THE MULTI-FACTOR PARTITIONING MODEL AND A SUGGESTION FOR ITS MODIFICATION

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Abstract

The multi-factor partitioning model (MFP) is one of the shift-share analysis models and constitutes an essential contribution to the effort of describing and understanding a region's growth. The purpose of the present paper is: 1) To present, the multi-factor partitioning model and its connection to traditional and homothetic one; 2) To explain why the use of standardized relative changes in the use of the MFP model ignores two effects: the distribution effect and the structure effect; 3) To propose a modification of multi-factor partitioning model to take into account the previous mentioned effects; 4) To apply the multi-factor partitioning and the proposed modified multi-factor partitioning model in order to identify growth regional patterns in thirteen Greek regions, and show that the use of multi-factor partitioning model instead the proposed modified model, misleads us to the results.

Keywords: Multi-Factor Partition, Shift-Share Analysis, Regional Planning, Homothetic Employment

Authors' individual contribution: Conceptualization – G.X. and K.P.; Methodology - G.X. and K.P.; Writing – G.X. and K.P.

1. INTRODUCTION

The shift-share analysis (Creamer, 1942; Dunn, 1960; Jones, 1940) is a descriptive statistical technique, which is exceptionally famous because of its simplicity and low demands in data. The primary purpose of the mentioned technique is to examine patterns of regional growth and divide a region's economic growth into result components. Although economic growth is the result of different variables that interact with one another when applying the technique to either employment or production, an added value is used (Loveridge, 1995). Of course, any other variable connected to a region's growth used as the total can be breaking into components (Dunn, 1960). In this study, the variable in use is employment. The central idea of the technique is simple. The observed change in the region's employment is decomposing into the sum of two parts, the national share part, and the shift part. The national share is the part of region total employment change that is ascribing to the relative change of nationwide employment. The shifting part is the part of the region's total employment change that is ascribing to regional factors. In the bibliography, there are many Shift share models or, correctly defined shift-share identities (Loveridge & Selting, 1998). The most common in use is the classical model and the Homothetic model (Dunn, 1960; Esteban-Marquillas, 1973). The multi-factor partitioning (MFP) or the Ray-Srinath model (Ray, 1990) is an improvement of the classical model. The MFP-model using standardized relative employment change results in partitions the observed employment change of the region into five components instead of the three components of the classical model or the four components of the Homothetic model. This study aims to explain why the MFP needs our proposed modification to improve its ability to describe and classify regions in which a nation has divided. The structure of this paper is as follows. Section 2 is a review of the central literature concerning shift-share analysis applications and fundamental criticism. Section 3 presents the classical and homothetic shift-share models, and the multi-factor partitioning one. Section 4 presents the proposed modification of the multi-factor partitioning model. Section 5 illustrates the difference between the multi-factor and the proposed modified multi-factor partitioning models and make comparisons between them following the eight classes of Biffignandi's typologies. Finally, a conclusion presented in the last Section 6. The mathematical proofs were needed to place in Appendix A.

2. LITERATURE REVIEW

We are going to present some of the wide range of shift-share model applications. First of all, shift-share models used commonly to describe regional growth and recognize regional growth patterns. As a consequence, the method used to group the regions in eight or six types (Boudeville, 1966; Bianchi & Biffignandi, 2014; Stilwell, 1969). Parallel Shift share analysis has been used to measure the impact of regional policies (Bartels, Nicol, & Van Duijn, 1982; Bartels & Van Duijn, 1984; Moore & Rhode, 1973) or to assess if specific regional policies are necessaries to support a regional economy (Stilwell, 1969). Shift-share identities and Arima modeling efforts are used in forecasting employment at the regional level (Brown, 1969; Hellman, 1976; Mayor, Lopez, & Perez, 2007). Also, they are used to predict regional patterns (Andrikopoulos, Brox, & Carvalho, 1990). The efficiency in regional manufacture (Dinc & Haynes, 1999) or the changes in employee productivity in the regions under study (Haynes & Dinc, 2006) examined via shift-share models. Another application uses the shift-share model as a tool that analyses the economy of a county or a region, (Chen & Xu, 2003; Labib, Zahidur, Bhuiya & Musfiqur, 2013) or to identify clusters (Matatkova & Stejskal, 2012). A shift-share model is also a useful tool for local authorities to benchmarking regional economy (Martinez-Prats & Armenta-Ramirez, 2018). The role of exports as a growth engine for a region and the relationship between Regional Development and exports (Nachnani & Swaminathan, 2017) examined under shift-share modeling. Also In the field of international trade studies, shift-share analysis has been applied (Chiang, 2012; Dinc & Haynes, 1998; Piezas-Jerbi & Coleman, 2009). Application in Tourism and tourism competitiveness of a region is also an exciting field of applying shift-share analysis (Chun & Yang, 2008; Shi, Zhang, Yang & Zhou, 2007) or the changes in employment connected to tourism (Sirakaya, Choi, & Var, 2002), or the tourist arrivals (Rex, Habibbullah, & Lim, 2004). Finally, it deserves to mention that the method also used as a health policy tool (Twardowska & Jewczak, 2018). Independence of the range of application, shift-share models, has been criticized. The main expressed criticism refers to the lack of theoretical base (Bartels et al., 1982; Houston, 1967) or to its usefulness and the fact that it provides no interpretation (Richardson, 1978). Also, the criticism focused on some more technical points that we refer to in the next section, three accompanying the model presentation. Despite the points of criticism mentioned above, the technique widely applied and different variations used as suggestions for solving the identified problems (Artige & Van Neuss, 2014; Arcelus-Francisco, 1984; Esteban-Marquillas, 1972; Kalbacher, 1979; Loveridge & Selting, 1998).

3. THE TRADITIONAL, THE HOMOTHETIC AND THE MULTI PARTITIONING FACTOR (MPF) SHIFT-SHARE ANALYSIS MODELS

The most common shift-share analysis models in use, as we have already referred, are the traditional model and the homothetic model. In the next paragraphs, we shall briefly present the classical and the homothetic model and, more extensively, the MFP model. More details concerning these models are presented in Appendix A (Paragraphs 1-5). Using the employment as a measure of growth at the national and regional level, we define at the base year t=0:

e^0_{ij} = employment in the *i-th* industry in the *j-th* region;
$\tilde{e}^0_{ij}$ = homothetic employment in the *i-th* industry in the *j-th* region;
$e^0_{i.}$ = total employment in the *i-th* industry at national level;
e^0 = total employment at national level;
$e^0_{.j}$ = total employment in *j-th* region;
r_{ij} = relative employment change in the *i-th* industry, in the *j-th* region;
$r_{.j}$ = relative total employment change in the *j-th* region;
$r_{i.}$ = relative employment change in the *i-th* industry at the national level;
r = relative to a total employment change

The components of classical Equation 1 and homothetic Equation 2 model in which is divided the total employment change $e^t_{.j} - e^0_{.j}$ in the *j-th* region between base year t=0 and final year t are:

$$e^t_{.j} - e^0_{.j} = \sum\nolimits_i^m e^0_{ij}\, r + \sum\nolimits_i^m e^0_{ij}\,(r_{i.} - r) + \sum\nolimits_i^m e^0_{ij}(r_{ij} - r_{i.}) \tag{1}$$

$$e^t_{.j} - e^0_{.j} = \sum\nolimits_i^m e^0_{ij}\, r + \sum\nolimits_i^m e^0_{ij}\,(r_{i.} - r) + \sum\nolimits_i^m \tilde{e}^0_{ij}(r_{ij} - r_{i.}) + \sum\nolimits_i^m (e^0_{ij} - \tilde{e}^0_{ij})(r_{ij} - r_{i.}) \tag{2}$$

The components are:
$NS_j = \sum_i^m e^0_{ij}\, r$ is the part of the *j-th* region total employment change ascribed to relative employment change at a national level. $IM_j = \sum_i^m e^0_{ij}\,(r_{i.} - r)$ is the industry mix component. Industry component is the part of the *j-th* region's total employment change. That part attributed to the difference between the region's and the nation's concertation of employment in the *i-th* industry. The regional share $RS_j =$

$\sum_i^m e_{ij}^0(r_{ij} - r_{i.})$ is the last component of the classical shift-share model. This component refers to the part *j-th* region total employment change that ascribed to specific regional factors. The homothetic shift-share model keeps the same national share and industry mix components but divides the regional share into a sum of two components. The first one is the allocation component $AE_j = \sum_i^m(e_{ij}^0 - \tilde{e}_{ij}^0)(r_{ij} - r_{i.})$. The allocation component refers to the part of the *j-th* region's total employment change that attributed to the *j-th* region specialization in the various industries (Herzog & Olsen, 1977). The homothetic competitive $\tilde{C} = \sum_i^m \tilde{e}_{ij}^0(r_{ij} - r_{i.})$ is the second component, which is part of the *j-th* region total employment change that attributed to unique regional factors. In Appendix A, Paragraphs 2 and 5, the alternatives expressions for the traditional and homothetic models are presented. For the above-mentioned models, the points of criticism are five. The first one is that in the traditional model the industry mix effect and the competitive effect results are interwoven and this, on the one hand, leads into them getting mixed, and the extension of the contribution of each result to the region's employment change not being correctly assessed and, on the other hand, the comparison between different regions resulting problematic (Rosenfeld, 1959). The second point of criticism refers to the arbitrary way of decomposition (Artige et al., 2014). The third point refers to the difficulties for comparison of results across regions and comparisons over time (Kochanowski, Wayne, & Joray, 1989). The fourth point of criticism focuses on weighing the results (Barff & Prentice, 1988; Dunn, 1960; Herzog & Olsen, 1977). The last fifth point of criticism is much of interest supposing that all the previous four points of criticism have successfully faced. For the same employee total change for a given region, the models presented above result in different numerical values for their component. In other words, the numerical result depends upon the model, and there is not any criterion to decide which decomposition is preferable (Houston, 1967; Artige et al., 2014). The MFP model offers a very satisfactory solution to the mentioned five points of criticism. That model decomposes the region's total employment change in a different way, resulting in the "correct partition" components (Lamarche, Srinath, & Ray, 2003). The critical point in the MPF model is that it replaces the relative employment change with standardized relative employment changes (Srinath & Ray, 1990). Now it is necessary to define the standardized relative employment changes in use in the MFP model:

$\hat{r}_{i.} = \sum_j^n \frac{e_{.j}^0}{e^0} r_{ij}$ is the standardized relative employment change in the *i-th* industry at a national level;

$\hat{r}_{.j} = \sum_i^m \frac{e_{i.}^0}{e^0} r_{ij}$ is the standardized relative total employment change in the *j-th* region;

$\hat{r} = \sum_i^m \frac{e_{i.}^0}{e^0}\hat{r}_{i.} = \sum_j^n \frac{e_{.j}^0}{e^0}\hat{r}_{.j} = \sum_i^m \sum_j^n \frac{e_{.j}^0}{e^0}\frac{e_{i.}^0}{e^0} r_{ij} = \sum_i^m \sum_j^n \frac{\tilde{e}_{ij}^0}{e^0} r_{ij}$ is the standardized relative total employment change at the national level. Using standardized relative employment changes, the MFP model removes two effects. The industry location effect $LE_i = \sum_j^n(\frac{e_{ij}^0}{e_{i.}^0} - \frac{e_{.j}^0}{e^0})r_{ij}$. That effect is associated with the dissimilar distribution between the region's share of the national employment in the *i-th* industry and the region's share of total employment. This dissimilarity affects relative employment change in the *i-th* industry at the national level. The second effect is the region specialization effect $SE_j = \sum_i^m(\frac{e_{ij}^0}{e_{.j}^0} - \frac{e_{i.}^0}{e^0})r_{ij}$. The region specialization effect is associated with the dissimilarity between the region's and the nation's concentration of employment in the *i-th* industry. In Appendix A, we show that it is possible to derive the MFP model starting not only from the classical model but also from the homothetic one or their alternatives. Furthermore, the result components have unique numerical values, and the fifth point of criticism has faced. The lack of theoretical base is still a problem but it is a problem for all the shift-share models. Having the above definitions, the MFP model divides the region under study total employment change $e_{.j}^t - e_{.j}^0$ into five components, as expressed by Equation 3:

$$e_{.j}^t - e_{.j}^0 = e_{.j}^0 r + e_{.j}^0(r - \hat{r}) + \sum\nolimits_i^m e_{ij}^0(\hat{r}_{i.} - \hat{r}) + \sum\nolimits_i^m e_{ij}^0(\hat{r}_{.j} - \hat{r}) + \sum\nolimits_i^m e_{ij}^0(r_{ij} - \hat{r}_{i.} - \hat{r}_{.j} + \hat{r}) \qquad (3)$$

The five components are:

$NS_j = e_{.j}^0 r$ is the national share component, common to above presented shift-share models. Second is the allocation component $\widehat{AE}_j = e_{.j}^0(\hat{r} - r)$. The allocation component refers to the part of total employment change in the *j-th* region that attributed to the distribution of regions shares of total employment. The third is the Ray-Srinath industry mix component $\widehat{IM}_J = \sum_i^m e_{ij}^0(\hat{r}_{i.} - \hat{r})$. The industry mix refers to the part of the *j-th* region's total employment change that attributed to the difference between the region's and the nation's concertation of employment in the *i-th* industry, given the standardized relative employment change in the *i-th* industry at the national level. The fourth $\widehat{R}_j = \sum_i^m e_{ij}^0(\hat{r}_{.j} - \hat{r})$ is the region component and refers to the part of the *j-th* region total employment change, attributed to the commons for all industries in the region under study characteristics. $\widehat{MR}_j = \sum_i^m e_{ij}^0(r_{ij} - \hat{r}_{i.} - \hat{r}_{.j} + \hat{r})$ is the industry-region interaction component, and it refers to the part of the *j-th* region total employment change that attributed to specific characteristics for the *j-th* region.

4. THE PROPOSED MODIFICATION OF THE MULTI-FACTOR PARTITIONING MODEL

The classical and the homothetic model presented in previous sections use observed employment in the base year and the relative employment changes to calculate the numerical values of the components. The MFP model also uses observed employment in the base year, but the standardized relative employment changes for calculations. The reason for using standardized relative changes is to remove two effects mentioned in Section 3: the industry location and the region's specialization effect. Doing this, the MFP model the components not interwoven anymore, and the comparison across regions is feasible. Before our proposed solution, which constitutes the modification of the multi-factor partitioning model, let us point that the standardized relative changes based on the hypothesis, that there is not any difference between the region's share of the nation's employment in the industry *i* and the region's share of total employment, i.e. $\frac{e^0_{ij}}{e^0_{i.}} = \frac{e^0_{.j}}{e^0}$. It also means that homothetic employment is in use in the base year. If we adopt the hypothesis that the total employment evenly distributed among them industries and *n* regions, then the base year employment in the *i-th* industry in *j-th* region is a simple arithmetic mean: $\bar{e}^0_{ij} = \frac{e^0}{nm}$. The consequences are that the concentration of employment in the *i-th* industry in the *j-th* region and nation are equal i.e. $\frac{e^0_{ij}}{e^0_{.j.}} = \frac{e^0_{i.}}{e^0} = \frac{1}{m}$. Also equal are *j-th* region share of total employment to nation's employment in the *i-th* industry, i.e. $\frac{e^0_{ij}}{e^0_{i.}} = \frac{e^0_{.j}}{e^0} = \frac{1}{n}$. Our propose for MFP model modification is to replace the standardized relative employment changes with the arithmetic averages of relative employment changes. So, instead of standardized relative employment change in the *i-th* industry at the national level, we use the simple arithmetic mean of relative employment changes in the *i-th* industry $\bar{r}_{i.} = \sum_j^n \frac{1}{n} r_{ij}$ and instead of standardized relative total employment change in the *j-th* region, we use the simple arithmetic mean of relative employment changes in the *i-th* industry in the *j-th* region $\bar{r}_{.j} = \sum_i^m \frac{1}{m} r_{ij}$. Combining the two means, we get the average relative total employment change $\bar{r} = \sum_i^m \sum_j^n \frac{1}{m}\frac{1}{n} r_{ij}$. After using the averages, the modified multi partitioning ($\overline{\text{MFP}}$) model is expressed by Equation 4.

$$e^t_{.j} - e^0_{.j} = e^0_{.j} r + \sum\nolimits_i^m e^0_{ij} (\bar{r} - r) + \sum\nolimits_i^m e^0_{ij} (\bar{r}_{i.} - \bar{r}) + \sum\nolimits_i^m e^0_{ij} (\bar{r}_{.j} - \bar{r}) + \sum\nolimits_i^m e^0_{ij} (r_{ij} - \bar{r}_{i.} - \bar{r}_j + \bar{r}) \quad (4)$$

In Appendix A, Paragraph 7, we show in details how the modified multi partitioning model ($\overline{\text{MFP}}$) derived from classical or homothetic or MFP models. The average of course is a number which depend on number of regions and number of industries but allows us not only to compare the region's concentration employment in the *i-th* industry with an even distribution which a more neutral comparison but also is giving as the opportunity to remove two more effects on the region except for the removal of industry location (LE) and region specialization (SE) as MFP model does. These effects are the distribution effect and the structure effect. Let us make it more clear. For the relative employment change in the *i-th* industry at the national level, we write: $r_{i.} = \bar{r}_{i.} + [(r_{i.} - \hat{r}_i) + (\hat{r}_i - \bar{r}_{i.})] = \bar{r}_{i.} + LE_i + (\hat{r}_i - \bar{r}_{i.})$. Every term of the sum is the relative contribution to the total employment change in the *i-th* industry at the national level. The last term, the difference between standardized relative employment changes in the industry *i* and the average employment change in the *i-th* industry $DE_i = \hat{r}_i - \bar{r}_{i.} = \sum_j^n \left(\frac{e^0_{.j}}{e^0} - \frac{1}{n}\right) r_{ij}$ is the distribution effect. This effect accounts in what way the distribution of total employment among the regions affects relative employment change in the *i-th* industry at a national level. Also, the relative total employment change in the *j-th* region written as $r_{.j} = \bar{r}_{.j} + \left[\left(r_{.j} - \hat{r}_{.j}\right) + \left(\hat{r}_{.j} - \bar{r}_{.j}\right)\right] = \bar{r}_{.j} + SE_j + \left(\hat{r}_{.j} - \bar{r}_{.j}\right)$. The last term $SE_j = \hat{r}_{.j} - \bar{r}_{.j} = \sum_i^m \left(\frac{e^0_{i.}}{e^0} - \frac{1}{m}\right) r_{ij}$ is the structure effect for the *j-th* region. This effect accounts how the employment concentration in the *i-th* industry at the national level affects the relative total employment change for the *j-th* region. As we show in Appendix A, Paragraph 8, these effects make the component result of the MFP model not to be the expected ones'. Now it is necessary slightly to reinterpret the components of the proposed modified model. Let us recall the central idea of shift-share analysis models. The observed change in a region's employment divided into the national part, i.e., the national share component and the shift part, which is the sum of the rest components. For the national share component, we have already mentioned it. The shift components are: the modified allocation component $\overline{AE}_j = \sum_i^m e^0_{ij} (\bar{r} - r)$, which is the part of total employment change in the *j-th* region, that would happen if the total employment evenly distributed among regions and industries. The modified industry mix component $\overline{IM}_j = \sum_i^m e^0_{ij} (\bar{r}_{i.} - \bar{r})$ uses the average relative employment changes in the *i-th* industry at the national level to capture the part of regions' total employment change attributed to the concentration of employment in the *i-th* industry in the *j-th* region. The modified regional component $\bar{R}_j = \sum_i^m e^0_{ij} \left(\bar{r}_{.j} - \bar{r}\right)$ is the part of the region's total employment change attributed to common in average characteristics in the region for all industries. The last component which is the modified industry-region interaction effect $\overline{MR}_j = \sum_i^m e^0_{ij}(r_{ij} - \bar{r}_{i.} - \bar{r}_j + \bar{r})$ is the part of the region's total employment change ascribed to specific regional factors and their interaction with *j-th* region employment concentration in the *i-th* industry and relative employment change in the *i-th* industry in the *j-th* region. For the proposed model, we have noticed two points. First, it is possible for the sum of negative and positive relative changes to be equal to zero. The second point is that the numerical result components calculated by the modified model are expected to be different in comparison to numerical results calculated by MFP. For the first point we have the following cases:

Case 1: if $\bar{r}_{i.} = 0$ and $\bar{r}_{.j} \neq 0$ then: $e^t_{.j} - e^0_{.j} = e^0_{.j}\bar{r}_{.j} + \sum_i^m e^0_{ij}(r_{ij} - \bar{r}_{.j})$. In Case 1, only the local factors are responsible for the formation of the region's total employment change.

Case 2: if $\bar{r}_{i.} \neq 0$ and $\bar{r}_{.j} = 0$ then: $e^t_{.j} - e^0_{.j} = \sum_i^m e^0_{ij}\bar{r}_{i.} + \sum_i^m e^0_{ij}(r_{ij} - \bar{r}_{i.})$. The region's total employment change divided into two components, which contribute to its formation. In this case, only domestic factors concerning the industries are responsible for the formation of the region's employment change.

Case 3: if $\bar{r}_{i.} = 0$ and $\bar{r}_{.j} = 0$ then we apply the Ray-Srinath multi-factor partitioning model and not the modified one.

For the second point: the second point is that the numerical result components calculated by the modified model are expected to be different in comparison to numerical results calculated by MFP.

$\widehat{AE}_j - \overline{AE}_j$ the difference in allocation effect results because given the relative employment change in the *i-th* industry in the *j-th* region, we compare the effect of homothetic employment versus the effect of evenly distributed employment among regions and industries on relative total employment change. In other words, what hypothetically could be better for the national economy when the region's share of nations employment in *i-th* industry distribution is similar to the region share of total employment distribution or when the total employment evenly distributed among regions and industries.

$\widehat{IM}_j - \overline{IM}_j = \sum_i^m e^0_{ij}DE_i - (\widehat{AE}_j - \overline{AE}_j)$ the difference between the modified MFP and MFP industry mix is due to the distribution effect. If the distribution effect is zero, i.e. total employment is equally distributed among regions, then $\widehat{AE}_j - \overline{AE}_j$ depends on difference $\frac{e^0_{i.}}{e^0} - \frac{1}{m}$.

$\widehat{R}_j - \overline{R}_j = \sum_i^m e^0_{ij}SE_i - (\widehat{AE}_j - \overline{AE}_j)$ the difference between the modified MFP and MFP region component is due to the structure effect. If the structure effect is zero, i.e. the total employment is equally distributed among industry at the national level, then $\widehat{AE}_j - \overline{AE}_j$ depends on difference $\frac{e^0_{.j}}{e^0} - \frac{1}{n}$.

$\widehat{MR}_j - \overline{MR}_j = (\widehat{AE}_j - \overline{AE}_j) - \sum_i^m e^0_{ij}CE_i - \sum_i^m e^0_{ij}NSE_j$ the difference between the modified MFP and the MFP industry region component is due to the interaction of distribution and structure effects. If both distribution and structure effect is zero, then $\widehat{MR}_j = \overline{MR}_j$. So, a question is raised: Why is a modification of the MFP model necessary, given that most parts of criticism referred to Sections 2 and 3 are satisfactorily solved? It answered: The answer is that the MFP model ignores the effects that have already mentioned, i.e., the concertation effect and the national economic structure effect.

5. ILLUSTRATING THE DIFFERENCE BETWEEN MFP AND THE MODIFIED MULTI-FACTOR MODEL

This section aims to elucidate the difference between the MFP and the $\overline{\text{MFP}}$. Model in our effort to describe the main characteristics of growth employment in the Greek regions, and identify patterns following Biffignandis typology (Ray, Lamarche, & Biffignandi, 2011; Biachi et al., 2014). For our purpose, we have used employment[1] data by industry, and region[2] for years 2010 and 2015. A general overview is necessary first of all. In the year 2015, the total employment change at the national level is negative relative to the base year 2010. Also, unfavorable is the total employment change in twelve out of thirteen Greek regions. For the same period, also negative is the employment change at the national level for the ten industries in use in this study without any exception. We must notice here that the distribution of nation total employment among regions and industries is highly concentrated. Two out of thirteen regions (Attiki and Kentriki, Makedonia) concentrates more than 54% of the number of employees. Another, 52% of the total number of employees at the national level concentrated in two industries. The first one is Wholesale and retail trade, repair of motor vehicles and motorcycles, and transportation, storage, accommodation, food service activities. The second one is public administration, defense, compulsory social security, education, human health, and social work activities for both years under study. The relative total employment change for Greek regions and for comparison reason, the standardized relative total employment change and the average relative total employment change for a region are placed in Table 1. In Table 2, relative employment change in the *i-th* industry at the national level and the corresponding standardized relative employment change and the average employment change placed. In the same table, the four mentioned effects in Sections 3 and 4 are also placed.

[1] Greece's national accounts employment data by industry NACE rev.2. Data are available in from the national statistical office http://www.statistics.gr/.
[2] Greece is divided in thirteen administrative regions (Nuts2 level). The full names of regions in the tables are: N.A. = Notio Aigaio. V.A= Voreio Aigaio. I.N=Ionia Nisia. Pe= Peloponnisos. A.MT=Anatoliki Makedonia, Thraki. Ip. = Ipeiros. Kr.= Kriti. S.E.= terea Ellada. D.M.=Dytiki Makedonia. K.M=Kentriki Makedonia. At. = Attiki. Th.= Thessalia. D.E.= Dytiki Ellada.

Table 1. Relative employment changes, region specialization, and structure effects

Nuts2	*Relative total employment change of a region*	*Standardized relative total employment change of a region*	*Average relative total employment change of a region*	*Region specialization effect*	*Structure effect*
N.A	0.0025	-0.0091	-0.0801	0.0116	0.0711
V.A	-0.0739	-0.0650	-0.1025	-0.0090	0.0375
I.N	-0.0899	-0.0797	-0.1204	-0.0102	0.0407
Pe.	-0.0987	-0.0896	-0.1133	-0.0090	0.0237
A.MT	-0.1062	-0.1107	-0.1251	0.0045	0.0144
Ip.	-0.1196	-0.1195	-0.1429	-0.0001	0.0235
Kr.	-0.1247	-0.1117	-0.1455	-0.0130	0.0338
S.E	-0.1339	-0.1255	-0.1512	-0.0085	0.0257
D.M	-0.1346	-0.1209	-0.1239	-0.0137	0.0030
K.M	-0.1489	-0.1499	-0.1611	0.0010	0.0111
At.	-0.1514	-0.1611	-0.1816	0.0097	0.0205
Th.	-0.1519	-0.1477	-0.1505	-0.0043	0.0029
D.E.	-0.1693	-0.1648	-0.1782	-0.0045	0.0134
GREECE	-0.1514	-0.1380	-0.1366		

Table 2. Relative employment changes, location and distribution effects

*Industry**	*Relative employment change for the industry*	*Standardized relative employment change for the industry*	*Average relative employment change for the industry*	*Location effect*	*Distribution effect*
A	-0.116	-0.141	-0.113	**0.025**	-0.028
B,C,D,E	-0.226	-0.221	-0.181	-0.004	-0.041
F	-0.373	-0.372	-0.346	-0.001	-0.025
G,H,I	-0.103	-0.107	-0.081	**0.004**	-0.026
J	-0.097	-0.084	-0.069	-0.013	-0.015
K	-0.216	-0.212	-0.207	-0.004	-0.005
L	-0.190	-0.189	-0.187	-0.001	-0.003
M,N	-0.007	-0.020	-0.022	**0.013**	**0.002**
O,P,Q	-0.110	-0.109	-0.087	-0.001	-0.022
R,S,T,U	-0.157	-0.126	-0.074	-0.031	-0.052
GREECE	-0.136	-0.138	-0.137		

*Note: *full names of industries: A = Agriculture, forestry and fishing. B, C, D, E = Mining and quarrying, manufacturing, electricity, gas, steam, air conditioning and water supply, sewerage, waste management and remediation activities. F = Construction. G, H, I = Wholesale and retail trade, repair of motor vehicles and motorcycles, transportation and storage, accommodation and food service activities. J = Information and communication. K = Financial and insurance activities. L = Real estate activities. M, N = Professional, scientific and technical activities, administrative and support service activities. O, P, Q = Public administration and defence, compulsory social security, education, human health and social work activities. R, S, T, U = Arts, entertainment, recreation, other service activities, activities of households as employers, undifferentiated goods and services producing activities of households for own use, activities of extraterritorial organisations and bodies.*

Now, before proceeding to apply MFP and the proposed modified $\overline{\text{MFP}}$. Model to identify growth patterns below the Biffignandi's typology, two comments. First of all the structure effect enhance the relative total employment change of the *j-th* region (last column of Table 1). On the contrary, to the region specialization effect in most of the cases does not enhance the relative total employment change of a region. Also, the Distribution effect and industry location effect does not enhance relative employment change in the *i-th* industry at the national level in most of the cases. Biffignandi's typology depends on the sign of components. So, the calculation of components is necessary as a first step. For our calculations, we expressed components of Equations 3 and 4 in terms of relative contribution to the change of total employment per region. From our calculation, the national share for every region is negative $\text{NS}_\text{j} = -0.136$ and also negatives are the numeric values of allocation effect no matter which the model used for its calculation. So, the allocation effect calculated from MFP model for every region is $\widehat{\text{AE}}_\text{j} = -0.0017$ and the allocation effect calculated from the modified model is $\overline{\text{AE}}_\text{j} = -0.00032$. The values are different, but the same sign combined with the national share component indicates that the macro area (Greece) is in recession. The same sign for both models can be by an acquaintance, and the question is which of the models is preferable. The answer is the modified one because it counts the interaction between regions share of total employment and concertation of employment in the *i-th* industry at the national level. On the contrary, MFP model counts only regions share of total employment in industry *i*. In the next Table 3, the numerical values of the relative contribution of industrial mix (IM) component, region component (R), and industry-region interaction component presented for both models. We shall use these numerical values to compare the models and to identify growth patterns.

Table 3. The relative contribution of each component to region total employment change

Nuts2	*N.A*	*V.A*	*I.N*	*Pe.*	*A.MT*	*Ip.*	*Kr.*	*S.E*	*D.M*	*K.M*	*At.*	*Th.*	*D.E.*
$\overline{MFP}$: IM	0.019	0.024	0.023	0.019	0.022	0.017	0.022	0.010	0.013	0.026	0.030	0.024	0.020
MFP: IM	-0.0056	0.0002	-0.0012	-0.007	-0.003	-0.008	-0.003	-0.017	-0.014	0.001	0.006	-0.001	-0.005
$\overline{MFP}$: R	0.056	0.034	0.016	0.023	0.012	-0.006	-0.009	-0.015	0.013	-0.024	-0.045	-0.014	-0.042
MFP: R	0.129	0.073	0.058	0.048	0.027	0.019	0.026	0.013	0.017	-0.012	-0.023	-0.010	-0.027
$\overline{MFP}$: MR	0.064	0.005	0.007	-0.004	-0.003	0.006	-0.001	0.008	-0.023	-0.014	0.000	-0.026	-0.011
MFP: MR	0.017	-0.009	-0.009	-0.002	0.007	0.008	-0.010	0.008	0.000	0.000	0.004	-0.004	0.001
$\overline{MFP}$: Typology	**5**	**5**	**5**	**5**	**5**	**7**	**7**	**7**	**5**	**7**	**7**	**7**	**7**
MFP: Typology	**6**	**5**	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**7**	**7**	**8**	**8**

We have to notice again that the sign of the numerical value is more critical in comparison to the numerical value. Of course, the numerical value shows the contribution size of every component to the change of total employment in the region, but the sign identifies the growth pattern of the region.

Industry mix calculation: The industry mix component signs are not identical to the models. That happens because the MFP model industry mix calculation does not account for the distribution effect (last column, Table 2). The concentration effect is negative, with only one exception for all industries. That is a strong indication that regions do not benefit from the given distribution of regions share of total employment. As a first result, the given distribution of total employment among Greek regions contributes to accelerating the negative change of the region's total employment change.

Regional component calculations: The sign is also different depending on the model. The MFP and the proposed modified model result in six regions with a positive sign and four with a negative, but for three regions, the models result in a different sign. That is because the MFP model does not account for the structure effect (last column Table 1), which is positive to all regions – indicating that regions do benefit from the way that total employment concentrated in the *i-th* industry at the national level.

Industry-region interaction component calculation: Only five regions have the same sign no matter the applied model. The rest eight regions have a different sign. Again, the difference in signs is since the MFP model does not account for the interaction of region's specific factors with distribution and national structure effects. The different component signs result in different typologies[3] in growth patterns. Given that all regions are under the recession of the macro area, typologies follow.

Two regions (Thessalia and Dytiki (Western) Ellada) are below Typology 7 instead of Typology 8, which is a result of the MFP model. The main difference between Typologies 8 and 7 is that, in Typology 8, both region and industry are in intense recessive phase, but in Typology 7 industry may be in a moderate recessive phase.

Two regions (Kentriki (Central) Makedonia and Attiki) are as they result from both models in the same Typology 7, i.e., the region in intense recessive phase, the industry is moderate decline.

Three regions (Ipeiros, Kriti, Sterea Ellada) are under Typology 7 as they result from the modified model instead of 6, which is a result of the MFP model. In Typology 6 the region is in expansion or moderate recession, but the industry is in steady decline.

Five regions (Notio Aigaio, Ionia Nisia, Peloponnisos, Anatoliki Makedonia Thraki, Dytiki Makedonia) are under Typology 5 as they result from the modified model instead of Typology 6 which is the result of MFP model. Regions in Typology 5 are in decelerating expansion, or moderate recession and industry are in moderate decline.

One region (Voreio Aigaio), as it results, is under Typology 5 for both models.

The main result of the above analysis is that three regions (Ipeiros, Kriti, Sterea Ellada) are in the worst typology as a result of the modified model. From the rest regions, only three are in the same typology for both models (Voreio Aigaio, Kentriki Makedonia, Attiki), and four are in an improved typology.

6. CONCLUSIONS AND FURTHER DISCUSSION

In the present paper, we have established that the multi-factor partitioning MFP model, which constitutes an improvement on the traditional and homothetic model of shift-share analysis, leaves room for improvement. The improvement suggested takes into account the concentration effect and the national economic structure effect. The ignorance of these effects misleads the decision-maker as to which regional policies must be applied to enforce the employment. As it has mentioned, the shift-share analysis method often used despite the fact that the method describes and does not interpret the causes of regional growth. So, an open issue is which explanatory variable in a regression model, for example, can predict the sign and the numerical size of the components. A second issue is the projection of the employment level for planning purposes. Which model the MFP or the modified one result in better projection at the regional level? The last issue is to examine the sensitivity in results under different levels of aggregation for variables in use.

[3] Typologies and the component with their sigh in brief: Typology 1: Generalized expansion [N (+), IM (+), MR (+ or -), R (+)]. Typology 2: Territorial expansion but not industrial. [N (+), IM (-), MR (+ or -), R (+)]. Typology 3: Local delay in the context of overall growth. [N (+), IM (+), MR (+ or -), R (-)]. Typology 4: Local and industrial delays in the context of macro area growth. [N (+), IM (-), MR (+ or -), R (-)]. Typology 5: Discrete local and industrial performance within a context of recession in the macro area. [N (-), IM (+), MR (+ or -), R (+)]. Typology 6: Local containment in the context of macro area and industrial recession. [N (-), IM (-), MR (+ or -), R (+)]. Typology 7: Industrial containment in the context of a macro area and area recession. [N (-), IM (+), MR (+ or -), R (-)]. Typology 8: Generalized recession. Corresponds to a situation of contraction at both industrial and territorial (level, national and local) [N (-), IM (-), MR (+ or -), R (-)].

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APPENDIX A

1. Definitions

i = number of industries $(i = 1,2,3,\dots,m)$ j = number of regions in geographical area or nation $(j = 1,2,3,\dots,n)$
t = 0 the base year t = the terminal year

$e_{ij}^0 =$	employment in the *i-th* industry in the jth region
$e_{i.}^0 = \sum_j^n e_{ij}^0$	total employment in the *i-th* industry at the national level
$e_{.j}^0 =$	total employment in the *j-th* region
$e^0 = \sum_i^m \sum_j^n e_{ij}^0$	total employment
$\tilde{e}_{ij}^0 = \frac{e_{i.}^0}{e^0} e_{.j}^0$	the homothetic employment in the *i-th* industry in the *j-th* region
$r_{ij} = \frac{e_{ij}^t - e_{ij}^0}{e_{ij}^0}$	relative employment change in the *i-th* industry in the *j-th* region (t = 0 the base year)
$r_{i.} = \sum_j^n \frac{e_{ij}^0}{e_{i.}^0} r_{ij}$	relative employment change in the *i-th* industry at the national level
$\hat{r}_{i.} = \sum_j^n \frac{e_j^0}{e^0} r_{ij}$	standardized relative employment change in the *i-th* industry at the national level
$\bar{r}_{i.} = \sum_j^n \frac{1}{n} r_{ij}$	average relative in employment change in the *i-th* industry at the national level
$r_{.j} = \sum_j^n \frac{e_{ij}^0}{e_{.j}^0} r_{ij}$	relative total employment change in the jth region
$\hat{r}_{.j} = \sum_i^m \frac{e_{i.}^0}{e^0} r_{ij}$	standardized relative total employment change in the *j-th* region
$\bar{r}_{.j} = \sum_i^m \frac{1}{m} r_{ij.}$	average relative in employment change in the *j-th* region

$r = \sum_i^m \frac{e_{i.}^0}{e^0} r_{i.} = \sum_j^n \frac{e_{.j}^0}{e^0} r_{.j} = \sum_i^m \sum_j^n \frac{e_{ij}^0}{e^0} r_{ij}$ relative total employment change

$\hat{r} = \sum_i^m \frac{e_{i.}^0}{e^0} \hat{r}_{i.} = \sum_j^n \frac{e_{.j}^0}{e^0} \hat{r}_{.j} = \sum_i^m \sum_j^n \frac{e_{.j}^0}{e^0}\frac{e_{i.}^0}{e^0} r_{ij} = \sum_i^m \sum_j^n \frac{\tilde{e}_{ij}^0}{e^0} r_{ij}$ standardized relative total employment change

$\bar{r} = \sum_j^n \frac{1}{n} \bar{r}_{.j} = \sum_i^m \frac{1}{m} \bar{r}_{i.} = \sum_i^m \sum_j^n \frac{1}{m}\frac{1}{n} r_{ij} = \sum_i^m \sum_j^n \frac{\bar{e}_{ij}^0}{e^0} r_{ij}$ average relative total employment change, where $\bar{e}_{ij}^0 = \frac{e^0}{mn}$

2. The classical shift-share analysis model (Dunn, 1960)

The total employment change $e_{.j}^t - e_{.j}^0$ between the terminal and the base year in the *j-th* region is divided into the sum of three components:

$$e_{.j}^0 - e_{.j}^0 = e_{.j}^0 r + \sum_i^m e_{ij}^0 (r_{i.} - r) + \sum_i^0 e_{ij}^0 (r_{ij} - r_{i.}) \tag{A.1}$$

where,
$NS_j = e_{.j}^0 r$ is the national share component;
$IM_j = \sum_i^m e_{ij}^0 (r_{i.} - r)$ is the industry mix component;
$RS_j = \sum_i^0 e_{ij}^0 (r_{ij} - r_{i.})$ is the regional share component.
We can express Equation A.1 in terms of the relative contribution of each component to the total employment change $e_{.j}^0 - e_{.j}^0$:

$$r_{.j} = r + \sum_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{e_{i.}^0}{e^0}\right) r_{i.} + \sum_i^0 \frac{e_{ij}^0}{e_{.j}^0} (r_{ij} - r_{i.}) \tag{A.2}$$

3. The alternative to the classical shift-share analysis model (Artige et al., 2013)

The total employment change $e_{.j}^t - e_{.j}^0$ between the terminal and the base year in the *j-th* region is divided into the sum of three components:

$$e_{.j}^{0} - e_{.j}^{0} = e_{.j}^{0} r + \sum_{i}^{m} (e_{ij}^{0} - \tilde{e}_{ij}^{0}) r_{ij} + \sum_{i}^{0} \tilde{e}_{ij}^{0} (r_{ij} - r_{i.}) \tag{A.3}$$

where,
$\tilde{e}_{ij}^{0} = \frac{e_{i.}^{0}}{e^{o}} e_{.j}^{0}$ is the homothetic employment (Esteban-Marquillas, 1972) and:
$NS_j = e_{.j}^{0} r$ is the national share component;
$IM_j = \sum_i^m (e_{ij}^{0} - \tilde{e}_{ij}^{0}) r_{ij}$ is the industry mix component;
$RS_j = \sum_i^0 \tilde{e}_{ij}^{0} (r_{ij} - r_{i.})$ is the regional share component.
We can express Equation A.3 in terms of the relative contribution of each component to the total change $e_{.j}^{0} - e_{.j}^{0}$:

$$r_{.j} = r + \sum_{i}^{m} \left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{e_{i.}^{0}}{e^{0}}\right) r_{ij} + \sum_{i}^{0} \frac{e_{i.}^{0}}{e^{0}} (r_{ij} - r_{i.}) \tag{A.4}$$

4. The homothetic shift-share analysis model (Esteban-Marquillas, 1972)

The total employment change $e_{.j}^{t} - e_{.j}^{0}$ between the terminal and the base year in the *j-th* region is divided into the sum of four components:

$$e_{.j}^{0} - e_{.j}^{0} = e_{.j}^{0} r + \sum_{i}^{m} e_{ij}^{0} (r_{i.} - r) + \sum_{i}^{m} (e_{ij}^{0} - \tilde{e}_{ij}^{0})(r_{ij} - r_{i.}) + \sum_{i}^{0} \tilde{e}_{ij}^{0} (r_{ij} - r_{i.}) \tag{A.5}$$

where,
$\tilde{e}_{ij}^{0} = \frac{e_{i.}^{0}}{e^{o}} e_{.j}^{0}$ is the homothetic employment (Esteban-Marquillas,1972) and:
$NS_j = e_{.j}^{0} r$ is the national share component;
$IM_j = \sum_i^m (e_{ij}^{0} - \tilde{e}_{ij}^{0}) r_{ij}$ is the industry mix component;
$AE_j = \sum_i^m (e_{ij}^{0} - \tilde{e}_{ij}^{0})(r_{ij} - r_{i.})$ is the allocation component;
$C_j = \sum_i^0 \tilde{e}_{ij}^{0} (r_{ij} - r_{i.})$ is the homothetic competitive component.
We can easily express Equation A.5 in terms of the relative contribution of each component to the total change $e_{.j}^{0} - e_{.j}^{0}$

$$r_{.j} = r + \sum_{i}^{m} \left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{e_{i.}^{0}}{e^{0}}\right) r_{i.} + \sum_{i}^{m} \left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{e_{i.}^{0}}{e^{0}}\right)(r_{ij} - r_{i.}) + \sum_{i}^{0} \frac{e_{i.}^{0}}{e^{0}} (r_{ij} - r_{i.}) \tag{A.6}$$

5. Alternative to the homothetic shift-share analysis model (Artige et al., 2013)

The total employment change $e_{.j}^{t} - e_{.j}^{0}$ between the terminal and the base year in the *j-th* region is divided into the sum of four components:

$$e_{.j}^{0} - e_{.j}^{0} = e_{.j}^{0} r + \sum_{i}^{m} (e_{ij}^{0} - \tilde{e}_{ij}^{0}) r_{ij} - \sum_{i}^{m} (e_{ij}^{0} - \tilde{e}_{ij}^{0})(r_{ij} - r_{i.}) + \sum_{i}^{0} e_{ij}^{0} (r_{ij} - r_{i.}) \tag{A.7}$$

where,
$\tilde{e}_{ij}^{0} = \frac{e_{i.}^{0}}{e^{o}} e_{.j}^{0}$ is the homothetic employment (Esteban-Marquillas,1972) and:
$NS_j = e_{.j}^{0} r$ is the national share component;
$IM_j = \sum_i^m (e_{ij}^{0} - \tilde{e}_{ij}^{0}) r_{ij}$ is the industry mix component;
$AE_j = \sum_i^m (e_{ij}^{0} - \tilde{e}_{ij}^{0})(r_{ij} - r_{i.})$ is the allocation component;
$C_j = \sum_i^0 e_{ij}^{0} (r_{ij} - r_{i.})$ is the homothetic competitive component.
We can easily express Equation A.7 in terms of the relative contribution of each component to the total change $e_{.j}^{0} - e_{.j}^{0}$:

$$r_{.j} = r + \sum_{i}^{m} \left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{e_{i.}^{0}}{e^{0}}\right) r_{ij} - \sum_{i}^{m} \left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{e_{i.}^{0}}{e^{0}}\right)(r_{ij} - r_{i.}) + \sum_{i}^{0} \frac{e_{ij}^{0}}{e_{.j}^{0}} (r_{ij} - r_{i.}) \tag{A.8}$$

6. The derivation of multi-factor partitioning model:

From classical to MFP model:

The standardized relative employment change $\hat{r}_{i.}$ is added and subtracted in the components IM and RS of the classical model (Equation A.1). This results that the employment change is the sum of three components:

$e_{.j}^0 r$, $\sum_i^m e_{ij}^0(\hat{r}_{i.} - r)$, and $\sum_i^o e_{ij}^0(r_{ij} - \hat{r}_{i.})$. Next step is to add and subtract the standardized relative employment changes $\hat{r}_{.j}$ in the component $\sum_i^o e_{ij}^0(r_{ij} - \hat{r}_{i.})$ and $\hat{r} = \sum_i^m \frac{e_{i.}^0}{e^0}\hat{r}_{i.} = \sum_j^n \frac{e_{.j}^0}{e^0}\hat{r}_{.j}$in the component $\sum_i^m e_{ij}^0(\hat{r}_{i.} - r)$. The result is Equation A.9 i.e., the MFP model.

$$e_{.j}^0 - e_{.j}^0 = NS_j + \widehat{AE}_j + \widehat{IM}_j + \hat{R}_j + \widehat{MR}_j \tag{A.9}$$

where,
$NS_j = e_{.j}^0 r$ is the national share component;
$\widehat{AE}_j = \sum_i^m e_{ij}^0(\hat{r} - r)$ is the allocation component;
$\widehat{IM}_j = \sum_i^m e_{ij}^0(\hat{r}_{i.} - \hat{r})$ is the industry mix component;
$\hat{R}_j = \sum_i^m e_{ij}^0(\hat{r}_{.j} - \hat{r})$ is the regional share component;
$\widehat{MR}_j = \sum_i^o e_{ij}^0(r_{ij} - \hat{r}_{i.} - \hat{r}_{.j} + \hat{r})$ is the industry-region interaction component.
We can express the MFP model in terms of the relative contribution of each component to the j-regions total employment change Equation A.10.

$$r_{.j} = r + (\hat{r} - r) + \sum\nolimits_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{e_{i.}^0}{e^0}\right)\hat{r}_{i.} + (\hat{r}_{.j} - \hat{r}) + \sum\nolimits_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{e_{i.}^0}{e^0}\right)(r_{ij} - \hat{r}_{i.}) \tag{A.10}$$

From the alternative to MFP model:

Given that $\tilde{e}_{ij}^0 = \frac{e_{i.}^0}{e^0} e_{.j}^0$ is the homothetic employment and doing similar steps, adding and subtracting standardized relative employment change $\hat{r}_{i.}$ from the alternative to classical model, i.e. from Equation A.3: $\sum_i^m (e_{ij}^0 - \tilde{e}_{ij}^0)(r_{ij} \pm \hat{r}_{i.}) + \sum_i^o \tilde{e}_{ij}^0(r_{ij} \pm \hat{r}_{i.} - r_{i.})$ results in the sum of three components for total employment change in j-region $\sum_i^m e_{ij}^0(\hat{r}_{i.} - r)$ and $\sum_i^o e_{ij}^0(r_{ij} - \hat{r}_{i.})$. Adding and subtracting the standardized relative changes $\hat{r}_{.j}$ and $\hat{r}$ the result is Equation A.9 again i.e. MFP model.

From the homothetic to MFP model:

The sum $AE + C = RS$ in Equation A.5. Following the same steps as in the classical one, the result is the MFP model. Also, from the alternative to the homothetic model the sum $AE + C = RS$ in Equation A.7. Following the same steps as in the alternative to the classical one, the result is the MFP model again.

7. The derivation of the modified multi-factor ($\overline{\mathbf{MFP}}$) partitioning model

Doing the same steps as we did to derive the MFP model from any of the above-presented models, and instead of standardized relative employment change, we use the averages. We have the modified multi partitioning model $\overline{\text{MFP}}$:

$$e_{.j}^t - e_{.j}^0 = NS_j + \overline{AE}_j + \overline{IM}_j + \bar{R}_j + \overline{MR}_j \tag{A.11}$$

where,
$NS_j = e_{.j}^0 r$ is the national share component;
$\overline{AE}_j = \sum_i^m e_{ij}^0(\bar{r} - r)$ is the allocation component;
$\overline{IM}_j = \sum_i^m e_{ij}^0(\bar{r}_{.j} - \bar{r})$ is the industry mix component;
$\bar{R}_j = \sum_i^m e_{ij}^0(\bar{r}_{.j} - \bar{r})$ is the regional component;
$\overline{MR}_j = \sum_i^m e_{ij}^0(r_{ij} - \bar{r}_{i.} - \bar{r}_j + \bar{r})$ is the industry-region component.
We can express the MFP model in terms of the relative contribution of each component to the j-regions total employment change Equation A.12.

$$r_{.j} = r + (\bar{r} - r) + \sum\nolimits_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{1}{m}\right)\bar{r}_{i.} + (\bar{r}_{.j} - \bar{r}) + \sum\nolimits_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{1}{m}\right)(r_{ij} - \hat{r}_{i.}) \tag{A.12}$$

8. Testing MFP and ($\overline{\mathbf{MFP}}$) partitioning models with a simple numeric example

We are going to apply the MFP and $\overline{\text{MFP}}$ models using the expression of the relative contribution of each component to *j-th* region total employment change:

$$\text{MFP: } r_{.j} = r + (\hat{r} - r) + \sum_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{e_{i.}^0}{e^0}\right)\hat{r}_{i.} + (\hat{r}_{.j} - \hat{r}) + \sum_i^m \left(\frac{e_{ij}^0}{e_{.j}^0} - \frac{e_{i.}^0}{e^0}\right)(r_{ij} - \hat{r}_{i.}) \tag{A.13}$$

and

$$\overline{\text{MFP}}: \mathrm{r}_{.j} = \mathrm{r} + (\bar{\mathrm{r}} - \mathrm{r}) + \sum_{i}^{m}\left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{1}{m}\right)\bar{\mathrm{r}}_{i.} + (\bar{\mathrm{r}}_{.j} - \bar{\mathrm{r}}) + \sum_{i}^{m}\left(\frac{e_{ij}^{0}}{e_{.j}^{0}} - \frac{1}{m}\right)(\mathrm{r}_{ij} - \hat{\mathrm{r}}_{i.}) \tag{A.14}$$

Next four tables present the same type of data: number e_{ij}^{0} of people employed in industry $i = 1,2,3,4,5$ in the region $j = A, B, C, D$, and the corresponding relative employment changes r_{ij}.

Table A.1. Case 1

	e_{iA}^{0}	e_{iB}^{0}	e_{iC}^{0}	e_{iD}^{0}	*nation*	r_{iA}	r_{iB}	r_{iC}	r_{iD}
i=1	**100**	200	90	300	690	0.080	0.020	0.030	0.028
i=2	**200**	150	100	80	530	0.095	0.070	0.086	0.167
i=3	100	70	80	50	300	0.070	0.059	0.021	0.028
i=4	250	250	200	100	800	0.013	0.096	0.073	0.077
i=5	350	180	280	160	970	0.024	0.033	0.019	0.028
e_{j}^{0}	1000	850	750	690	3290				

Table A.1. Case 2

	e_{iA}^{0}	e_{iB}^{0}	e_{iC}^{0}	e_{iD}^{0}	*nation*	r_{iA}	r_{iB}	r_{iC}	r_{iD}
i=1	**200**	200	90	300	790	0.080	0.020	0.030	0.028
i=2	**100**	150	100	80	430	0.095	0.070	0.086	0.167
i=3	100	70	80	50	300	0.070	0.059	0.021	0.028
i=4	250	250	200	100	800	0.013	0.096	0.073	0.077
i=5	350	180	280	160	970	0.024	0.033	0.019	0.028
e_{j}^{0}	1000	850	750	690	3290				

Table A.2. Case 3

	e_{iA}^{0}	e_{iB}^{0}	e_{iC}^{0}	e_{iD}^{0}	*nation*	r_{iA}	r_{iB}	r_{iC}	r_{iD}
i=1	**100**	**200**	90	300	690	0.080	0.020	0.030	0.028
i=2	**200**	**150**	100	80	530	0.095	0.070	0.086	0.167
i=3	**100**	**70**	80	50	300	0.070	0.059	0.021	0.028
i=4	**250**	**250**	200	100	800	0.013	0.096	0.073	0.077
i=5	**350**	**180**	280	160	970	0.024	0.033	0.019	0.028
e_{j}^{0}	**1000**	**850**	750	690	3290				

Table A.2. Case 4

	e_{iA}^{0}	e_{iB}^{0}	e_{iC}^{0}	e_{iD}^{0}	*nation*	r_{iA}	r_{iB}	r_{iC}	r_{iD}
i=1	**200**	**100**	90	300	790	0.080	0.020	0.030	0.028
i=2	**150**	**200**	100	80	430	0.095	0.070	0.086	0.167
i=3	**70**	**100**	80	50	300	0.070	0.059	0.021	0.028
i=4	**250**	**250**	200	100	800	0.013	0.096	0.073	0.077
i=5	**180**	**350**	280	160	970	0.024	0.033	0.019	0.028
e_{j}^{0}	**850**	**1000**	750	690	3290				

The above Cases 1 and 2 differ only in the number of employees between industries 1 and 2 for region A. (bold), and Cases 3 and 4 differ only in the number of employees between region A and B (bold). In the next Table A.3, we have placed the numerical component results after applying MFP and $\overline{\text{MFP}}$ models. The results from the MFP model are different between the two cases, even though we would expect for regions B, C, D the same numerical results, given that with the exception of region A; the regions B, C, D are identical in both cases. The results from the modified model are the anticipated ones, as we can see with a simple inspection. The numerical results differ only for region A and similar for regions B, C, D. The weakness of the MFP model is due to the structure effect. Again, we apply both models for data in Table A.2. Now we have switched between regions A and B between cases. The results placed in Table A.4. Even the regions C, D are identical for both Cases 3 and 4; the results from the MFP model are not the anticipated ones due to the distribution effect. On the contrary, the results of the modified MFP model expected. Only regions A and B result in different numerical values for their components.

Table A.3. Results due to structure effect

	Application of MFP					***Application of modified MFP***			
Case$_1$_MFP	A	B	C	D	Case$_1$_$\overline{MFP}$	A	B	C	D
NS	0.049	0.049	0.049	0.049	NS	0.049	0.049	0.049	0.049
AE	0.003	0.003	0.003	0.003	AE	0.006	0.006	0.006	0.006
IM	0.002	0.003	-0.002	-0.004	IM	-0.001	0.0001	-0.005	-0.008
R	-0.003	0.002	-0.007	0.010	R	0.001	-0.0003	-0.010	0.010
MR	-0.005	0.000	0.001	-0.007	MR	-0.009	0.001	0.004	-0.006
Case$_2$_MFP	A	B	C	D	Case$_2$_$\overline{MFP}$	A	B	C	D
NS	0.048	0.048	0.048	0.048	NS	0.048	0.048	0.048	0.048
AE	0.002	0.002	0.002	0.002	AE	0.008	0.008	0.008	0.008
IM	-0.003	0.005	-0.001	-0.002	IM	-0.008	0.0001	-0.005	-0.008
R	-0.002	0.002	-0.007	0.008	R	0.001	-0.0003	-0.010	0.010
MR	-0.002	0.000	0.001	-0.005	MR	-0.005	0.001	0.004	-0.006
Numerical difference	A	B	C	D	Numerical difference	A	B	C	D
NS	0.002	0.002	0.002	0.002	NS	0.002	0.002	0.002	0.002
AE	0.000	0.000	0.000	0.000	AE	-0.002	-0.002	-0.002	-0.002
IM	0.004	-0.002	-0.002	-0.002	IM	0.007	**0.000**	**0.000**	**0.000**
R	-0.001	0.000	0.000	0.002	R	**0.000**	**0.000**	**0.000**	**0.000**
MR	-0.003	0.000	0.000	-0.002	MR	-0.005	**0.000**	**0.000**	**0.000**

Table A.4. Results due to distribution effect

	Application of MFP					***Application of modified MFP***			
Case$_3$_MFP	A	B	C	D	Case$_3$_$\overline{MFP}$	A	B	C	D
NS	0.049	0.049	0.049	0.049	NS	0.049	0.049	0.049	0.049
AE	0.003	0.003	0.003	0.003	AE	0.006	0.006	0.006	0.006
IM	0.002	0.003	-0.002	-0.004	IM	-0.001	0.0001	-0.005	-0.008
R	-0.003	0.002	-0.007	0.010	R	0.001	-0.0003	-0.010	0.010
MR	-0.005	0.000	0.001	-0.007	MR	-0.009	0.001	0.004	-0.006
Case$_4$_MFP	A	B	C	D	Case$_4$_$\overline{MFP}$	A	B	C	D
NS	0.0512	0.0512	0.0512	0.0512	NS	0.0512	0.0512	0.0512	0.0512
AE	0.0012	0.0012	0.0012	0.0012	AE	0.0047	0.0047	0.0047	0.0047
IM	0.0033	0.0019	-0.0020	-0.0046	IM	0.0001	-0.0013	-0.0055	-0.0079
R	-0.0036	0.0015	-0.0070	0.0099	R	0.0006	-0.0003	-0.0101	0.0097
MR	-0.00172	0.00163	0.00045	-0.00642	MR	-0.0063	0.0031	0.0035	-0.0064
Numerical difference	A	B	C	D	Numerical Difference	A	B	C	D
NS	-0.0018	-0.0018	-0.0018	-0.0018	NS	-0.0018	-0.0018	-0.0018	-0.0018
AE	0.0016	0.0016	0.0016	0.0016	AE	0.0018	0.0018	0.0018	0.0018
IM	-0.0017	0.0013	-0.0004	0.0010	IM	-0.0014	0.0014	0.0000	0.0000
R	0.0002	0.0002	0.0002	0.0002	R	0.0000	0.0000	0.0000	0.0000
MR	-0.0029	-0.0016	0.0004	-0.0010	MR	-0.0032	-0.0017	0.0000	0.0000