Jim Gustafsson (Denmark)

A strategic decision-making model executing the use test under Solvency II

Abstract

Solvency II allows insurers to determine their own solvency capital requirements (SCR) under a one-year Value-at-Risk (VaR), and using of the internal models to quantify risks can therefore replace parts, or even all risks, of the standard formula. A major challenge for the industry is the use test under which companies will need to convince regulators that senior management understands, trusts and takes appropriate account of internal model outputs within its key decisions. This paper presents a conceivable solution that links capital allocation, pricing, performance and strategy together. The model presented in the article will execute the use test under Solvency II and provides senior management with drivers to make strategic decisions based on the defined risk appetite and the internal model output. The model will be presented under the scope with a partial internal model for reserve risk, but could easily be extended to a full internal model environment. The model will be interpreted as a premium reserve model with intention to assess the ratio of the risk return of the SCR held.

Keywords: Solvency II, use test, strategic decision-making model, internal model, premium reserve, capital reserve, capital allocation.

Introduction

Solvency II constitutes a fundamental review of the capital adequacy regime for the European insurance industry and aims at establishing a revised set of EU-wide capital requirements and risk management standards which will replace the requirements of the present European regulatory regime. The new legislation was underway well before the recent financial crisis, yet the crisis had reinforced the need for improvement of risk and capital management and brought this topic to the forefront of the senior management agenda. Ideally, Solvency II allows insurers to determine their own solvency capital requirements (SCR) under a one-year Value-at-Risk (VaR), and using of the internal models to quantify risks can therefore replace parts, or even all risks, of the standard formula.

The implementation of Solvency II brings several key hurdles to overcome for insurance companies. Many boards and business teams may be unfamiliar with the nature, implications and, not least, limitations of the risk and capital evaluations that they will need to build into their strategy and business planning. A risk-based approach to strategic and performance assessment, along with the weight of documentation needed to verify it, may demand a significant cultural shift within many companies. The biggest headaches is the so called use test, under which companies will need to convince regulators that senior management understands, trusts and takes appropriate account of model outputs within its key decisions.

One of the main reasons why firms are finding it so difficult to overcome these challenges may be that they are actually concentrating too closely on the regulatory aspects of how to demonstrate that the internal model is appropriately used and compliant with the six tests (use test, statistical quality, calibration standards, validation standards, statement of profit and loss, documentation standards). A more effective approach would be to focus on the development of risk and capital evaluation capabilities that can help to make the business safer, nimbler and more consistently profitable in short, thereby a model that boards and business teams will want to use. In addition to the evident business benefits, development and embed a clear articulated risk appetite will provide a valuable bridge between the model and business decisions by creating benchmarks against which senior management can judge the firms risk profile and risk-based performance. The advantages include a more informed basis for capital allocation, understand risk diversification benefits and setting triggers for action and escalation.

Companies that will utilize an internal model need to demonstrate the relevance and important role of the model based on the integration of the model output into the daily risk-management function. The idea behind this is that the tools, methods, and assumptions used in the model are also used in other parts of the organization. This paper present a conceivable solution that linking capital allocation, pricing, performance and strategy together. The idea originates from the paper by Nielsen, Poulsen and Mumford (2010), while they focus on hedging financial risks, this paper demonstrate the solution in a Solvency II context. The model presented in the study will execute the use test under Solvency II and provides senior management with drivers to make strategic decisions based on the defined risk appetite and the internal model output. The model will be presented under the scope with a partial internal model for reserve risk, but could easily be extended to a full internal model environment. The model will be interpreted as a premium reserve model with intention to assess the ratio of the risk return of the SCR held. The general idea behind the model is that old underwriting should not be the driver of a negative portfolio performance, and consequently release more capital for new underwriting.

The model brings a company advantages meeting its performance targets, having better investment and development strategies, and execute the use test by helping senior management make strategic decision – making of model outputs. Further, by using this model the senior management could easily control which business units to shrink or grow, adjust individual business units return based on historical and expected performance, and also it would be easier to adapt customer value propositions. Compared to Nielsen, Poulsen and Momford (2010) we will present a detailed description of the theory behind the model, and we will derive evidence on real data that the model work in practice. To give a clear picture of the proposed premium reserve model we first need to present some reserving methodology and discussion on how different decisions on risk measure and allocation principle affect the business. In section 2 we discuss the background theory and concept to reserving, risk measures, internal model and allocation methodologies. In Section 3 we present the premium reserve model. Section 4 demonstrate the method discussed by using three real data sets with different tail characteristics, and provide a clear link to a companies risk appetite and how the decision that is taken will execute the use test. The final section summarizes and concludes.

1. Understanding the risk contained by capital management

An insurance contract is, at its core, a promise from the company to deliver in the future. The ability to make good on its promises is vital for an organization and capital adequacy is critical under Solvency II to deliver its liabilities. These promises should also be combined with the request on a high return from the shareholders. In general, the higher is the risk, the higher is the return, which should satisfy the shareholders more than the policyholders since they are by definition more risk averse. On the other hand, the risk a company should take to satisfy the shareholders is a contradiction to what the policyholders expect a company to act. For that reason, a company should decide their purpose before choosing reserving model, risk measure and allocation principle. There is not a one size fits all methods that can be used for all purposes. The risk measures and allocation principles discussed in this section by no means define the universe of possible methods. These are some prominent measures and principles that have emerged in the literature. There is no single measure that is recognized as the best one, but some have appealing properties that make them more relevant to the discussion on capital adequacy under Solvency II. Many different models have been proposed during the years, e.g, Urban, Dittrich, Kluppelberg and Stolting (2003). They discuss risk measures and allocation principles such as the risk measures Value-at-Risk (VaR) and Tail-Value-at-Risk (TVaR). The VaR allocate capital according to how a business unit contribute to a specific percentile. This measure is not robust and it can be extremely sensitive to tail behavior. The TVaR is defined as the VaR plus the average loss in those cases where the losses exceed VaR. The method is more robust than VaR and this can be a useful measure for insurance companies where the extent of the tail can be important. Under Solvency II the unexpected part of the total loss distribution have been widely discussed, what measure to operate with to fulfill its obligations and how this should be allocated to business units with a restriction to follow decided risk criteria? Of course, managing economic capital depends highly on the expected part as well. Capital needs to be held against risk and there is clearly considerable uncertainty in the claims reserves, and the capital is held to support this uncertainty.

Since reserving is defined as the expected value of claims to be paid in the future for accidents incurred on policyholders in the past, the exact size of these future liabilities is subject to uncertainty and highly affecting the understanding of economic capital. For that reason it has always been a challenge to predict a companies future expenses, and using the most standard chain ladder methodology alone to forecast these liabilities arise several problems. One of the problems is that the chain ladder method gives a deterministic estimate rather than being based directly on a stochastic model. In the past, various stochastic models associated with the chain ladder have been studied, where to our knowledge, only two models reproduces the chain ladder estimates exactly. The first one was presented by Renshaw and Verrall (1994) using a generalized linear model with underlying over-dispersed Poisson error distribution. The second model was presented by Mack (1993) and is a distribution free approach. The main difference between those models, besides the distribution assumption, is the variance structure where Mack (1993) introduce a time dependent setup. For a deeper discussion around the dissimilarities, see England and Verrall (1999). There have also been a large number of other papers investigating stochastic reserving which do not reproduce the chain ladder, see, e.g, England and Verrall (1999) for details. The next section introduce the method by Renshaw and Verrall (1994), this is then forming the basis when we proceed with the analysis and introduce the bootstrap methodology by England and Verrall (1999) building a partial internal model under Solvency II for reserving risk. Thereafter, the SCR could be determined and allocation to business units could be made.

2. Background theory and concepts to reserving

Let the random variable C_{ijk} be the k^{th} amount, $k = 1,..., N_{ij}$, paid to a policyholder in year $i = 1, ..., m$ with $j = 1, ..., m$ years delay. We assume for simplicity that all incurred claims are settled after m

years. The random variables N_{ij} describe the number of claims occurred in year i, paid during development year j. The cumulated amount paid to all policyholders for all incurred claims in accident year i during development year j is thus:

$$C_{ij\cdot} = \sum_{k=1}^{N_{ij}} C_{ijk}. \quad (1)$$

Let $C_{ij1}, C_{ij2}, \ldots, C_{ijN_{ij}}$ be identically distributed random variables and the random variables [(N_{ij}), $(C_{ijk})_k$] are mutually independent $\forall_{ij}$. Incorporate this in a more common setup, all data should be placed in a triangle and each element is an aggregated amount, cumulated yearly as:

$$D_{it\cdot} = \sum_{t=1}^{j} C_{it\cdot}. \quad (2)$$

Triangle 1 gives a visual interpretation of this setup and is normally phrased as the run-off triangle:

Triangle 1 (cumulative data)

$D_{11\cdot}$	$D_{12\cdot}$	$D_{13\cdot}$	…	$D_{1,m-1\cdot}$	$D_{1,m\cdot}$
$D_{21\cdot}$	$D_{22\cdot}$	$D_{23\cdot}$	…	$D_{2,m-1\cdot}$	
$D_{31\cdot}$	$D_{32\cdot}$	$D_{33\cdot}$	…		
…	…	…			
$D_{m-1,1\cdot}$	$D_{m-1,2\cdot}$				
$D_{m,1\cdot}$					

The key assumption using the chain ladder methodology is that the development of claims in a given accident year is stable and is the same for all accident years. The forecasts procedure is obtained by calculating the average rate of increase (development factors) from each development year to the next and utilize these to predict future payments. The routine is such that we seek the development factors $\{f_j : j \in 2\ [2, m]\}$ that tells us how much we should increase the mean value of $D_{ij\cdot}$ for the next development year $j + 1 \le m$. This could be formulated by:

$$\mathrm{E}D_{ij+1\cdot} = f\mathrm{E}D_{ij\cdot}.$$

An heuristic argument is that one could estimate the development factor as:

$$\hat{f}_j = \frac{\sum_{i=1}^{m-j} D_{ij+1}}{\sum_{i=1}^{m-j} D_{ij}},$$

and since all values up to $D_{i,m-i+1\cdot}$ are known we could make use of $\hat{f}_{m-i+1}$ to predict the next year value on the same accident year by

$$\hat{D}_{i,m-i+2\cdot} = \hat{f}_{m-i+1} D_{i,m-i+1\cdot}.$$

A step by step procedure is done to obtain the general chain ladder predictor

$$\hat{D}_{ij\cdot} = \hat{f}_{m-i+1}\hat{f}_{m-i+2} \cdot \ldots \cdot \hat{f}_{j-1} D_{i,m-i+1\cdot}. \quad (3)$$

As seen above the chain ladder model (3) is applied on cumulated data. When using stochastic models it is more appealing to work with incremental form $C_{ij\cdot}$, defined in evaluation (1). The idea in Renshaw and Verrall (1994) is to use a multiplicative structure for the expected value:

$$\mu_{ij} = d\alpha_i\beta_j$$

and thereby assuming that:

$$\mathrm{E}C_{ij\cdot} = \mu_{ij} \text{ and } VC_{ij\cdot} = \phi\mu_{ij}$$

such that

$$\log(\mu_{ij}) = \eta_{ij} \text{ for } \eta_{ij} = d + \alpha_i + \beta_j$$

with $\alpha_1 = \beta_1 = 0$.

If we let $C_{ij\cdot}$ be compound Poisson distributed with Poisson distributed random variable N_{ij} and lognormal distributed random variable C_{ijk}, the model reproduce the chain ladder outcomes exactly. The underlying motivation for this assumption is since:

$$\mathrm{E}C_{ij\cdot} = \sum_{k=0}^{\infty} E\left[C_{ijN_{ij}} \mid N_{ij} = k\right]P(N_{ij} = k) =$$

$$= \sum_{k=0}^{\infty} \mathrm{E}C_{ijk} P(N_{ij} = k) = \sum_{k=0}^{\infty} k\mathrm{E}C_{ij1} P(N_{ij} = k) =$$

$$= \mathrm{E}(C_{ij1})\sum_{k=0}^{\infty} kP(N_{ij} = k) = \mathrm{E}C_{ij1}\mathrm{E}N_{ij} = \mu_{ij}$$

and the variance:

$$VC_{ij\cdot} = \mathrm{E}\left[V(C_{ij\cdot} \mid N_{ij})\right] + V\left[\mathrm{E}(C_{ij\cdot} \mid N_{ij})\right] =$$

$$= \mathrm{E}N_{ij}VC_{ij1} + (\mathrm{E}C_{ij1})^2 VN_{ij} = \mathrm{E}N_{ij}\left(VC_{ij1} + (\mathrm{E}C_{ij1})^2\right) =$$

$$= \mathrm{E}N_{ij}\mathrm{E}C_{ij1}\left(\frac{\mathrm{E}C_{ij1}^2}{\mathrm{E}C_{ij1}}\right) = \mathrm{E}N_{ij}\mathrm{E}C_{ij1}\phi = \mathrm{E}C_{ij\cdot}\phi = \mu_{ij}\phi$$

with $\phi > 0$. The over-dispersion ϕ is important for the model and could for example come up if there is clustering of events in the Poisson process. More discussion on this, and a more precise theory on generalized linear models, see McCullagh and Nelder (1989).

When the future liabilities are estimated using the model above, the next step is to make a solid appraisal of the results. It is natural that there occur a prediction error that includes both variability due to estimation (estimation variance) and also a variability in data (process variance). By re-sampling with replacement the residuals from the forecasted triangle gives pseudo data and could be used to re-fit our model and thereby obtain a new version of future payments. This increase our understanding and it encapsulate the variability when forecasting the reserve.

The bootstrap procedure bring a full distribution for each *cell* (i, j) for $i + j - 1 > m$ and $i, j \le m$, and is obtained by re-sample the Pearson's residuals

$$\varepsilon_{ij} = \frac{C_{ij\cdot} - \mu_{ij}}{\left(\mu_{ij}\phi\right)^{1/2}},$$

where the errors ε_{ij} are assumed *iid*. For other model assumptions on the residuals, see England and Verrall (1999). The procedure begin by estimate (μ_{ij}, ϕ) and then obtain the estimated residuals $\hat{\varepsilon}_{ij}$. These residuals are then re-sampled B times, dented by, $\hat{\varepsilon}^{b}_{ij}$ for $b = 1, ..., B$, to obtain the pseudo set required:

$$C^{b}_{ij\cdot} = \hat{\mu}_{ij} + \hat{\varepsilon}^{b}_{ij}\left(\hat{\mu}_{ij}\hat{\phi}^{b}\right)^{1/2}$$

with over-dispersion factors estimated as:

$$\hat{\phi}^{b} = \frac{\sum_{i,j \le n-i+1}\left(\hat{\varepsilon}^{b}_{ij}\right)^{2}}{\frac{1}{2}(n-1)(n-2)}.$$

2.1. Risk measure and solvency capital requirements. The most intuitive risk measure when discussing capital adequacy is probably probability of ruin, or ruin theory. This is since the question arise on how likely it is that the company will be able to stay in business over a given time horizon. Ruin theory is closely related to the risk measure VaR which only takes into account the probability of insolvency and not the magnitude of ruin. Another measure, the expected policyholder deficit (EPD), incorporates the fact that not all insolvencies are the same. Both these ad hoc measures are appealing since they are logical risk measures providing the likelihood when the company might be insolvent. Nevertheless, none of these measures was developed by an axiomatic approach. This mathematical approach leads to coherent risk measures, see Jarrow (2002) for a brief introduction on coherent risk measures. The TVaR is coherent and combines the ideas from the VaR and EPD into a single measure.

To distinguish between these measures one could say that for a risk tolerance α, the measure VaR is a threshold in the loss distribution, the EPD is a function of the area above the threshold, and the TVaR is the expectation of this area. We focus on the ad hoc measure VaR and the coherent measure TVaR, where VaR will be used for estimating the SCR and TVaR for allocating this estimated SCR number. Therefore, refering to the previous sections notations, we have that $C^{b}_{ij\cdot}$ is a vector with B different bootstrap amounts on each future *cell* (i, j) forming a loss distribution. The measure VaR is defined for $i + j - 1 > m$, $i, j \le m$, with risk tolerance α,

$$VaR^{b}_{\alpha ij} = \sup\left\{c \in \Re \mid P\left(C^{b}_{ij\cdot} \le c\right) \le \alpha\right\}. \quad (4)$$

The TVaR is expressed for each future *cell* (i, j) by:

$$TVaR^{b}_{\alpha ij} = \mathrm{E}\left[C^{b}_{ij\cdot} \mid C^{b}_{ij\cdot} \ge VaR^{b}_{\alpha ij\cdot}\right]$$

In a business an overall capital needs to be decided and under Solvency II this refers to the SCR and is calculated by adding up individual risks for a specific risk tolerance. The legislation require that enough capital is held to ensure that there is only a 1 in 200 chance (99.5%) that the insurance company is unable to honour its claims next year. The pseudo number for the total capital held is obtained by adding all future expences for each business unit l together, thereby obtain the total sample amount $C^{bl}_{\cdots}$ forming the loss distribution. The total amount for a business unit is then the VaR for a specific risk tolerance α of that loss distribution. The overall loss distribution for the company is created similar for the individual businesses, adding the units exposure together for each bootstrap sample resulting in $C^{b\cdot}_{\cdots}$. More precise, for a partial internal model covering reserve risk, with risk tolerance α = 99.5%, the total SCR summed over each future *cell* (i, j) on the pseudo data set b, takes the following form:

$$SCR^{b} = VaR^{b}_{99.5\%\cdot\cdot} = \sup\left\{c \in \Re \mid P\left(C^{b}_{\cdots} \le c\right) \le 99.5\%\right\}.$$

In reality a company will hold even more capital, both due to the overall solvency needs (the ORSA), and to maintain a specific S&P rating. Further, improvements to the partial internal model described might include a correlation framework between business units, however this is not a part of the content of this paper but would be a natural extension of the model, see Denuit, Dhaene, Goovaerts and Kaas (2005) for details.

2.2. Capital allocation. The choice of allocation method is heavily dependent on the underlying strategy of the company. There is not a one size fits all method that can be used for all purposes. So, for example, you would certainly use a different allocation basis for strategic decisions than the one used for technical pricing. When allocating capital to different business units in an insurance company, one has to remember to center the variables. If this is forgotten one would assign a disproportionate amount to lines of businesses of big size, disregarding the risk involved, see Nielsen, Poulsen and Mumford (2010). Therefore, we have some yearly random payments $C^{b}_{ij\cdot}$ on a pseudo set b, $R^{b}_{ij\cdot} = C^{b}_{ij\cdot} - \mathrm{E}C^{b}_{ij\cdot}$ denotes the centered version of these payments, i.e, the unexpected part of the loss distribution. By $R^{b}_{\cdots} = \sum_{i,j} R^{b}_{i,j\cdot}$ we denote the total present value of unexpected payments for a business unit. There exists a number of allocation principles used for different purposes, a natural way is to split them into two broader classes, namely "top-down" and "bottom-up" principles. In the top-down method one optimizing in a global way and assume all points of the allocation tree knowing what's happening in the rest of the tree, whereas in the bottom-up method optimization is done locally. In this paper we apply the top-down approach with a TVaR proportional allocation method. This approach is a simple and intuitive way of allocating capital, see, e.g., Venter (2004) for a deeper discussion on this.

Let $R^{b\cdot}_{\dots} = \sum_b \left(\sum_{l=1}^{L} R^{bl}_{\dots} \right)$ be the overall losses, then the capital allocated to each business unit is obtained through:

$$\psi^l = \frac{\mathrm{E}\left[R^{bl}_{\dots} \mid R^{b\cdot}_{\dots} \geq VaR_\alpha\left(R^{b\cdot}_{\dots}\right)\right]}{\mathrm{E}\left[R^{b\cdot}_{\dots} \mid R^{b\cdot}_{\dots} \geq VaR_\alpha\left(R^{b\cdot}_{\dots}\right)\right]} \cdot SCR_\alpha , \tag{6}$$

where the SCR^b_a is calculated by $VaR_{99.5\%}\left(R^{b\cdot}_{\dots}\right)$. Throughout the paper we illustrate the allocation principle with two different risk criteria α = 50% and α = 95%. When the business unit have received their allocated capital SCR^{bl}_a , this should be allocated to each *cell* (i, j) for future payments by:

$$\psi^l_{ij\cdot} = \frac{\mathrm{E}\left[R^{bl}_{ij\cdot} \mid R^{bl}_{\dots} \geq VaR_\alpha\left(R^{bl}_{\dots}\right)\right]}{\mathrm{E}\left[R^{bl}_{\dots} \mid R^{bl}_{\dots} \geq VaR_\alpha\left(R^{bl}_{\dots}\right)\right]} \cdot \psi^l . \tag{7}$$

Comparing the top-down approach with the bottom-up principle under the TVaR measure, the form would instead be $\psi^l_{ij\cdot} = \lambda \mathrm{E}\left[R^{bl}_{ij\cdot} \mid R^{bl}_{ij\cdot} \geq VaR_\alpha\left(R^{bl}_{ij\cdot}\right)\right]$. Here λ is chosen at the end so that the total amount of capital allocated sum-up to the total solvency capital requirements. By comparing these two ways of capital allocation there exist minor differences. The most important dissimilarity is in the condition. More precise, the top-down formula have a global condition of the entire company and then individual business unit makes a loss, whereas in the bottom-up formula we have the localized condition of a particular cells future payments. Note that the total capital held in a business unit l for each future *cell* (i, j) is the sum between the old underwriting amount $EC_{ij\cdot}$ and the SCR^{bl}_a.

The next section demonstrates the calculation of the so-called premium reserve model. The section covers the definition of the model as well as the complement model – the capital reserve model. It brings interpretation of the models and an explanation of the impact in a business implementation. Apart from this, a discussion surrounding performance and strategy is reflected throughout the section, and the execution of the use test and the link to a companies risk appetite is articulated.

3. The premium reserve model

Utility curve theory states that the hurt from losing is far greater than the pleasure from gaining an equal amount, see Haugen (2001). An example of this is that when companies make a large profit the gain is not as great as the loss involved when they loose the same amount of money and have to sell part of the business. Thus, it is vital to develop decision-making models, and this is also an element behind the use test under Solvency II. Here companies will need to convince regulators that management understands, trusts and takes appropriate account of model outputs within its key decisions. The premium reserve model could be characterized as a strategic decision-making model. The model could follow the recommendation from Froot and Stein (1999) and not invest in stocks but stick to totally reliable financial instruments, i.e., government bonds. The Froot and Stein model is widely accepted and although the analysis in the original paper is not fully correct, Hogh, Linton and Nielsen (2006) suggest the alternative paths that may lead to a slightly weaker conclusion. However, if some risk should be taken for strategical purposes this could also be achieved by the premium reserve model. The model gives a desirable advantage in controlling the exact risk and return that senior management asked for based on their strategic agenda. Besides this, the model gives an advantage meeting performance targets and gives a foundation for development new strategies. Using the model, a company should expect better combined operating ratio and have more capital for new underwriting. The shareholders should be satisfied since the model assure that a company could delivered the required return corresponding to the hurdle rate. The main issue is to find out how the policyholders are affected. One result for policyholder when a company uses this method is that long-tailed insurance will become more expensive, since they now have to pay upfront for capital to be held for a long time. On the other hand, short-tailed insurance may get cheaper since they no longer have to contribute towards cost of holding capital for long-tailed businesses. See Nielsen, Poulsen and Mumford (2010) for illustration of these scenarios.

3.1. A mathematical formulation for the premium reserve model. Let P^l_{i1} be the total premium income for a business unit l paid by the policyholders in accident year i. The index j = 1 states that the premium is always paid during the first development year. Further let r_f be the risk-free interest rate available and the rate of return required from investors, the hurdle rate, is denoted by r_h. For simplicity, the risk-free rate is the same for all future years, however one could easily change this to a yearly increasing vector by adding a new index to r_f. Assume further that the expected value $\mathrm{E}C^l_{ij\cdot}$ and the variance $VC^l_{ij\cdot}$ exists. Since the premium should cover future claims that could happen in the period $i + j - 1 > m$ for $i, j \le m$, and by leaning on that the first moment exists, the total risk premium for next year $(m + 1)$ with a loading $\theta \ge 0$, for profit and expenses, is given by the formula:

$$P^l_{(m+1)1\cdot} = (1+\theta)\sum_{j=1}^{m} EC^l_{(m+1)j\cdot}. \tag{8}$$

The premium reserve method assess the return on the capital required by adding an amount on the premium formula corresponding to the exact return of the capital it is invested in r_f. Capital should be held for a number of years in the future and the decreasing amount will earn risk-free interest. Therefore, the present value for next year $(m + 1)$ for a business unit l of the interest we make by investing all our capital risk-free take the following form:

$$PV\left(\psi^l_{(m+1)\cdot\cdot}(r_f)\right) = \sum_{j=1}^{m} \frac{r_f \cdot \psi^l_{(m+1)j\cdot}}{(1+r_f)^j}.$$

On the other hand, the shareholders require a return on capital corresponding to r_h%. Hence, the present value of the return under shareholders requirement could be formulated as:

$$PV\left(\psi^l_{(m+1)\cdot\cdot}(r_h)\right) = \sum_{j=1}^{m} \frac{r_h \cdot \psi^l_{(m+1)j\cdot}}{(1+r_f)^j}.$$

The loading on the premiums to make the shareholders required return for year $(m + 1)$ is denoted by δ, phrased as the premium reserve model, and presented as:

$$\delta^l_{(m+1)} = PV\left(\psi^l_{(m+1)\cdot\cdot}(r_h - r_f)\right) = = \sum_{j=1}^{m} \frac{(r_h - r_f)\cdot \psi^l_{(m+1)j\cdot}}{(1+r_f)^j}. \tag{9}$$

Nielsen, Poulsen and Mumford (2010) described this adjustment to a pricing formula with the following equation:

$$\overline{P}^l_{(m+1)1\cdot} = (1+\theta)\sum_{j=1}^{m} \mathrm{E}C^l_{(m+1)j\cdot} + \delta^l_{(m+1)}. \tag{10}$$

By using the formula (10) instead of (8) to set premium will improve the capital management in the organization, helps senior management to execute strategic plans, helps a company meeting Solvency II use test, and it bridging risk and capital management. For new underwriting the premium reserve model match to a specific policy and the portfolio amount (9) is splitted into individuals. After a year the first year capital has been released and the remaining capital held for this year is then presented by old underwriting. This capital should of course also have a return, but this should not be taken from the policyholders. The expected extra amount from policies written in year $(m + 1)$ to assess the required return for year $(m + 2)$ then becomes:

$$\sum_{j=1}^{m} \frac{(r_h - r_f)\cdot \psi^l_{(m+1)(j+1)\cdot}}{(1+r_f)^j}.$$

The capital reserve model bring this return on capital for old underwriting that should be held with the capital $\psi^l_{ij\cdot}$ for $i \le m$, placed risk-free interest to obtain the required return. The capital reserve model is denoted by τ and have the following form:

$$\tau^l_{(m+1)} = PV\left(\psi^l_{(m+1)\cdot\cdot}(r_h - r_f)\right) = = \sum_{i=2}^{m} \sum_{j=m-i+2}^{m} \frac{(r_h - r_f)\cdot \psi^l_{ij\cdot}}{(1+r_f)^j}. \tag{11}$$

The model (9) will free capital in exactly such a way that a business unit get subsidized with an amount corresponding to an expected return on capital equal to a return on capital of r_h% for the next year $(m + 1)$. The only requirement is that the capital $\psi^l_{(m+1)j\cdot}$, $\forall_j$, at least have a return of r_f%. The model τ ensures instead that the required return on old underwriting is fulfilled. This amount should be contributed by itself to ensure the return is obtained to satisfy the shareholders. However, this could be financed in many ways, one could be with the earned one year interest from δ amount, the τ amount, and the business units reserve as they are placed in risk-free bonds. In other words, τ should be held for year $(m + 1)$, paid to the shareholders, and satisfy the equation:

$$\tau^l_{(m+1)} \le \frac{r_f}{1+r_f}\left(\tau^l_{(m+1)} + \delta^l_{(m+1)} + (1+\theta)\sum_{i=2}^{m+1}\sum_{j=1}^{m-i+2} \mathrm{E}C^l_{ij\cdot}\right).$$

If this hold, then at the end of year $(m + 1)$, the τ amount needed for old underwriting to be always financed. From an average perspective, both the δ and τ models ensures that old underwriting does not harm a companies performance. One result for a company using the model δ is that long-tailed insurance will become more expensive for policyholders. This will also be the situation for the company itself since τ will be larger than for light-tailed business units. The reason for this is that a company now should finance this in the start for the capital to be held in the future. The other side of the coin will be that short-tailed products will have a reduction in prices as they no longer have to supply the cost of holding capital for long-tailed business.

In summary, the required return from the shareholders could be assessed with the two described models, δ and τ, with the condition that all capital have at least a return on r_f% at the end of year $(m + 1)$.

We have showed that for a business unit l the following equation holds

$$\sum_{i=2}^{m+1} \sum_{j=m-i+2}^{m} \frac{r_h \cdot \psi_{ij\cdot}^{l}}{\left(1+r_f\right)^j} = \delta_{(m+1)}^{l} + + \tau_{(m+1)}^{l} + \sum_{i=2}^{m+1} \sum_{j=m-i+2}^{m} \frac{r_h \cdot \psi_{ij\cdot}^{l}}{\left(1+r_f\right)^j}. \quad (12)$$

In the next section we raise the question on how much should increase the premium increase if δ is implemented in a company, how much should the amount τ be for different business units, and how could senior management apply this in strategic decision-making.

4. Application on three business units with different tail characteristics

This section analyzes the capital structure in three business units by making use of the theory presented in the previous sections. The data used comes from a global insurance company and we assume that the presented three business units make the whole company cooperation. We further assume that the reserve risk underpin the approach by the partial internal model described, and the SCR presented ignoring other risks for simplicity. The three business units used in the study are commercial indemnity (CI), care group accident (CGA) and commercial property (CP). As to be showed the CI business unit is the most heavy tailed business, while the CP unit is characterized as a short-tailed business. However, the CP has instead the highest turnover, while CGA has the lowest one. For all three business units the historical information is presented by run-off triangles with cumulative data as in Triangle 1. We have that m = 12, meaning that the information given is the companies payments to policyholders during the period of 1998-2010 for all three business units. Tables 1-3 (Triangles 2-4) presented in the Appendix represent these payments in millions and in the same triangle the yearly earned premium is visualized for each accident year.

As seen in Triangle 2 the CI business unit had an increase in premiums in the period of 1998-2009 by 66.6%. If we try to analyze the historical payments flow, notable years was the seventh and eighth development year for policies written in year 1999. Here the company paid 33.59 million and 34.55 million, respectively. In Triangle 3 there is a much more stable payments flow in the historical information. The largest payments is always happened in the second and third development year which indicate its light-tailed distribution. Further, this business unit has the largest historical increase in premium. Triangle 4 brings the CP business unit. Here we could see that the business unit has suffered some years. Notable the payment is in accident year 2003, second development year. Here the company paid losses to an amount of 664.03 million.

As it is seen in these triangles there exists information on year 2010. This is normal in the industry to include one year new underwriting in the estimation when standing at the end of the year 2009. This number is not the focus in an estimation point of view, therefore we just take the same as in the previous year, but beware that this information is stochastic and therefore should have risk capital to cover the years fluctuations. In Figure 1 we illustrate each unit volatility in the payments flow between the development years.

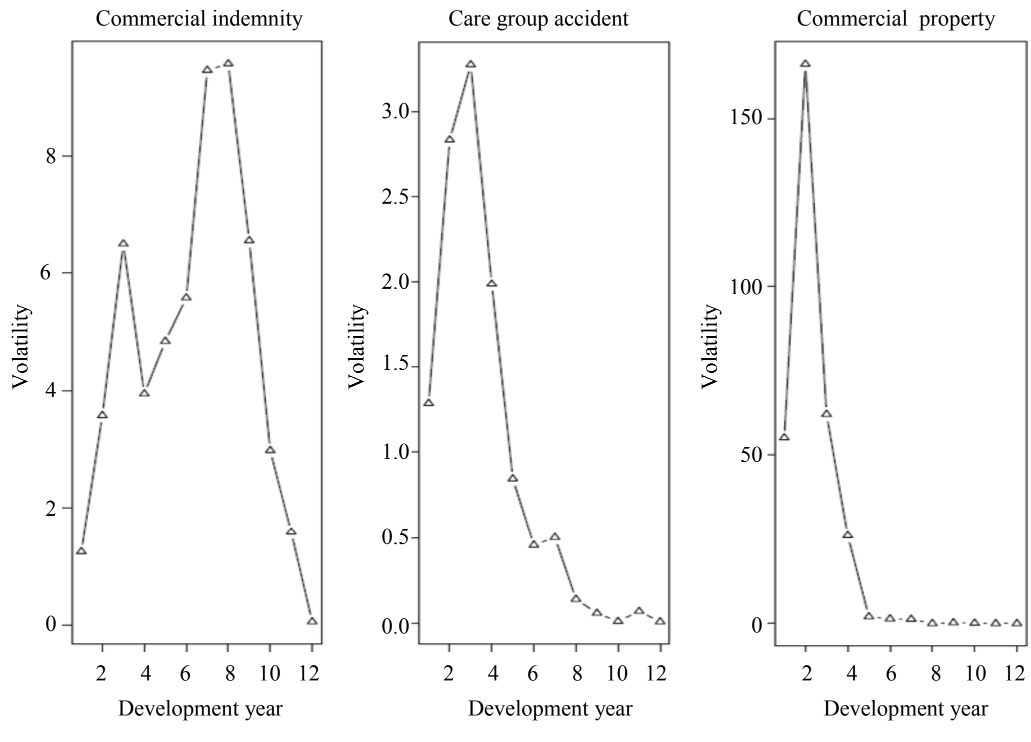
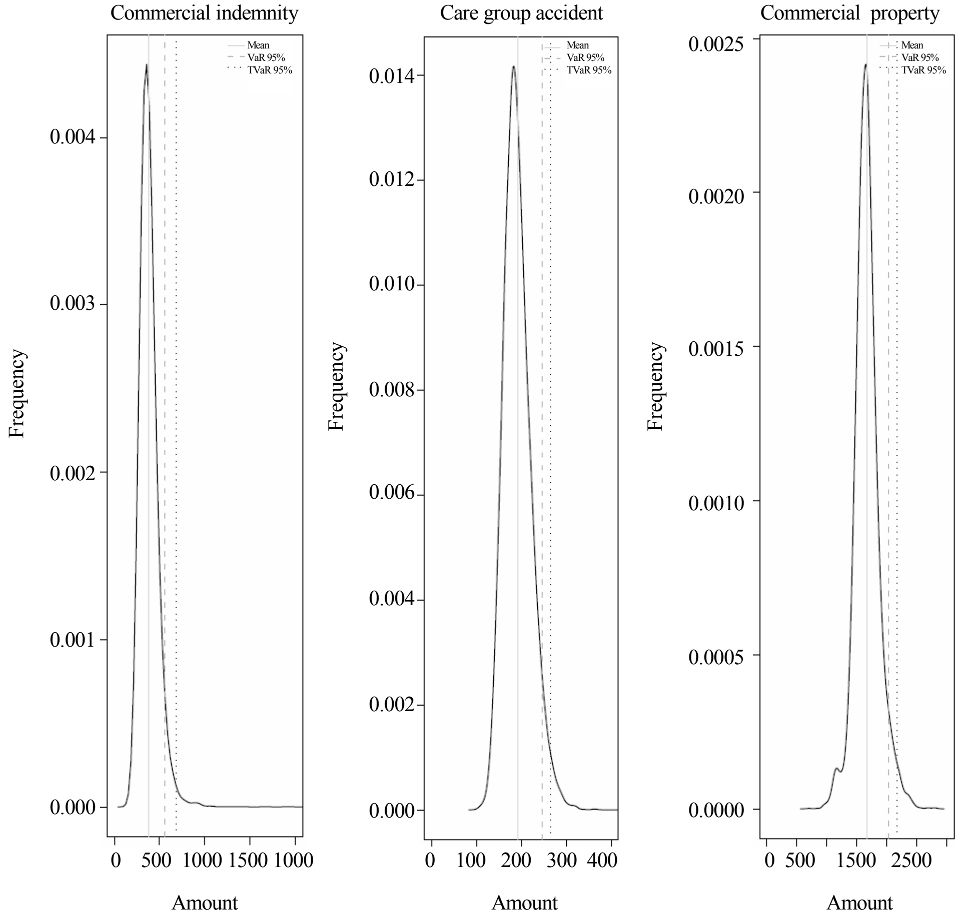


Fig. 1. Volatility in development year for all three business units

The analysis begin with the estimation the expected future payments for each business units liabilities. Thereafter, we use the bootstrap technique explained above to get the loss distribution. The expected amount is calculated based on 10.000 simulated outputs, and thereafter a summation is done to get a total value of the liabilities. Figure 2 presents the loss distributions for each business unit. Note that the expected value is indicated with a solid line. Also, by using the formulas (4) and (5) for $\alpha = 0.95$ we could estimate the VaR and TVaR for each loss distribution. Here the broken line represent the VaR value and the dotted line is the risk measure of TVaR.



Notes: The mean value is given by the whole line, the VaR is given by the broken line, and the TVaR is given by the dotted line.

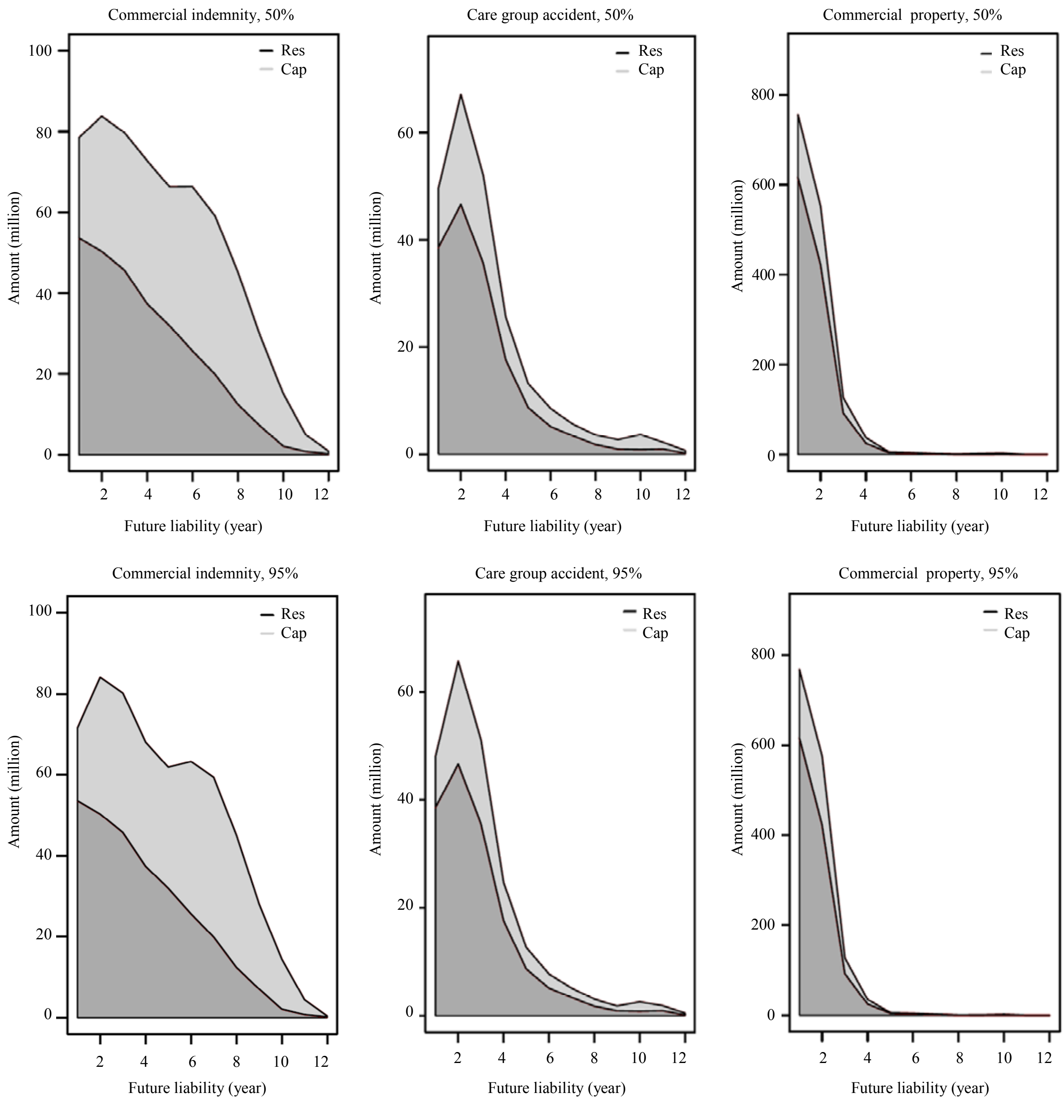
Fig. 2. The loss distribution for each business unit

At this stage in the process we have a perception of the expected amount of the future liabilities for each business unit. Next we focus on the unexpected part. Starting with centralizing the random variable $C^{l}_{ij\cdot}$ to obtain the random variable $R^{l}_{ij\cdot}$. Then, $\forall ijl$, we have 10.000 values in each future cell. For each iteration b we add all unexpected future liabilities in the whole organization together. This result in an overall distribution, and based on that aggregation we have the data to calculate SCR_{α} by $\mathrm{VaR}_{99.5\%}$ ($R^{l}_{\cdots}$). Thereafter, capital is allocated down to each business unit using a proportional allocation with risk measure TVaR, reflecting individual exposure ψ^{l} of each units. For the allocation exercise to individual units we use two different α-values, $\alpha = 50\%$ and $\alpha = 95\%$ to stress the differences in the allocation with various risk appetite. Table 1 presents both the overall capital figure which is the same in both situations as well as allocated capital to individual units.

Table 1. Proportional allocation to business units of the overall capital

	Commercial indemnity	Care group accident	Commercial property	SCR_{α}
$\alpha = 50\%$	305.5	75.6	329.2	710.3
$\alpha = 95\%$	285	65.2	360.1	710.3

The capital figures from Table 1 is then allocated to each business units individual *cell* $\psi^{l}_{ij\cdot}$ defined by equation (7) to cover fluctuations around yearly expected future payments. Based on this individual allocation, Figure 3 illustrates the companies liabilities over time for each business unit. The first row present the three business unit when we have used $\alpha = 50\%$ in the allocation formula, the second row with $\alpha = 95\%$.



Notes: The first row have capital allocated with $\alpha = 50\%$, and the second row use $\alpha = 95\%$.

Fig. 3. The reserve and capital held over time for each business unit

By using this individual capital allocation we could calculate δ and τ defined by equations (9) and (11). In this calculation the risk-free interest rate r_f equal 5%, and the hurdle rate r_h is chosen to 15%. These thresholds could of course be chosen in any way depending on senior managements risk appetite and the shareholders requirement. Table 2 brings various calculated numbers using the developed models.

Table 2. Statistics for all three business units

	Total reserve	Total capital		Premium 2010	δ, 2010		τ, 2010		% increase in premium	
		α = 95%.	α = 95%.		α = 50%	α = 95%.	α = 50%	α = 95%.	α = 50%	α = 95%.
IN	251.3	305.5	285	121.3	8.5	8.8	16.0	13.9	7.0%	7.3%
CGA	146.2	75.6	65.2	55.6	2.7	2.6	3.5	2.9	4.9%	4.7%
CP	868.4	329.2	366.1	815.9	16.6	22.4	12.9	10.1	2.0%	2.7%

For the CP business unit the premium should be increased by 2% under allocation with $\alpha = 95\%$. Doing that and finance the capital reserve of 12.9 million, a company will meet the expected return on capital. The capital reserve τ is just 3.5% of the total capital held which is not massive and could easily be collected for a company. Also the total reserve will have a risk free return that could be used for finance τ. For the long-tailed business IN an increase in premium by 7% is suggested. Here senior management might consider the impact on customer retention before executing the premium increase. By increasing the risk-free rate will reduce δ, τ and the increase in premium, but doing this the company need a higher return on capital than the risk free rate. A company could doing analysis by changing the risk-free rate and the hurdle rate and thereby create a foundation for senior management to make strategic decisions in line with its risk appetite. Thereby a company have a tool to execute the use test under Solvency II.

Conclusion

This paper has presented a conceivable solution that linking capital allocation, pricing, performance and strategy together. The article has demonstrated one way of building a partial internal model for reserving risk, and based on that a strategic decision-making model has been presented to execute the use test under Solvency II. We have shown that based on senior management risk appetite the model outputs was linked to risk appetite and the guidelines on willingness of taking risks. The techniques was demonstrated on three different lines of business and we concluded how the capital reserve model and the premium reserve model were calculated and interpreted. The model could easily be implemented in a company and will provide drivers to make strategic decisions based on the defined risk appetite and the internal model output, and most of all executing the use test under Solvency II.

References

1. M. Denuit, J. Dhaene, M. Goovaerts, R. Kaas, D. Vyncke (2005). Actuarial Theory for Dependent Risks Measures, Orders and Models, Wiley.
2. England, P.D. and Verrall, R.J. (1999). Analytic and bootstrap estimates of predicting errors in claims reserving, *Insurance: Mathematics and Economics*, 25, pp. 281-293.
3. Froot, K.A. and Stein, J.C. (1998). Risk Management, Capital Budgeting and Capital Allocation for Financial Institutions: an Integrated Approach, *Journal of Financial Economics*, Vol. 47 (1), pp. 55-82.
4. Haugen, R.A. (2001). *Modern Investment Theory*, Prentice Hall.
5. Hogh, N., Linton, O. and Nielsen, J.P. (2006). The Froot and Stein Model Revisited, *Annals of Actuarial Science*, Vol. 1, pp. 37-47.
6. Jarrow, R. (2002). Put option premiums and coherent risk measures, *Mathematical Finance*, Vol. 12, No. 2, pp. 135-142.
7. Mack, T. (1993). Distribution-free calculation of the standard error of chain-ladder reserve estimates, *ASTIN Bulletin*, 23, pp. 213-225.
8. McCullagh, P., and Nelder, J.A. (1989). *Generalized Linear Models*, Second Edition, Chapman and Hall, London.
9. Nielsen, J.P, Poulsen, R. and Mumford, P. (2010). Capital Allocation for Insurance Companies: Issues and Methods. Belgien Actuarial Bulletin, Vol. 9, No. 1.
10. Renshaw, A.E. and Verrall, R.J. (1998). A stochastic model underlying the chain-ladder technique, *B.A.J.*, 4, pp. 903-923.
11. Urban, M., Dittrich, J., Klüppelberg, C., Stlölting, R. (2003). Allocation of risk capital to insurance portfolios, *Blätter der DGVFM*, Vol. 26, pp. 389-406.
12. Bibitemven Venter, G.G. (2004). A survey of capital allocation methods with commentary, *North American Actuarial Journal*, Vol. 8, No. 2, pp. 96-107.

Appendix

Table 1. Triangle 2 (commercial indemnity)

	1	2	3	4	5	6	7	8	9	10	11	12	Premium
1998	0	0	8.45	15.62	21.29	24.02	27.55	29.37	34.38	35.08	35.14	34.92	72.79
1999	0	6.07	29.20	35.99	47.92	67.18	100.77	135.32	157.02	167.82	173.54	0	71.62
2000	2.02	13.20	24.20	36.44	51.30	61.01	74.50	83.80	95.27	96.19	0	0	71.22
2001	2.13	9.92	17.26	22.04	25.93	32.96	36.42	38.97	38.98	0	0	0	75.97
2002	0.79	8.38	17.90	22.29	29.68	32.84	35.57	38.08	0	0	0	0	78.03
2003	1.12	7.31	12.58	17.12	21.53	25.09	29.46	0	0	0	0	0	74.07
2004	1.56	8.72	24.26	33.66	36.01	39.94	0	0	0	0	0	0	74.30
2005	4.84	13.85	20.92	28.30	30.71	0	0	0	0	0	0	0	71.42
2006	1.53	6.56	11.87	17.89	0	0	0	0	0	0	0	0	76.99
2007	1.45	6.25	12.41	0	0	0	0	0	0	0	0	0	86.34
2008	0.42	7.84	0	0	0	0	0	0	0	0	0	0	86.95
2009	0.80	0	0	0	0	0	0	0	0	0	0	0	121.26
2010	(0.80)	0	0	0	0	0	0	0	0	0	0	0	(121.26)

Table 2. Triangle 3 (care group accident)

	1	2	3	4	5	6	7	8	9	10	11	12	Premium
1998	0.43	2.84	6.69	8.68	9.43	9.85	9.98	10.43	10.63	10.65	10.90	10.93	1.60
1999	0.30	2.39	5.70	7.76	8.54	8.58	9.01	9.16	9.17	9.14	9.14	0	1.45
2000	0.56	2.99	6.15	7.77	8.40	9.08	9.13	9..38	9.46	9.46	0	0	1.51
2001	0.53	8.08	12.22	13.77	14.97	15.17	15.38	15.47	15.53	0	0	0	1.45
2002	3.20	8.18	15.01	22.21	24.81	26.09	27.90	28.10	0	0	0	0	2.43
2003	1.82	9.93	19.77	23.49	25.51	26.46	26.96	0	0	0	0	0	29.54
2004	0.62	6.23	13.41	16.20	17.65	18.56	0	0	0	0	0	0	29.54
2005	0.61	6.19	11.53	13.72	14.30	0	0	0	0	0	0	0	23.50
2006	1.20	5.79	11.96	14.56	0	0	0	0		0	0	0	25.30
2007	0.69	5.93	14.84	0	0	0	0	0	0	0	0	0	35.99
2008	2.40	10.91	0	0	0	0	0	0	0	0	0	0	52.20
2009	3.97	0	0	0	0	0	0	0	0	0	0	0	55.63
2010	(3.97)	0	0	0	0	0	0	0	0	0	0	0	(55.63)

Table 3. Triangle 4 (commercial property)

	1	2	3	4	5	6	7	8	9	10	11	12	Premium
1998	41.97	101.47	120.14	123.86	124.00	124.00	124.00	124.02	124.02	124.02	124.02	124.02	527.36
1999	56.59	185.33	195.26	197.15	197.54	197.50	197.92	197.92	198.09	198.49	198.49	0	518.69
2000	139.55	285.50	289.77	291.12	292.01	292.18	292.18	292.21	291.41	291.41	0	0	502.10
2001	185.08	370.25	404.64	404.41	405.49	405.67	405.62	405.61	405.61	0	0	0	528.87
2002	142.42	320.70	334.34	335.98	336.82	337.57	338.99	338.99	0	0	0	0	559.27
2003	171.08	835.11	1069.05	1164.43	1171.64	1176.67	1181.20	0	0	0	0	0	555.64
2004	221.94	469.83	504.76	510.57	510.27	511.23	0	0	0	0	0	0	564.31
2005	140.45	303.11	325.95	326.97	326.35	0	0	0	0	0	0	0	565.91
2006	149.92	418.90	471.01	485.63	0	0	0	0		0	0	0	676.82
2007	168.71	357.79	378.61	0	0	0	0	0	0	0	0	0	796.35
2008	153.85	380.43	0	0	0	0	0	0	0	0	0	0	799.72
2009	216.23	0	0	0	0	0	0	0	0	0	0	0	815.91
2010	(216.23)	0	0	0	0	0	0	0	0	0	0	0	(815.91)