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ON THE NON-LINEAR RELATIONSHIP BETWEEN VIX AND REALIZED SP500 VOLATILITY

Abstract

VIX, a ticker symbol for Volatility Index, measures the implied annual volatility of at-the-money SP500 Index Options. Conventional wisdom presumes VIX to measure the magnitude (positive or negative) of possible movements in future equity prices, with movements being a positive function of VIX. This research investigates the nature of the relationship between VIX and SP500 volatility, and answers the question as to whether that relationship is linear or nonlinear. Based on this research paper, the authors conclude that the realized SP500 volatility is nonlinear, and grows with the level of VIX at an increasing rate. The nonlinearity relationship between VIX and SP500 has enormous implications for investment management and hedging in the financial markets.

Keywords VIX, SP500 volatility, SP500 options, implied volatility

JEL Classification G11, G13, G17

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INTRODUCTION

VIX, a ticker symbol for Chicago Board Options Exchange Volatility Index, measures the implied 30-day volatility of at-the-money SP500 Index Options. Since its first publication on Jan. 2, 1990, VIX is regularly quoted in the broadcast and print financial media. Technically, it measures the magnitude, positive or negative, of possible movements in SP500, but it is largely viewed as measuring the magnitude and probability of a possible downward movement in the SP500. As such, the VIX is quoted as an indicator of "fear" in the equity markets. Since the VIX is a quantitative, scalable metric, futures and options contracts on VIX futures have been developed for hedging of equity portfolios.

This research investigates the functional relationship of realized SP500 return volatility (SPR) to levels of VIX. We conclude that

a) the level of the VIX does, in fact, act as a proxy for future actual volatility;

b) the proxy for future actual volatility is without up- or down-bias;

c) the functional relationship between VIX and realized volatility is nonlinear with realized volatility growing with VIX at an increasing rate.

The results of this research should be of interest to the entire investment community in general, but to the portfolio management and hedging community, in particular.

1. PRIOR RESEARCH

Modeling financial security and Volatility Index has existed for as far back as information has been available. Research into volatility took a huge leap forward following the introduction of the Black Scholes option pricing model (1973) and the Black Scholes Index Option Pricing Model (1976), both of which allowed for computation of implied volatility. VIX, the ticker symbol for implied volatility of at-the-money SP500 Index Options was first published in 1990, and the VIX as a tradable futures and option contract in its own right began in 2010. Since then, VIX and its relationship to different indexes and markets have been the focus of many studies. Several of these studies that are relevant to the current research are noted below.

French et al. (1987) found evidence that the exh pected market risk premium (the expected return on stock portfolio minus the Treasury bill yield) is positively associated to the predictable volatility of stock return. Chan et al. (1991) used the intraday relationship between returns and return volatility in the stock index and stock index futures markets to conclude that an inter-market dependency exists in the volatility of the cash and futures returns. But that research did not address the predictive nature of current volatility to subsequent actual volatility. Andersen and Bollerslev (1998) demonstrate that volatility models produce a precise intra-daily forecast for the latent volatility factor that would be of interest in most financial applications. Fleming (1998) examines the performance of the SP500 implied volatility as a forecast of future stock market volatility. His results indicated that implied volatility does predict future volatility, but the forecasts contain an upward bias. However, that research included current period underlying asset movement in the computation of the volatility index. Hence, forecasting subsequent period volatility using an index that included that subsequent price movement amounts to including a "look ahead bias". This may have accounted for his finding of an upward bias in implied option volatility.

Several research studies have explored Volatility Index as a predictor of subsequent volatility with the focus on financial prices being the outcome of a "jump process". In contrast to a diffusion process, upon which Black Scholes is based which assumes constant arrival intervals and continuous price movements, a jump process has discrete price movements, called jumps, with random arrival times. Concurrent research since 2000 has attempted to apply neural network technologies to enhance the modeling process. For example, Eraker et al. (2003) used neural network methodology to examine continuous-time stochastic volatility models incorporating jumps in returns and prices. Their results indicated that volatility forecasts based on neural networks outperform implied volatility forecasts and are able to closely approximate actual volatility. But that conclusion rests on neural network technologies which are not known to be robust, and jump modeling processes, which, by its very nature, is extremely short term, e.g., less than per hour. Hence, conclusions regarding longer horizon are not able to be addressed. Poon and Granger (2003) reviewed major research studies, which focused on the using different models to forecast volatility. However, many of these models resulted in a major bias and an inefficient forecast of the volatility. Banerjee et al. (2007) investigate the relationship between future returns and current implied volatility levels. Furthermore, they examine the common risk factors in the returns on stocks and bonds and concluded that the VIX-related variables have strong predictive ability. Giot (2014) investigates the effect of implied volatility and its possible signaling power for the future stock index by using the Mexican Stock Index (MEXBOL). Chow et al. (2014) established that without imposing any structure on the underlying forcing process, the Volatility Index (VIX) does not measure the market expectation of volatility. Furthermore, they proposed a generalized Volatility Index (GVIX) based on the log-return variance where VIX would be considered as its special case. To address the inefficiency and bias of forecasting the volatility, Baruník and Hlínková (2016) used the wavelet band least squares model to explore the long memory of volatility and attempted to address the cause of bias. Their proposed model resulted in reducing biasness and improving the forecast of future volatility.

The question of whether VIX predicts subsequent volatility over a longer period, i.e., one day to the life of the option remains unsettled. Furthermore, the possibility that the relationship is nonlinear has never been addressed. This research adds nicely to the literature and settles those two open-ended issues. Correctly removing the look ahead bias, thus, allows a VIX to be investigated as a genuine forecast of future volatility and the functional relationship is found to be nonlinear.

2. METHODOLOGY

Daily time series data for the SP500 and VIX were obtained for the period January 1, 1990 until March 1, 2017 and was taken from Federal Reserve Bank of St. Louis, Federal Reserve Economic Data facility. The functional specification is given in Eqn. 1, below.

$$Stdev\left(SPR_t\right) = f\left(VIX_{t-1}\right),$$

where:

SPR_t – log SP 500 returns for the current time period;

VIX_{t-1} – SP 500 Volatility Index in previous (day) time period.

Note that the independent variable, VIX, is lagged one time period so as to remove the "look-ahead bias". Using the current period VIX would amount to including current day SPR movement into the current computation of VIX.

Graphical methods, i.e., histograms, time series plots and scatterplots, and analytical methods, i.e., descriptive statistics, correlation and regression methods will be used to analyze the data on a daily basis. The data will be analyzed using the R statistical program.

3. RESULTS

Histograms and time series plots are presented in Figures 1-4 for SP500 daily returns and VIX daily levels. The histogram in Figure 1 shows SP500 daily returns to be basically normally distributed, but the time series plot in Figure 2 shows the returns to be quite heteroscedastic. The histogram in Figure 3 shows daily VIX to be highly skewed to the right. The skewness and heteroscedasticity of VIX is seen in the time series plot of daily VIX in Figure 4.

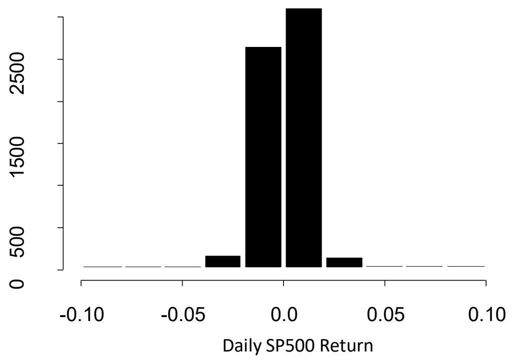

Figure 1. Daily SP Return

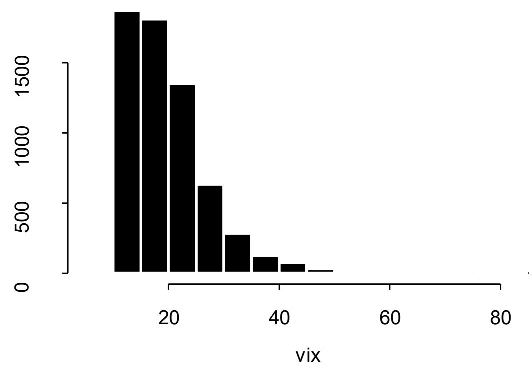

Figure 3. Daily VIX

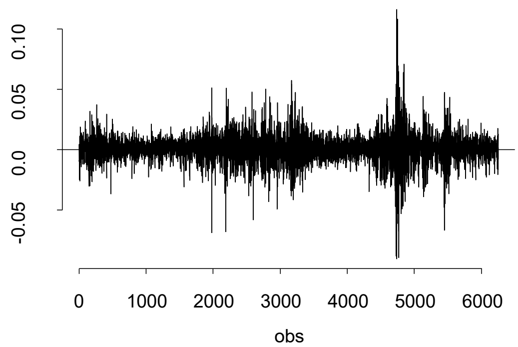

Figure 2. Dailly Sequence SP500 Return

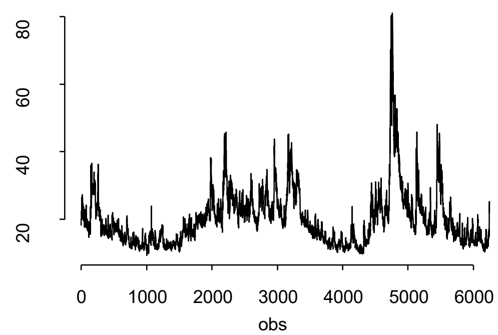

Figure 4. Dailly Sequence VIX

Table 1. Descriptive statistics – daily SP 500 returns and VIX

Series	Mean	Med	Stdev	Skew	Kurt	Min	Max	n
SPR	0.00033	0.0005	0.01119	–0.08897	8.82072	–0.09035	0.1158	6850
VIX	19.61201	17.7500	7.81478	2.07242	7.39297	9.31000	80.0600	6850

Table 1 displays descriptive statistics for daily SP 500 returns and VIX.

The VIX skew shown in Fig. 3 is confirmed by the skewness metric shown in Table 1. Note that while the SP500 returns are essentially symmetric, they are highly leptokurtic.

To investigate the relationship between realized volatility and anticipated volatility as measured by VIX, Figures 5 and 6 display scatterplots of SP500 daily returns with previous period (day) VIX and absolute value of return to previous period (day) VIX, respectively. Both scatterplots show SP 500 returns to be heteroscedastistic to VIX.

Descriptive statistics for the 6850 observations are presented in Table 2 for 10 equal size groups, i.e., bins, based on ascending values of VIX. SP500 volatility, as measured by standard deviation of daily log returns, increases with mean VIX. The number of bins being 10 is somewhat arbitrary. It could easily have been larger or smaller. Notice the SPR skew for bin 10 (underlined) is highly skewed to the right and kurtosis (underlined) uncharacteristically high. We characterize this as an outlier, and remove that observation from subsequent regression procedures.

Scatterplots of the SPR standard deviation vs mean VIX from Table 2, with and without the outlier from bin 10, is displayed in Figures 7 and 8, respectively, with linear model overlay.

The scatterplot of SPR Standard deviation to mean VIX is positive for Figures 7 and 8 scatterplots. The first evidence in support of the nonlinear relationship between SPR volatility and the VIX is evident in both plots. SPR volatility appears to be increasing at an increasing rate to VIX for both plots. Notice in both plots the systematic under-, over-, and, then, under- prediction of the linear overlay to the realized volatility.

The out-of-character datapoint in bin 10 is evident in Figure 7. As this datapoint might distort regression inferences, it will be deleted as an outlier, reducing the dataset to 9 observations (bins) aggregating 6165 datapoints. Table 3 displays the results of linear regression of standard deviation of mean bin SPR on mean bin VIXt-1 for 9 observations (bins).

The F-statistic is easily significant at the 1 percent level of significance. R-squared is very high, and the t-statistic is also easily significant at the 1 percent level.

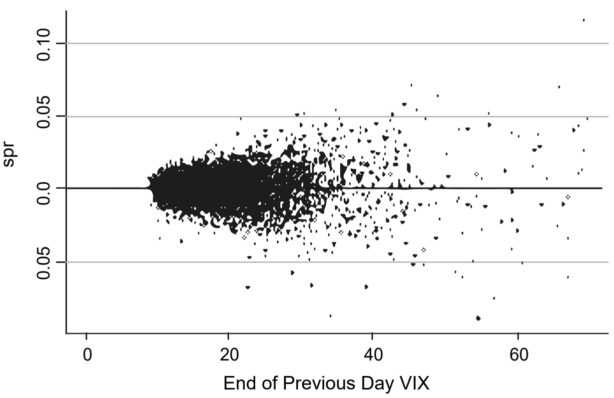

Figure 5. Dailly SP500 Return v VIX

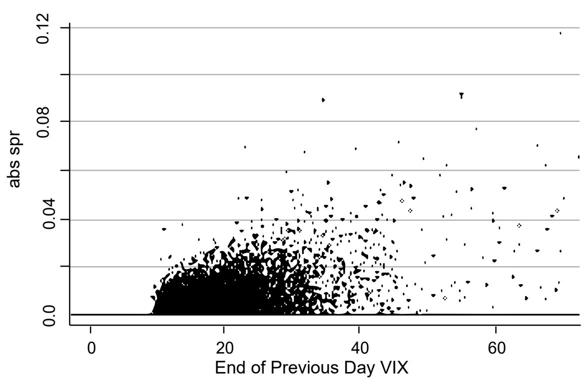

Figure 6. Dailly Fabdolute SP500 Return v VIX

Table 2. Descriptive statistics By VIX group mean – daily

	Bin	Bin	Mean	Stdev	Skew	Kurt	n
Bin	Min	Max	VIX	SPR	SPR	SPR	
1.	9.31	12.14	11.426	0.005	–0.920	0.583	685
2.	12.14	13.31	12.723	0.006	0.018	–1.206	685
3.	13.31	14.67	13.953	0.006	0.092	–1.227	685
4.	14.67	16.17	15.422	0.007	–0.040	–1.206	685
5.	16.18	17.75	16.913	0.008	0.164	–1.171	685
6.	17.75	19.72	18.727	0.009	0.022	–1.229	685
7.	19.72	21.76	20.729	0.010	0.026	–1.173	685
8.	21.77	24.39	23.056	0.012	0.104	–1.216	685
9.	24.39	28.90	26.335	0.014	0.308	–1.149	685
10.	28.90	80.06	36.834	0.023	2.033	4.479	685

The second evidence of the nonlinear relationship between SPR volatility and the VIX is seen in the intercept of the linear regression equation which is both negative (–.0021) and highly significant (t = –5.26). The negative intercept implies that a sufficiently low level VIX would be associated with a negative predicted SPR volatility, clearly an impossibility and not realistic. A more realistic intercept would be something near, but not significantly different than. On the oft chance that it were negative, it should not be significant.

Table 4 displays the results of quadratic nonlinear regression. A quadratic VIX(t-1) term is added to the specification.

The results present a third set of evidence of the nonlinear relationship between SPR and VIX. As with the linear specification, the F-statistic is easily significant at the 1 percent level of significance. R-squared is significantly higher than the linear specification, and SE is significantly lower. Note also that the t-statistic for the intercept – although slightly positive – is not significant (t = .75). This is clearly a superior result compared to the intercept in the linear model which was negative and significant. Hence, in the limit, a zero VIX would lead to a zero expected SPR volatility, and not a negative (impossible) volatility as was the case with the linear model. Also, notice that the coefficients for VIX and VIX^2 are positive and easily significant at the .065 and .024 percent one-sided level, respectively. This is the evidence indicating the nonlinearity between SPR and VIX.

The fourth and last set of evidence supporting the non-linear relationship is displayed in Figyres 9 and 10, which display scatterplots of residuals to

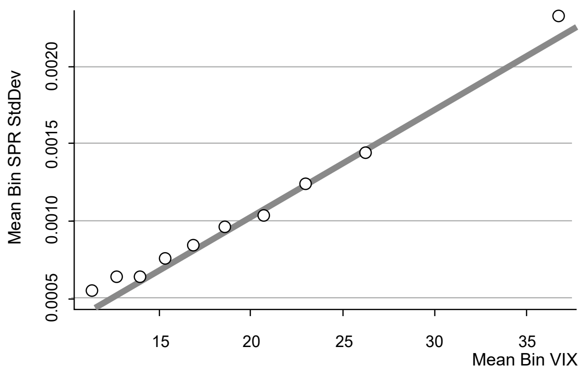

Figure 7. SPR Bin Std. dev v. Mean Bin VIX Outlier Included

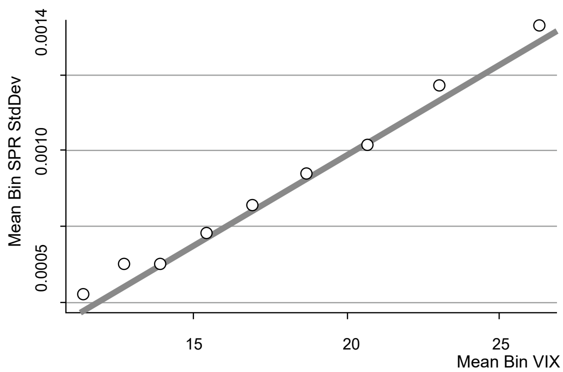

Figure 8. SPR Bin Std. dev v. Mean Bin VIX Outlier Omitted

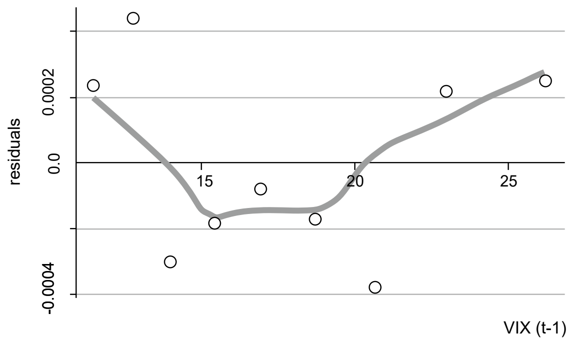

Figure 9. Residuals v VIX (t-1) – Linear

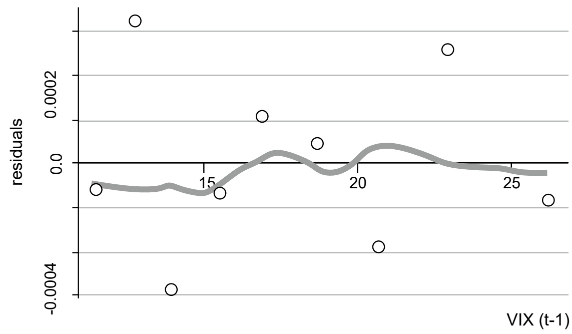

Figure 10. Residuals v VIX (t-1) – Non-linear

previous period VIX for both the linear and non-linear, i.e., quadratic specifications. Smooth-spline curves are overlaid on both to accentuate the non-linearity and absence of non-linearity. Notice that the bowl-shaped configuration of the residuals in the linear model, i.e., Figure 9, which indicates a nonlinear bias, is eliminated in the non-linear model, Figure 10.

Recall that the above analysis does not include the outlying observation (observation 10) from Table 1, which was deemed to be an outlier. The results arguing for nonlinear specification are even more compelling when that outlying observation is included. Said another way, observation 10, when evaluated in a nonlinear model, is not an outlier at all.

The conclusion of this analysis is quite inescapable: the function modeling next period SPR volatility to VIX nonlinear, and increases at an increasing rate. This important conclusion regarding stdev(SPR) being a non-linear function of VIX has enormous implications for the investment community. On a casual basis, low and high levels of VIX slightly underestimate next period SPR volatility, while mid-levels of VIX overestimate next period SPR volatility. On a more formal, quantitative and algorithmic information basis, at the money, and perhaps the entire SP500 options series for any one day, may be mis-priced, and this mis-pricing might be traded to financial advantage. On the basis of this research, a new SP500 Index Option Arbitrage might just now have been born. Traders take note.

Table 3. Linear regression results of stdev(SPR) = f(VIX_{t-1})

Coefficients	Value std.	Error	t-value	Pr(>|t|)
(Intercept)	0.002099	0.000399	–5.260799	0.001172
VIX(t-1)	0.000602	0.000022	27.622000	0.000000

Residual standard error: 0.000306265 on 7 degrees of freedom
Multiple R-squared: 0.990909
Adjusted R-squared: 0.98961
F-statistic: 762.975 on 1 and 7 degrees of freedom, the p-value is 2.09242e-008

Table 4. Nonlinear regression results of stdev(SPR) = f(VIX_{t-1} , VIX_{t-1}^2)

Coefficients	Value std.	Error	t-value	Pr(>|t|)
(Intercept)	0.000968	0.001279	0.756744	0.477856
VIX(t-1)	0.000251	0.000143	1.750140	0.130665
VIX(t-1)^2	0.000009	0.000004	2.467993	0.048590

Residual standard error: 0.000233032 on 6 degrees of freedom
Multiple R-squared: 0.995489
Adjusted R-squared: 0.993985
F-statistic: 661.983 on 2 and 6 degrees of freedom, the p-value is 9.1819e-008

CONCLUSION

This research concludes that the current VIX level does serve as a proxy for future SP500 volatility, but that the SP500 return volatility increases at an increasing rate with previous period VIX. Thus, portfolio managers who use VIX as a predictor of portfolio volatility over the near future should adjust for a non-linear bias to control risk. This has further implications for the risk profile of the portfolio and degree of leverage to be utilized.

Further research could utilize methods such as ARCH (Autoregressive Conditional Heteroscedasticity) and GARCH (Generalized Autoregressive Conditional Heteroscedasticiy) with quadratic terms to model VIX. But the inescapable and incontrovertible conclusion drawn here is that VIX as a raw volatility metric is a biased measure of subsequent period volatility. SPR volatility increases with VIX at an increasing rate.

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