^2}b_{020} + \frac{x_1^2}{a_1^2}b_{002}\right)dt + s_{200},$$

wherefrom, with known coefficients $b_{200}, b_{020}, b_{002}$ we find

$$s_{200} = \frac{5d_cC_{00} + (3b_{200} + b_{020} + b_{002})}{5},$$

$$s_{020}=\frac{5d_c C_{00}+\left(b_{200}+3b_{020}+b_{002}\right)}{5},$$

$$s_{002}=\frac{5d_c C_{00}+\left(b_{200}+b_{020}+3b_{002}\right)}{5},$$

Using equality (15), we determine s_{000}.

Stokes constants of the first order allow us to record for $s_{pqs}\left(p+q+s=3\right)$ following

$$s_{201}=s_{021}=C_{10}, \qquad s_{210}=s_{012}=\frac{S_{11}}{2},$$

$$s_{120}=s_{102}=C_{11}, \qquad s_{300}=2C_{11},$$

$$s_{030}=S_{11}, \qquad s_{003}=2C_{10}.$$

Considering that the beginning of the coordinate system is located in the center of the masses, we find that $s_{pqs}=0$ when $p+q+s=3$. Hence

$$s_{100}=s_{300}+s_{120}+s_{102}=0,$$

$$s_{010}=s_{030}+s_{210}+s_{012}=0,$$

$$s_{001}=s_{003}+s_{201}+s_{021}=0.$$

We proceed to determine the values s_{pqs} of the fourth order when one of the numbers p, q, or s are odd. Using Stokes constants C_{21}, C_{22}, S_{22} from [G. Meshcheryakov, 1991] we, as a result, obtain the following equality

$$s_{103}=4C_{21}, \quad s_{121}=C_{21},$$

$$s_{310}=s_{130}=4S_{22}, \quad s_{112}=S_{22},$$

and

$$s_{101}=s_{301}+s_{121}+s_{103},$$

$$s_{110}=s_{310}+s_{130}+s_{112}.$$

To find the degree moments s_{pqs} of even degrees of the second and fourth orders we take Stokes constants C_{20} and C_{22}:

$$C_{20}=\frac{1}{Ma_e^2}\int_t d\left(x_3^2-\frac{1}{2}\left(x_1^2+x_2^2\right)\right)dt = =\frac{1}{Ma_e^2}\int_t dU_{20}dt, \tag{16}$$

$$C_{22}=\frac{1}{Ma_e^2}\int_t d\left(\frac{1}{4}\left(x_1^2-x_2^2\right)\right)dt = =\frac{1}{Ma_e^2}\int_t dU_{22}dt. \tag{17}$$

The consistent application of (11) for the values (16), (17) in variables x_1, x_2, x_3 for a spherical planet determines the system equation:

$$Ax=b \tag{18}$$

$$A=\begin{pmatrix} 2 & -3 & -3 & 0 & 0 & 0 \\ 0 & 6 & 0 & -1 & -3 & 0 \\ 0 & 0 & 6 & 0 & -3 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 & 1 \end{pmatrix},$$

$$b=\begin{pmatrix} 6C_{20} \\ 6C_{20} \\ 6C_{20} \\ 4C_{22} \\ 12C_{22} \\ 12C_{22} \end{pmatrix}, \qquad x=\begin{pmatrix} s_{004} \\ s_{202} \\ s_{022} \\ s_{400} \\ s_{220} \\ s_{040} \end{pmatrix},$$

that has many solutions (det (A)=0). We transform it to the form

$$DY=C_{20}b_1+C_{22}b_2+s_{040}b_3, \tag{19}$$

де

$$D=\begin{pmatrix} 2 & -3 & -3 & 0 & 0 \\ 0 & 6 & 0 & -1 & -3 \\ 0 & 0 & 6 & 0 & -3 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}, \quad b_1=\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$b_2=\begin{pmatrix} 0 \\ 0 \\ 0 \\ -2 \\ -1 \end{pmatrix}, \qquad b_3=\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \qquad Y=\begin{pmatrix} s_{004} \\ s_{202} \\ s_{022} \\ s_{400} \\ s_{220} \end{pmatrix}.$$

Present system is defined (det $(D)\neq 0$) and its solution is determined by value s_{040}. For its calculation we take identities

$$s_{400}+s_{220}+s_{202}=s_{200},$$

$$s_{040}+s_{220}+s_{022}=s_{020},$$

$$s_{004}+s_{202}+s_{022}=s_{002},$$

which we transform in following way:

$$s_{400}+s_{040}+s_{004}+2\left(s_{220}+s_{022}+s_{202}\right)= =s_{200}+s_{020}+s_{002}=s_{000}. \tag{20}$$

Consistently solving the system equation $DY_i=b_i\left(i=1,2,3\right)$, we find [Fys M., 2016] a general solution in the form $Y=C_{20}Y^1+C_{22}Y^2+s_{040}Y^3$, a substitute in (20), hence s_{040} is calculated

$$s_{040}=\frac{s_{000}-\sum_{i=1}^{2}r_i\left(y_1^i+y_4^i+2\left(y_2^i+y_3^i+y_5^i\right)\right)}{\left(y_1^3+y_4^3+1+2\left(y_2^3+y_3^3+y_4^3\right)\right)}$$ and

the rest $s_{2p2q2s}\left(p+q+s=2\right)$.

According to the formula (6)–(8) we find I_{pqs}^i and, further, expansion coefficients a_{mnk}^i in (5). Using the expression (5) we find derivatives of the density function and then a presentation for function d:

$$d_{in}\left(x_1,x_2,x_3\right)=\int_0^{x_1}\frac{\partial d}{\partial x_1}\left(x_1x_2,x_3\right)dx_1+ +\int_0^{x_2}\frac{\partial d}{\partial x_2}\left(0,x_2,x_3\right)dx_2+\int_0^{x_3}\frac{\partial d}{\partial x_3}\left(0,0,x_3\right)dx+d_0, \tag{21}$$

where d_0 – is the density value for the center of the planet [Fys M., 2013].

We note that summands of the formula (21) take the following form after integration

$$\int_0^{x_1} \frac{\partial d}{\partial x_1} dx_1 = \sum_{m+n+k=0}^{3} a_{mnk}^1 \int_0^{x_1} W_{mnk}\left(x_1, x_2, x_3\right) dx_1 ,$$

$$\int_0^{x_2} \frac{\partial d}{\partial x_2} dx_2 = \sum_{m+n+k=0}^{3} a_{mnk}^2 \int_0^{x_2} W_{mnk}\left(0, x_2, x_3\right) dx_2 ,$$

$$\int_0^{x_3} \frac{\partial d}{\partial x_3} dx_3 = \sum_{m+n+k=0}^{3} a_{mnk}^3 \int_0^{x_3} W_{mnk}\left(0, 0, x_3\right) dx_3 ,$$

And the analytical image's last relations will be written as follows:

$$\int_0^{x_1} W_{mnk}\left(x_1, x_2, x_3\right) dx_1 =$$

$$= \frac{N!}{2^N m! n! k! a_1^m a_2^n a_3^k} \sum_{l=\frac{N}{2}}^{N} \frac{(-1)^l}{2^l l!} \sum_{t_1+t_2+t_3=N-l} \frac{(2t_1-1)!!(2t_2-1)!!(2t_3-1)!!}{(2t_1-m+1)(2t_1-m)!(2t_2-n)!(2t_3-k)!} \left(\frac{x_1}{a_1}\right)^{2t_1-m+1} \left(\frac{x_2}{a_2}\right)^{2t_2-n} \left(\frac{x_3}{a_3}\right)^{2t_3-k},$$

$$\int_0^{x_2} W_{mnk}\left(0, x_2, x_3\right) dx_2 =$$

$$= \frac{N!}{2^N \left(\frac{m}{2}\right)! n! k! a_1^m a_2^n a_3^k} \sum_{l=\frac{N}{2}}^{N} \frac{(-1)^l}{2^l l!} \sum_{t_1+t_2+t_3=N-l-\frac{m}{2}} \frac{(2t_2-1)!!(2t_3-1)!!}{(2t_2-n+1)(2t_2-n)!(2t_3-k)!} \left(\frac{x_2}{a_2}\right)^{2t_2-n+1} \left(\frac{x_3}{a_3}\right)^{2t_3-k},$$

$$\int_0^{x_3} W_{mnk}\left(0, 0, x_3\right) dx_2 = \frac{N!}{2^N \left(\frac{m}{2}\right)! \left(\frac{n}{2}\right)! k! a_1^m a_2^n a_3^k} \sum_{l=\frac{N}{2}}^{N-\frac{m+n}{2}} \frac{(-1)^{N-l-\frac{m+n}{2}} 2^l}{\left(N-l-\frac{m+n}{2}\right)!(2l-k+1)} \left(\frac{x_3}{a_3}\right)^{2l-k+1}.$$

Thus the constructed three-dimensional density model, as seen in Fig. 1, keeps all basic properties of the reference PREM model: jumps value, the depth of their occurrence, and the character of density changes.

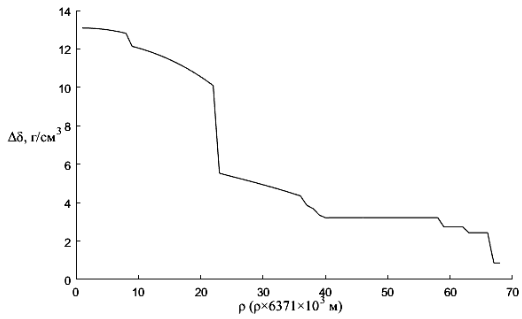


Fig. 1. *Graph of density distribution of the Earth on the model PREM, which is obtained by the method described in the article*

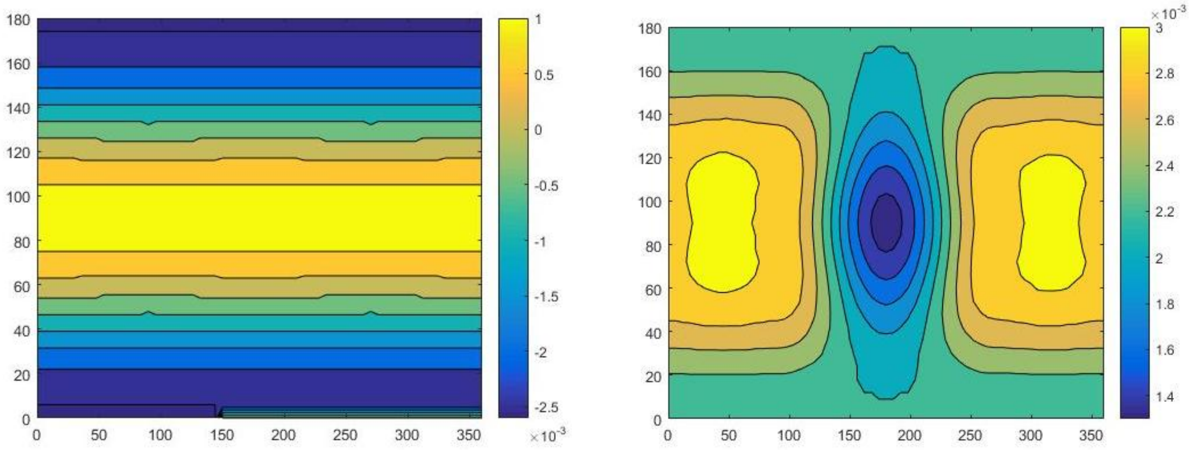
However, in contradistinction to the model d_2 (Fig. 2, *a*, Fig. 3, *a*, Fig. 4, *a*), the density anomaly d_{in} (Fig. 2, *b*, Fig. 3, *b*, Fig. 4, *b*) is more structured, and gives a more detailed picture of the placement of the masses. First of all, there is a redistribution of masses at different depths. Thus, from obtained "izoden" maps the property movement of the masses toward the surface is followed, which is caused by the rotating motion of the planet. We note that such displacements are characteristic throughout the radius of the Earth. On the contrary, at the axis of rotation, dilution of the mass is observed in depth. This is illustrated by the "izoden" map in Fig. 3, *b* (2 891 km depth, core-mantle boundary) and Fig. 4, *b* (5150 km depth, inner-outer core boundary).

Consequently, using the method described above, and considering classical and iterative methods, we obtain a density model, which gives a more detailed picture of the mass distribution inside the planet, using the same initial data.

Originality and practical significance

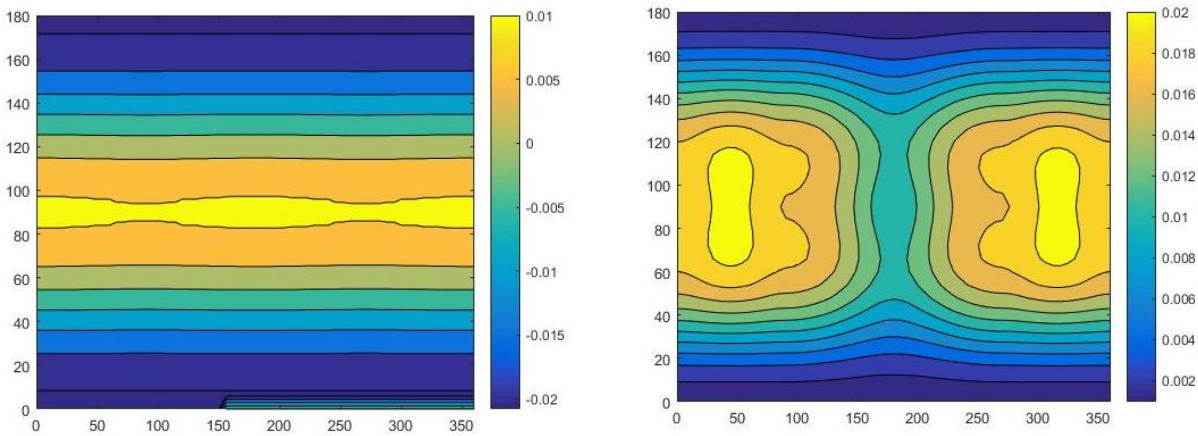
This work is in contrast to the classical results which have been obtained from the Adams-William's equations for the derivatives of the density of one variable (depth), that made an attempt to obtain derivatives in Cartesian coordinates. Using the described method of the gravitational field parameters up to second order increases the order of approximation of the distribution function of the masses of three variables from two to four by the possibility of restoring the planet's mass distribution by its derivatives. At the same time, in contrast to previous research, geophysical information accumulated in reference PREM model is used, therefore, are taking into account features of the internal structure.

The received function of mass distribution of the Earth can be used as a zero-order approximation when used in the presented algorithm Stokes constant of higher order. Their applications give the possibility of interpretation of the global anomalies of the gravitational field, and explore the geodynamic processes deep inside the Earth.



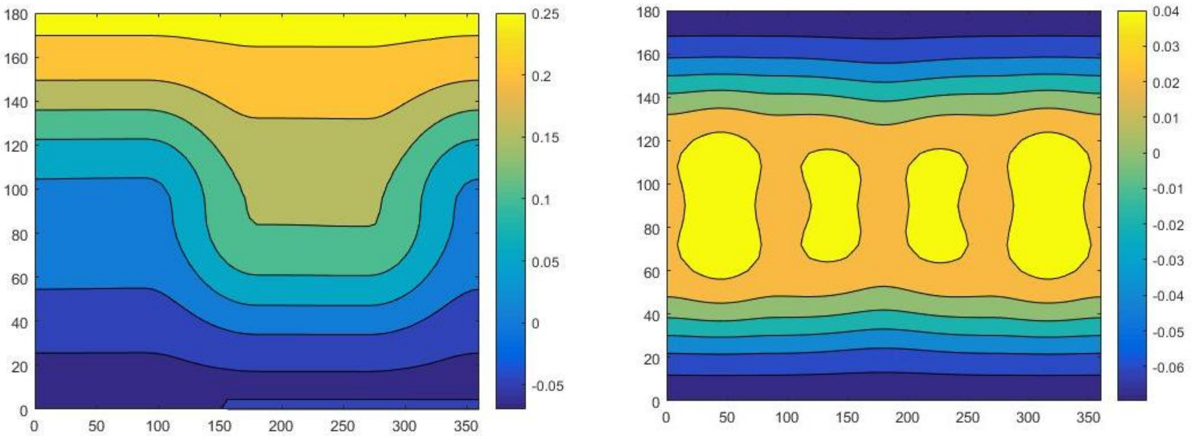
a *b*

Fig. 2. *Map of density anomalies of the Earth's interior from the surface*



a *b*

Fig. 3. *Map of density anomalies of the Earth's interior at a depth of 2891 km*



a *b*

Fig. 4. *Map of density anomalies of the Earth's interior at a depth of 5150 km*

Conclusions

1. The proposed method for approximate construction of the mass distribution of the Earth, in contrast to the conventional, allows better use of information about the gravitational field of the planet.

2. In contrast to the classical approximate method for determining the density, the proposed technique allows us to control the calculation process, and, therefore, to estimate the degree of reliability of such construction.

3. The constructed "izoden" map at some depths allow us to previously conclud about the

moving mass of the planet due to the rotating component of gravity.

References

Byzov D. D., Cidaev A. G. *Metodika postroienia 2D plotnostnoi modeli vierhniei mantii s uchetom uslovia izostaticheskoi kompensacii na glubinie* [Method for construction of a 2D density model of the upper mantle in conditions of isostatic compensation at different depths]. *Uralskii geophizicheskii vestnik [Ural Geophysical messenger]*. 2015, no. 1(25), pp. 33–36.

Bullen K.Ye. Plotnost' Zemli [The density of the Earth]. Moscow: *Mir*, 1978. – 437 p.

Garkov V. N., Trubicyn V. P. *Phizika planetarnyh nedr* [The physics of planetary subsurface]. – M.: Nauka, 1980, 448 p. – (Glav. red. Phiz.-mat. Lit.).

Korbunov A. I. *Metod funkcionalnyh predstavlenii pri reshenii obratnyh zadach gravimetrii* [A functional representations method in solving inverse problems of gravimetry]. *Phizika Zemli [Physics of the Earth]*. 2015, vol. 4, pp. 3–14.

Mashimov M. M. *Teoreticheskaia geodezia: Spravochnoie posobie* [Theoretical Geodesy: Reference Guide]. Moscow: *Nedra,* 1991, 268 p.

Meshcheriakov G. A. *Ispolzovanie stoksovyh postoiannyh Zemli dlia utochnenia jeio mehanicheskoi modeli* [Using Stoke's constants of the Earth to refine its mechanical model]. *Geodezia, kartographia i aerophotosiomka [Geodesy, cartography and aerial photography],* 1975, vol. 21, pp. 23–30.

Meshcheriakov G. A., Fys M. M. *Opredelenie plotnosti Zemnyh nedr riadami po biortogonalnym sistemam mnogochlenov* [Determination of the density of the Earth's interior by rows of biorthogonal systems of polynomials] *Teoria i metody interpretacii gravitacionnyh i magnitnyh anomalii [Theory and methods of interpretation of gravity and magnetic anomalies]*. 1981, pp. 329–334.

Meshcheriakov G. A., Fys M. M. *Trehmernaia i referencnaia plotnostnyie modeli Zemli* [The three-dimensional and reference density models of the Earth]. *Geophizicheskii Jurnal [Geophysical Journal]*. Kyiv, 1986, vol. 8, issue 4, pp. 68–75.

Meshcheriakov G. A. *Zadachi teorii potenciala i obobshchenaia Zemlia* [Tasks of potential theory and generalized Earth]. Moscow: *Nauka,* 1991, 216 p. (Glav.red. Phiz.-mat. Lit.).

Meshcheriakov G. A., Zazuliak P. M., Kulko O. V., Fys M. M., Shtabliuk P. I. *Variant mekhanicheskoi modeli nizhnei mantii* [Option of mechanical model of the lower mantle]. *Trudy III Orlovskoi konferencii "Izuchenie Zemli kak planet metodami astronomii, geophiziki i geodezii" [Proceedings III Orel conference "Study of the Earth as a planet by astronomy, geophysics and geodesy methods"]*. 1994, pp. 172–177.

Molodenskii S. M., Molodenskii M. S., Begitova T. A. *3D-modeli medlennyh dvijenii zemnoi kory I verhnei mantii v ochagovyh zonah seismoaktivnyh oblastei I ih sravnenie s vysokotochnymi dannymi nabliudenii* [3D-model of the slow movements of the crust and upper mantle in the focal zones of seismically active areas and compare them with the high precision data of observations]. *Phizika Zemli [Physics of the Earth]*. 2016, no. 5, pp. 25–50.

Moritc G. *Figura zemli: Teoreticheskaia geodezia i vnutrenniee stroienie Zemli* [The figure of the Earth: Theoretical Geodesy and internal structure of the Earth]. Kyiv, 1994, 240 p.

Pankov V. L., Garkov V. I. *O raspredilenii plotnosti v nedrah zemli* [About the density distribution in the Earth's interior]. *Zemnyie prilivy I vnutrieniee stroienie Zemli [Earth tides and the internal structure of the Earth]*. 1967, pp. 44–61.

Fys M. M., Fotca R. S., Sohor A. R., Volos V. O. *Metod znahodjennia gustyny rozpodilu mas planet z urahuvanniam stoksovyh stalyh do chetvertogo stepenia* [Method for planets density distribution construction with using of stoke's constants to fourth order]. *Geodynamika [Geodynamics]*. 2008, vol. 1(7), pp. 25–34.

Fys M. M., Zazuliak P. M., Cherniaha P. G. *Znachennia ta variacii gustyny u centri mas elipsoidalnyh planet* [The value and variations of density in mass centers of ellipsoidal planets]. *Kinematika i phizika nebesnyh tel [Kinematics and physics of celestial bodies]*. 2013, vol. 29, no. 2, pp. 62–68.

Fys M. M., Holubinka Yu. I., Yurkiv M. I. *Porivnialnyi analiz formul dlia potencialu ta joho radialnyh pohidnyh* trysharovyh kuliovyh ta elipsoidalnyh planet [Comparative analysis of formulas for the potential and its radial derivative three-layered spherical and ellipsoidal planets] *Suchasni dosjaghnennja gheodezychnoji nauky ta vyrobnyctva [Modern achievements in geodetic science and industry]*. 2014, vol. I (27), pp. 46–51.

Fys M. M., Yurkiv M. I., Brydun A. M. *Nablygenyi metod pobudovy pohidnyh ta znachennia rozpodilu mas nadr planet na prykladi spherychnoi planet Zemlia* [Approximate method for construction of derivatives and values of mass distribution of the planet in case of spherical Earth]. *New technologies in geodesy, land management, forest inventory and nature management* [*Novi tehnologii v geodezii, zemlevporiadkuvanni ta pryrodokorystuvanni]:* Materials VIII Intern. Scientific and Practical Conference, 6–7 October, 2016, Uzgorod, Ukraine, pp. 47–52.

Cerklevych A. L., Zaiatc O. S., Fys M. M. *Gravitaciini modeli tryvymirnoho rozpodilu gustyny planet zemnoi grupy* [Earth group planets gravitational models of 3-d density distributions]. *Geodynamika [Geodynamics]*. 2012, vol. 1 (12), pp. 42–53.

Cherniaha P. G., Fys M. M. *Novyi pidhid do vykorystannia stoksovyh stalyh dlia pobudovy* *funkcii ta ii pohidnyh rozpodiliv mas planety* [The new approach for using of stoke's constants to build functions and its derivatives mass distribution of planets]. *Suchasni dosjaghnennja gheodezychnoji nauky ta vyrobnyctva [Modern achievements in geodetic science and industry]*. 2012, vol. II (24), pp. 40–43.

Shcherbakov A. M. *Obiemnoie raspredilenie plotnosti Luny* [Volumetric density distribution of the Moon]. *Astronomicheskii vestnik [Astronomical messenger]*. 1978, vol. XII, no. 2, pp. 88–95.

Yatskiv Ya. S. *Nutacia v sistieme astronomicheskih postoiannyh* [Nutation in the system of astronomical constants]. Kyiv, 1980, 59 p. (Preprint AnN USSR;ITF–80–95P).

Dzewonski A., Anderson D. Preliminary reference Earth model. *Physics of the Earth and Planet Inter*. 1981, no. 25, pp. 297–356.

Martinenc Z., Pec K. Three – Dimensional Density Distribution Generating the Observed Gravite Field of planets: Part II. The Moon. Proc. Int. Symp. Figure of the Earth, the Moon and other Planets. Czechoslovakia, Prague. 1986, no. 1, pp. 153–163.

Moritz G. Computatson ellipsoidal mass distributions Department of Geodetic Science, The Ohio State University. 1973, no. 206, p. 20

Willamson E. D., Adams I. H. Density distribution in the Earth. *Journal of the Washington **Academy** of Sciences*. 1923, vol. 13, No 19, pp. 413–428.

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ОДИН ВАРІАНТ ПОБУДОВИ МОДЕЛІ ТРИВИМІРНОГО РОЗПОДІЛУ МАС НАДР ТА ЙОГО ПОХІДНИХ ДЛЯ СФЕРИЧНОЇ ПЛАНЕТИ ЗЕМЛЯ

Мета. За параметрами (стоксовими постійними до другого порядку включно) зовнішнього гравітаційного поля Землі побудувати тривимірні функції розподілу мас надр Землі без умови про мінімальне її відхилення від відомої в геофізиці моделі густини та встановити внесок коефіцієнтів розкладу потенціалу в разі їх уточнення. **Методика**. Класичні методи побудови розподілу мас використовують тільки стоксові постійні нульового та другого порядків. У ітераційних способах визначення модельних розподілів за нульове наближення береться референцна модель густини, узгоджена зі стоксовими постійними до другого порядку включно. Далі враховують коефіцієнти розкладу потенціалу до визначеного порядку, але при цьому не досліджено їхній внесок у функцію густини мас. У роботі зроблено спробу отримання такої оцінки. Запропонований метод також наближений, але в ітераційному процесі використовується не лише функція густини, але також її похідні. Зведення степеневих моментів густини до контрольованих значень (величин, визначених на поверхні кулі) дає можливість аналізувати процес послідовних наближень. **Результати**. На відміну від моделі другого порядку, яка описує грубі глобальні неоднорідності, отримана функція розподілу дає детальнішу картину розміщення аномалій густини (відхилення тривимірної функції від усередненої по сфері – "ізоденс"). Аналіз карт на різних глибинах (2891 км ядро-мантія, 5150 км внутрішнє-зовнішнє ядро) дає змогу зробити попередні висновки про глобальний перерозподіл мас за рахунок обертової складової сили тяжіння по всьому радіусу: її розрідження вздовж осі обертання та скупчення при відхиленні від неї. Це особливо проявляється для екваторіальних областей. Навпаки, в полярних частинах Землі спостерігається мінімум такого відхилення, що також має своє пояснення: величина сили обертання зменшується при відході від екватора. Побудована за допомогою запропонованого методу функція розподілу мас повніше описує розподіл мас. **Наукова новизна**. На відміну від класичних результатів, отриманих з рівняння Адамса-Вільямса для похідних густини однієї змінної (глибини), в роботі зроблено спробу одержати похідні за декартовими координатами. Використання в описаному методі параметрів гравітаційного поля до другого порядку включно збільшує порядок апроксимації функції розподілу мас трьох змінних з двох до чотирьох за рахунок можливості відновлення розподілу мас надр планети за її похідними. На відміну від попередніх досліджень, тут використовується геофізична інформація, акумульована в реферецній моделі PREM, а тому враховуються особливості внутрішньої структури. **Практична значущість.** Отриману функцію розподілу мас Землі можна використати як нульове наближення в разі вживання в поданому алгоритмі стоксових постійних вищих порядків. Її застосування дає можливість інтерпретувати глобальні аномалії гравітаційного поля та вивчати глибинні геодинамічні процеси всередині Землі.

Ключові слова: потенціал, гармонічна функція, гравітаційне поле Землі, модель розподілу мас, стоксові постійні.

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ОДИН ВАРИАНТ ПОСТРОЕНИЯ ТРЕХМЕРНОГО РАСПРЕДЕЛЕНИЯ МАС НЕДР И ЕГО ПРОИЗВОДНЫХ ДЛЯ СФЕРИЧЕСКОЙ ПЛАНЕТЫ ЗЕМЛЯ

Цель. По параметрам (стоксовым постоянными до второго порядка включительно) внешнего гравитационного поля Земли построить трехмерные функции распределения масс недр Земли не учитывая условия о минимальном ее отклонении от известной в геофизике модели плотности и установить вклад коэффициентов разложения потенциала при их уточнении. **Методика.** Классические методы построения распределения масс используют только стоксовые постоянные нулевого и второго порядков. В итерационных методах определения модельных распределений нулевым приближением считается референцная модель плотности, которая согласована со стоксовыми постоянными до второго порядка включительно. Далее, учитываются коэффициенты разложения потенциала определенного порядка, но при этом не исследуется их вклад в функцию плотности масс. В работе приведена попытка получения такой оценки. Предложенный метод также приближенный, но в итерационном процессе используется не только функция плотности, но и ее производные. Приведение степенных моментов плотности к контролируемым значениям (величинам, что определены на поверхности шара) дает возможность анализировать процесс последовательных приближений. **Результаты.** В отличие от модели второго порядка, описывающей грубые глобальные неоднородности, полученная функция распределения дает подробную картину размещения аномалий плотности (отклонения трехмерной функции от усредненной по сфере – “изоденс”). Анализ карт на разных глубинах (2891 км ядро-мантия, 5150 км внутреннее-внешнее ядро) позволяет сделать предварительные выводы о глобальном перераспределении масс за счет вращающейся составляющей силы тяжести по всему радиусу: ее разжижения вдоль оси вращения и скопления при отклонении от нее. Это особенно проявляется для экваториальных областей. Напротив, в полярных частях Земли наблюдается минимум такого отклонения, что также имеет свое объяснение: величина силы вращения уменьшается при отходе от экватора. Построена с помощью предложенного метода функция распределения масс, полнее описывает распределение масс. **Научная новизна.** В отличие от классических результатов, полученных из уравнения Адамса-Вильямса для производных плотности одной переменной (глубины), в работе предпринята попытка получить производные по декартовым координатам. Использование в описанном методе параметров гравитационного поля до второго порядка включительно увеличивает порядок аппроксимации функции распределения масс трех переменных с двух до четырех за счет возможности восстановления распределения масс недр планеты по ее производными. При этом, в отличие от предыдущих исследований, здесь используется геофизическая информация, аккумулированная в реферецной модели PREM, а потому, учитываются особенности внутренней структуры. **Практическая значимость.** Полученная функция распределения масс Земли может быть использована как нулевое приближение при наличии в представленном алгоритме стоксовых постоянных высших порядков. Ее применение дает возможность интерпретировать глобальные аномалии гравитационного поля и изучать глубинные геодинамические процессы внутри Земли.

Ключевые слова: потенциал, гармоническая функция, гравитационное поле Земли, модель распределения масс, стоковые постоянные.

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