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THERMO-RHEOLOGICAL HEREDITARY ELEMENTS AND DISCRETE CONTINUUM MODEL DYNAMICS

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In this paper we shall define a piezo- and thermo-rheological discrete continuum model as a system of material particles interconnected by light standard constraint elements (elastic, hereditary or creep) and which are, in piezo- or thermo-rheological modified dynamical state, on defined interdistances (when constraint elements are unstressed and stressed). By using standard creep light elements with fractional derivative order constitutive relations, as a constraints, for a discrete systems with many degrees of freedom the system of the integro-differential equations of fractional order and in covariant tensor form are composed. Also, we take into consideration more precise definition of the thermomodification of the standard hereditary element taking into account rheonomic constitutive relation of the thermo - modification of the standard hereditary rheological light element and possibilities of appearance of parametrically excited phenomena in the discrete system. Two examples: a thermorheological oscillator and a thermorheological coupled pendulums excited by deterministic and stochastic temperature excitation are considered, and a series of the systems of the averaged differential equations are derived for deterministic and stochastic cases.

discrete continuum, piezo and thermo-rheological hereditary elements, discrete system dynamics, sistem of material particles, discrete homogenous chain, discrete homogenous material net, integro-differential equations, rheological kernel, relaxational kernel, fractional derivtive order differential equations, tensor equations.

1. Inyroduction. Discrete continuum is *ideally elastic* if it's material particles are interconnected by light standard ideally elastic constraint elements. Discrete continuum is a *standard hereditary continuum* if it's material particles are interconnected by light standard hereditary elements. Discrete continuum is a *standard creep continuum* (see Refs. [9], [15]) if its material particles are interconnected by light standard creep elements (see Refs. [2], [3] and [4]). Discrete continuum is a *thermo- or piezo-modified continuum* if its material particles are interconnected by light standard thermo- or piezo-rheological hereditary elements.

We shall define *discrete chain system* [15] as a system of discrete material particles that can move along a line and are interconnected by standard constraint light elements. The chain is ideally elastic if material particles are interconnected by ideally elastic elements. The chain is standard hereditary if material particles are interconnected by standard hereditary elements. The chain is standard creep if material particles are interconnected by standard creep elements. The number of degrees of freedom of each of these chains is equal to the number of particles in it, since we hypothesize that each material particle moves in the direction of the chain line.

If all material particles of the discrete continuum move in the same plane they have two degrees of motion freedom and are interconnected by standard constraint light elements; such material system we shall call *the plane material net* [9]. The net can be elastic, standard hereditary or standard hereditary plane material net depending on the type of constraint elements that interconnects its material particles. It is a *plane discrete material surface* through which we can follow the propagation of deformation waves, which can be elastic, hereditary or creep excited by thermo- or piezo modifications in two orthogonal directions.

Fig. 1 shows two material particles, with masses m_1 and m_2 , the positions of which are determined by following vectors position \mathbf{r}_1 and \mathbf{r}_2 , constrained by light standard piezo-thermo-rheological hereditary element, negligible mass. At ends of the hereditary element in the extension stress-strain state, the force of deformation $P(t)$ appears, which depends on generalized coordinate of element deformation $\mathbf{r} - \mathbf{r}_0$, as well on corresponding temperatures, $T_K(t)$ and $T_M(t)$ and polarisations voltages $U_K(t)$ and $U_M(t)$ of piezo-thermo-modification of hereditary element. This system was studied in the papers [20], [7] and [8] and in monograph [2].

The paper [20] presents the discrete continuum method on examples of homogenous discrete systems' with limited number of degrees of motion freedom dynamics. These systems are in the form of homogenous chains [15] and nets in space and plain. Material points of these nets and chains are tied by elastic, standard hereditary or creep elements. These systems' dynamics is described by a system of integro-differential equations or differential equations with fractional derivatives. A light standard creep element is defined by a constitutive relation of stress-strain state, for the creation of which fractional order derivatives were used.

The fast development of science of material and experimental mechanics, as well as methods of numerical analysis, led to the creation of different models of real material bodies and methods for studying dynamics and processes which happen in them during the transduction of disturbance through deformable bodies (see Refs. [5], [6], [10], [11], [12], [16-20]).

There also exist different approaches to creating real body models. One such approach is represented by a model of discrete system of material particles which are connected by certain ties, and the number of which is then increased to

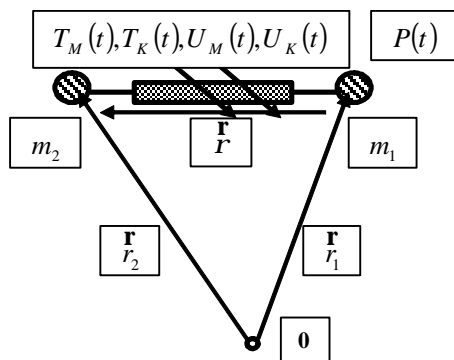


Fig. 1. Discrete system consisted of two material particles constrained by light standard piezo-thermo-rheological hereditary element

create a continuum, the motion and deformable wave propagation of which was then described by using partial differential equations. And then, due to the impossibility of solving them analytical, the approximation method was used for the purpose. Methods of discretization of systems of partial differential equations and methods of physical discretization of continuum were used.

2. Equations of dynamics of a discrete system with finite constraints and standard creep-elements. We investigate dynamical system (see Fig. 2.) of N material particles with masses m_n , $n = 1, 2, 3, \dots, N$, the position vectors of which are $\mathbf{r}_n = y_n^i \mathbf{e}_i$, $i = 1, 2, 3$; $n = 1, 2, 3, \dots, N$. Material particles are constrained by S finite constraints:

$$\bar{f}_m(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = f_m(y^1, y^2, \dots, y^{3N})$$

$$m = 1, 2, 3, \dots, S \quad (1)$$

and where we introduce the following notations: $y_v^k =: y^{3v-(3-k)}$, $k = 1, 2, 3$; $m_{3v-k} = m_{3v}$, $k = 1, 2, 3$, $v = 1, 2, 3, \dots, N$; as well as by K standard hereditary elements neglected mass and material properties parameters of which are: $n_{(v,v+1)k}$, $k = 1, 2, 3, \dots, K$, are times of relaxation, and $c_{(n,n+1)k}$ and $\rho_{(v,v+1)k}$ are an instantaneous rigid stiffness modulus as prolonged ones; and as well as by C standard creep elements neglected mass and material properties parameters of which are: $\alpha_{(v,v+1)c}$, $c = 1, 2, 3, \dots, C_v$, are proper (own) material constants of the characteristic creep law of creep elements materials, $E_{0(n,n+1)c}$, $c = 1, 2, 3, \dots, C_n$ and $E_{a(n,n+1)c}$, $c = 1, 2, 3, \dots, C_n$ are modulus of elasticity and creeping properties of standard creep elements.

Relations between reactions and deformations of the standard hereditary light element in the discrete system can be defined in the relaxational forms by using integral stress strain state relations [2], [8]:

$$P_{(v,v+1)k} = c_{(v,v+1)k} \left[\bar{\rho}_{(v,v+1)k}(t) - \int_0^t \mathbf{R}_{(v,v+1)k}(t-\tau) \bar{\rho}_{(v,v+1)k}(\tau) d\tau \right],$$

$$n = 1, 2, 3, \dots, N, k = 1, 2, 3, \dots, K_n \quad (2)$$

where are

$$\mathbf{R}_{(n,n+1)k}(t-t) = \frac{c_{(n,n+1)k} - \tilde{c}_{(n,n+1)k}}{n_{(n,n+1)k} c_{(n,n+1)k}} e^{-\frac{t-t}{n_{(n,n+1)k}}},$$

$$v = 1, 2, 3, \dots, N, k = 1, 2, 3, \dots, K_v \quad (3)$$

kernels of relaxation (see Refs. [2], [3], [4] and [7]), and

$$r_{(n,n+1)k} = |\mathbf{r}_{(n,n+1)k}| = |\mathbf{r}_{(n+1)k} - \mathbf{r}_{(n)k}|, \quad (a)$$

$$\bar{r}_{(n,n+1)k} = |\mathbf{r}_{(n,n+1)k} - r_{(n,n+1)k0}| = |\mathbf{r}_{(n+1)k} - \mathbf{r}_{(n)k}| - r_{(n,n+1)k0}.$$

and $\rho_{(v,v+1)k0}$ is length of a hereditary element in natural stress-strain state, when the strain and stress in the element are equal to zero.

Relations between reactions and deformations of the creep light element in the discrete system (see Ref. [6]) can be defined in the form by using fractional derivative order stress strain state relations:

$$m_n \ddot{\mathbf{r}}_n = \mathbf{F}_n(t) + \sum_{m=1}^{m=S} I_m \text{grad}_n f_m(\mathbf{r}_1, \dots, \mathbf{r}_N) + \mathbf{R}_{nT} +$$

$$+ \sum_{j=1}^N \left[\sum_{k=1}^{k=K_n} P_{(n,n+j)k}(t) \frac{\mathbf{r}_{(n+j)k} - \mathbf{r}_{(n)k}}{|\mathbf{r}_{(n+j)k} - \mathbf{r}_{(n)k}|} + \sum_{c=1}^{c=C_n} P_{(n,n+j)c}(t) \frac{\mathbf{r}_{(n+j)c} - \mathbf{r}_{(n)c}}{|\mathbf{r}_{(n+j)c} - \mathbf{r}_{(n)c}|} \right]$$

$$n = 1, 2, 3, \dots, N. \quad (8)$$

3. Covariant integro-differential equations of fractional order derivatives, of the motion of the discrete creep-hereditary system. The virtual displacements can be expressed by using generalized coordinates in the form:

$$\delta \mathbf{r}_v = \sum_{\alpha=1}^{\alpha=n} \frac{\partial \mathbf{r}_v}{\partial q^\alpha} \delta q^\alpha \text{ and introduced into the previous equation}$$

(7) for the work of the active and reactive forces on the virtual displacements, we obtain the following system of equations in the covariant coordinates:

$$a_{\alpha\beta} \frac{D\mathcal{Q}^\beta}{dt} = Q_\alpha + Q_\alpha^* + P_\alpha^H + P_\alpha^C ;$$

$$\alpha = 1, 2, 3, \dots, n; \quad n = 3N - S, \quad (9)$$

where, by analyzing the members, we have the following expressions of the fictive, active and reactive forces:

$$I_\alpha = - \sum_{v=1}^{v=N} m_v \left(\frac{\partial \mathbf{r}_v}{\partial q^\alpha}, \frac{\partial \mathbf{r}_v}{\partial q^\alpha} \right) = - \sum_{v=1}^{v=N} m_v \left(\frac{d}{dt} \sum_{\beta=0}^{\beta=n} \frac{\partial \mathbf{r}_v}{\partial q^\beta} \mathcal{Q}^\beta, \frac{\partial \mathbf{r}_v}{\partial q^\alpha} \right) =$$

$$- \left[a_{\alpha\beta} \left(\mathcal{Q}^\beta + \Gamma_{\gamma\delta}^\alpha \mathcal{Q}^\gamma \mathcal{Q}^\delta \right) \right] = - a_{\alpha\beta} \frac{D\mathcal{Q}^\beta}{dt}, \quad (10)$$

$$\alpha = 1, 2, 3, \dots, n; \quad n = 3N - S,$$

$$Q_\alpha = \sum_{v=1}^{v=N} \left(\mathbf{F}_v(t), \frac{\partial \mathbf{r}_v}{\partial q^\alpha} \right), \quad (11)$$

$$Q_a^f = \sum_{n=1}^{n=N} \sum_{m=1}^{m=S} I_m \left(\text{grad}_n f_m(\mathbf{r}_1, \dots, \mathbf{r}_N), \frac{\mathbf{r}_n}{|q^a} \right) = 0, \quad (12)$$

$$\mathbf{P}_\alpha^H = \sum_{v=1}^{v=N} \left\{ \sum_{j=1}^{j=N} \sum_{k=1}^{k=K_v} P_{(v,v+j)k}(t) \frac{\left(\frac{\mathbf{r}_{(v,v+j)k}}{|\mathbf{r}_{(v,v+j)k}|}, \frac{\partial \mathbf{r}_v}{\partial q^\alpha} \right) \right\}, \quad (13)$$

$$\mathbf{P}_a^C = \sum_{n=1}^{n=N} \left\{ \sum_{j=1}^{j=N} \sum_{c=1}^{c=C_n} P_{(n,n+j)c}(t) \frac{\left(\frac{\mathbf{r}_{(n,n+j)c}}{|\mathbf{r}_{(n,n+j)c}|}, \frac{\mathbf{r}_n}{|q^a} \right) \right\}, \quad (14)$$

$$Q_\alpha^* = \sum_{v=1}^{v=N} \left(\mathbf{R}_{vT}(t), \frac{\partial \mathbf{r}_v}{\partial q^\alpha} \right). \quad (15)$$

4. Light standard thermo-rheological hereditary element. When standard hereditary element is modified by

two temperatures $T_K(t)$ and $T_M(t)$, which are introduced by thermo-modification of visco-elastic properties by temperature $T_K(t)$, and by thermo-modification of elasto-viscous properties by temperature $T_M(t)$, than constitutive relation between stress and strain state of the thermo-rheological hereditary element is [2]:

$$n\mathbf{R}(t) + P(t) + n\mathbf{R}_M(t) + F_K(t) = nc\mathbf{R}(t) + \mathcal{R}[\rho(t) - \rho_0] \quad (16)$$

in which (see Ref. [1], [5] and [10])

$$F_M(t) = c_M \alpha_M T_M(t), \quad F_K(t) = c_K \alpha_K T_K(t) \quad (17)$$

are thermoelastic forces, and $\rho(t)$ is rheological coordinate, c_M, c_K are coefficients of thermo-elastic rigidity, α_M, α_K are coefficients of thermo-elastic dilatations, n is time of relaxation, and c, \mathcal{R} an instantaneous rigidity and a prolonged one of an element.

Constitutive relation (17) thermo-rheological hereditary element from differential form we can rewrite in two integro-differential form. By using explicit form with respect to the force $P(t)$ we can write:

$$P(t) = c \left[\rho(t) - \rho_0 - \int_0^t [\rho(\tau) - \rho_0] \mathbf{R}(t - \tau) d\tau \right] - F_M(t) +$$

$$+ \frac{c}{c - \mathcal{R}_0} \int_0^t [F_M(\tau) - F_K(\tau)] \mathbf{R}(t - \tau) d\tau \quad (18)$$

in which

$$\mathbf{R}(t - \tau) = \frac{c - \mathcal{R}_0}{nc} e^{-\frac{t-\tau}{n}} \text{ is a kernel of relaxation} \quad (19)$$

5. Light standard piezo- and thermo-rheological hereditary elements. When standard hereditary element is modified by two polarization voltages $U_K(t)$ and $U_M(t)$, which are introduced by piezo-modification of visco-elastic properties of subelement of piezoceramics, by $U_K(t)$ and by piezo-modification of elasto-viscous properties by $U_M(t)$, and thermo-modified by two temperatures $T_K(t)$ and $T_M(t)$, than constitutive relation between stress and strain state of the piezo-rheological hereditary hybrid element is in the form (16) in which (see Ref. [1], [10], and Fig. 3.) are:

$$F_M(t) = c_{UM} \mathbf{a}_{UM} U_M(t) + c_{TM} \mathbf{a}_{TM} T_M(t),$$

$$F_K(t) = c_{UK} \mathbf{a}_{UK} U_K(t) + c_{TK} \mathbf{a}_{TK} T_K(t) \quad (20)$$

are thermoelastic forces, and $\rho(t)$ is rheological coordinate, c_{TM}, c_{TK} are coefficients of thermo-elastic rigidity, α_{TM}, α_{TK} are coefficients of thermo-elastic dilatations, c_{UM}, c_{UK} are coefficients of piezo-elastic rigidity, α_{UM}, α_{UK} are coefficients of piezo-elastic dilatations n is time of relaxation, and c, \tilde{c} an instantaneous rigidity and a prolonged one of an hereditary element.

6. Thermo-rheological oscillator. In Fig. 4. a thermo-rheological oscillator is presented, containing material particle of mass m with one degree of freedom defined by generalized coordinate x , and a standard light thermo-visco-elastic element termomodified by temperature $T(t)$.

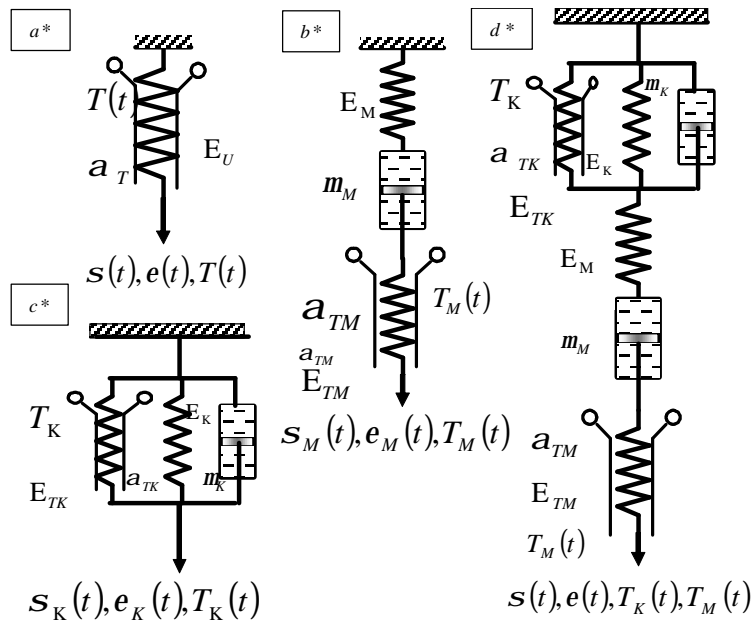


Fig. 3. Schematic notation of a thermo-elastic light element (a*); schematic presentation of the thermo-modified Maxwell elasto viscosic light hereditary element (b*); schematic presentation of the thermo-modified Kelvin-Foight visco-elastic light hereditary element (c*); schematic presentation of the thermo-modified Burgers light hereditary element (d*)

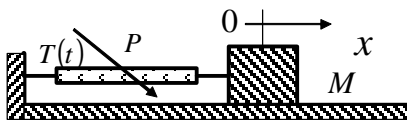


Fig. 4. Thermorheological oscillator

Now, we take into account that this standard light thermo-visco-elastic element termomodified by temperature $T(t)$ is in the dynamic state, and that we didn't neglect thermomodification of the element strain, then we can write that is $\Delta \mathbf{l} = \alpha_T T(t)(\mathbf{l}_0 + x)$, and that the constitutive relation of the thermo-visco-elastic stress-strain state is in the following form:

$$P(t) = c(\Delta \mathbf{l} + x) + b \dot{x} = cx[1 + \alpha_T T(t)] + \alpha_T \mathbf{l}_0 T(t) + b \dot{x} \quad (21)$$

In previous considerations and presented models the rheolinear member $\alpha_T T(t)x(t)$ was neglected as a small member with comparison with other members in constitutive relations. This is acceptable if we research dynamics of the thermorheological oscillator around main resonance motion, but we can show that it is not acceptable if we investigate phenomena around parametric resonance state.

Differential equation of the thermo-rheological oscillator presented in Figure 4. is:

$$m \ddot{x} = -cx[1 + \alpha_T T(t)] - \alpha_T \mathbf{l}_0 T(t) - b \dot{x} \quad (22)$$

or in the form:

$$\ddot{x} + \omega_0^2 [1 + \gamma \mathcal{P}(t)] x + 2\delta \dot{x} = -h_0 \mathcal{P}(t) \quad (23)$$

where

$$\omega_0^2 = \frac{c}{m}, \quad \gamma = \alpha_T T_0, \quad 2\delta = \frac{b}{m},$$

$$h_0 = \frac{\alpha_T \mathbf{l}_0 T_0}{m}, \quad \mathcal{P}(t) = \frac{1}{T_0} T(t). \quad (24)$$

From (23) we can conclude that dynamics of the thermorheological oscillator presented in Figure 4. is described by Mathieu-Hill rheolinear differential equation, if temperature $T(t)$ is deterministic function, and for the case that is stochastic, random function, we have stochastic differential. rheolinear equation.

For solving this equation (23), and for investigation we can use Mthie-Hill functions [23], and known Ince-Strutt stability cart [23]. But we can use different way. We intend to investigate role of the temperature $T(t)$ for dynamic phenomena of the appearance of the main resonance regime, and parametric resonance regime. For that reason we take into account temperature excitation in the form: $\mathcal{P}(t) = \sin(\Omega t + \beta)$ with deterministic constant value frequency Ω and constant deterministaic phase β and Π^* we propose that random, bonded noise temperature excitation

is taken in the following form $\mathcal{P}(t) = \sin\left(\frac{\Omega}{2}t + \sigma B(t) + \beta\right)$

with deterministic constant value frequency $\Omega/2$ and: that $B(t)$ is the standard Wiener process, and β is a random uniformly distributed variable in interval $[0, 2\pi]$, then $\mathcal{P}(t)$

is a stationary process having autocorrelation function and spectral density function (see Ref. [1]):

$$R(\tau) = \frac{1}{2} \mu^2 e^{-\frac{\sigma^2 \tau}{2}} \cos \Omega \tau \quad (25)$$

and

$$S(\omega) = \int_{-\infty}^{+\infty} R(\tau) e^{i\omega \tau} d\tau = \frac{1}{2} \mu^2 \frac{\omega^2 + \Omega^2 + \frac{\sigma^2}{4}}{\left[\left(\omega^2 - \Omega^2 - \frac{\sigma^2}{4} \right)^2 + \sigma^2 \omega^2 \right]}. \quad (26)$$

Stochastic process $|\dot{P}(t)| \leq 1$ is bounded for all values of time t . For first, and for both cases, we take into consideration asymptotic approximation of the amplitude and phase of the dynamic process close around I^* main resonance when $\Omega \approx \omega_0$ and Π^* around parametric resonance when $\Omega \approx 2\omega_0$.

Solution of the basic linear equation $\ddot{x} + \omega_0^2 x = 0$ is $x(t) = R \cos(\omega_0 t + \phi)$ where, amplitude R and phase ϕ are constant.

I^* For this first case with deterministic temperature function $P(t) = \sin(\Omega t + \beta)$, we can introduce that:

$$\Delta_{det} = \omega_0 - \Omega, \text{ and } \Phi(t) = \Omega t + \phi.$$

Now, we take into consideration that solution is in the form $x(t) = R(t) \cos \Phi(t)$, and that first derivative of this solution is same as in case that amplitude R and phase ϕ are constant. By this condition and from equation (23) we obtain two equations in the following forms:

$$\begin{aligned} \dot{R} \cos \Phi - \dot{\Phi} R \sin \Phi &= -\Delta_{det} R \sin \Phi \\ \dot{R} \sin \Phi + R \dot{\Phi} \cos \Phi &= -2\delta R \sin \Phi + \omega_0 \gamma \dot{P}(t) R \cos \Phi + \\ &+ \frac{h_0}{\omega_0} \dot{P}(t) + R \Delta_{det} \cos \Phi. \end{aligned} \quad (27)$$

After solving with respect to \dot{R} and $\dot{\Phi}$ we obtain:

$$\begin{aligned} \dot{R} &= -\delta R (1 - \cos 2\Phi) + \frac{1}{2} \omega_0 \gamma R \dot{P}(t) \sin 2\Phi + \frac{h_0}{\omega_0} \dot{P}(t) \sin \Phi \\ \dot{\Phi} &= \Delta_{det} (1 - \cos 2\Phi) - \delta \sin 2\Phi + \frac{1}{2} \omega_0 \gamma \dot{P}(t) (1 + \cos 2\Phi) + \\ &+ \frac{h_0}{\omega_0 R} \dot{P}(t) \cos \Phi. \end{aligned} \quad (28)$$

Now, we must apply the method of averaging (see Refs. [21], [22] and [13]) to the right-hand sides of equations with respect to the full phase $\Phi(t)$. After averaging the right-hand sides of both equations with respect to the full phase $\Phi(t)$, we obtain the system of averaged differential equations of the first approximation in the following form:

$$\dot{R} = -\delta R + \frac{h_0}{2\omega_0} \cos(\phi - \beta), \quad \dot{\Phi} = \Delta_{det} - \frac{h_0}{2\omega_0 R} \sin(\phi - \beta),$$

$$\Delta_{det} = \omega_0 - \Omega. \quad (29)$$

From averaged system equations (29) we can see that in first approximation parametric temperature excitation is small and negligible, and in first approximation haven't influence to the vibrations closed to the one frequency vibrations with $\Omega \approx \omega_0$.

Π^* For second case with stochastic temperature function

$P(t) = \sin\left(\frac{\Omega}{2} t + \sigma B(t) + \beta\right)$, we can introduce that:

$$\Delta_{st} = \omega_0 - \frac{\Omega}{2}, \text{ and } \Phi(t) = \frac{\Omega}{2} t + \phi.$$

Now, we take into consideration that solution is in the form $x(t) = R(t) \cos \Phi(t)$, and that first derivative of this solution is same as in case that amplitude R and phase ϕ are constant. By this condition and from equation (23) we obtain two equations in the following forms:

$$\begin{aligned} \dot{R} \cos \Phi - \dot{\Phi} R \sin \Phi &= -\Delta_{st} R \sin \Phi \\ \dot{R} \sin \Phi + R \dot{\Phi} \cos \Phi &= -2\delta R \sin \Phi + \omega_0 \gamma \dot{P}(t) R \cos \Phi + \\ &+ \frac{h_0}{\omega_0} \dot{P}(t) + R \Delta_{st} \cos \Phi. \end{aligned} \quad (30)$$

After solving with respect to \dot{R} and $\dot{\Phi}$ we obtain:

$$\begin{aligned} \dot{R} &= -\delta R (1 - \cos 2\Phi) + \frac{1}{2} \omega_0 \gamma R \dot{P}(t) \sin 2\Phi + \frac{h_0}{\omega_0} \dot{P}(t) \sin \Phi \\ \dot{\Phi} &= \Delta_{st} (1 - \cos 2\Phi) - \delta \sin 2\Phi + \frac{1}{2} \omega_0 \gamma \dot{P}(t) (1 + \cos 2\Phi) + \\ &+ \frac{h_0}{\omega_0 R} \dot{P}(t) \cos \Phi. \end{aligned} \quad (31)$$

Now, we must apply the method of averaging (see Refs. [21], [22] and [13]) to the right-hand sides of equations with respect to the full phase $\Phi(t)$. After averaging the right-hand sides of both equations with respect to the full phase $\Phi(t)$, we obtain the system of averaged differential equations of the first approximation in the following form:

$$\begin{aligned} \dot{R} &= -\delta R + \frac{\omega_0 \gamma R}{2} \cos(\phi - \sigma B(t) - \beta), \\ \dot{\Phi} &= \Delta_{det} - \frac{\omega_0 \gamma}{2} \sin(\phi - \sigma B(t) - \beta), \quad \Delta_{st} = \omega_0 - \frac{\Omega}{2}. \end{aligned} \quad (32)$$

From averaged system equations (32) we can see that in first approximation parametric temperature excitation is main and not neglectable, and in first approximation have main role, and that this influence to the vibrations closed to the one frequency vibrations with $\Omega \approx 2\omega_0$. Comparing obtained results, we can conclude that temperature excitation frequency to the standard thermorheological element have important role for decision which member in differential equation is possible neglect.

The change of variables [1] in the following ways and in the following forms: $\rho(t) = \ln R$, $\vartheta(t) = \phi(t) - \frac{1}{2}\psi$, where $\psi = \sigma B(t) + \beta$ and transforming the averaged system of differential equations (32) into the system of averaged stochastic differential equations Ito-type with respect to the unknown amplitude $R(t)$ and unknown phase $\phi(t)$, results in the following forms:

$$d\rho(t) = \frac{1}{R} \mathfrak{R} dt, \quad d\vartheta(t) = \mathfrak{C}(t) dt - \frac{1}{2} d\psi \quad (33)$$

or

$$d\rho(t) = -\delta dt - \frac{1}{2} \omega_0 \gamma R \cos 2\vartheta dt, \\ d\vartheta_s^k(t) = \Delta_{st} dt - \frac{1}{2} \omega_0 \gamma \sin 2\vartheta_s^k dt - \frac{1}{2} \sigma dB(t). \quad (34)$$

The Lyapunov exponent of system mode process λ , (see Ref. [1]) may be introduced by using the time mode and which by making use of the averaged equations become:

$$\lambda = \lim_{t \rightarrow \infty} \frac{1}{2t} \ln \left\{ [x(t)]^2 + \frac{1}{\omega_0^2} [\mathfrak{R}(t)]^2 \right\} = \lim_{t \rightarrow \infty} \frac{1}{t} \rho(t) \quad (35)$$

Now, we can use Lyapunov exponent as a measure of the average exponential growth of the amplitude $R(t)$ of process.

The Lyapunov exponent λ is a deterministic number with probability one (w. p. 1) for the system given by averaged equations. Solutions of the averaged differential equations depend on initial values.

In order to calculate the expression and values for Lyapunov exponent, it is necessary to integrate both sides of first stochastic differential equation of the system, so that, from expression for Lyapunov exponent λ , we can write the following expression:

$$\lambda = -\delta - \frac{1}{2} \omega_0 \gamma R E \left[\cos 2\vartheta_s^2 \right] \quad \text{with probability 1.} \quad (36)$$

Using equations and results by Ariaratnam [1] and Stratonovich (1967) [24] and also, previous stochastic differential equation (34), the values of the mathematical

expectation $E \left[\cos 2\vartheta_s^k \right]$, for the process $\vartheta(t) = \phi(t) - \frac{1}{2}\psi$,

and $\psi = \sigma B(t) + \gamma$ is found to be in the form of expressions:

$$E \left[\cos 2\vartheta_s^k \right] = \mathbf{F} \left(\frac{\omega_0 \gamma R}{\sigma^2}, \frac{4\Delta_{st}}{\sigma^2} \right), \quad (37)$$

where

$$\mathbf{F}(z, q) = \frac{1}{2} \left[\frac{\mathbf{I}_{1+iq}(z)}{\mathbf{I}_{iq}(z)} + \frac{\mathbf{I}_{1-iq}(z)}{\mathbf{I}_{-iq}(z)} \right]. \quad (38)$$

Hence, using previous expression for the Lyapunov exponents λ , with the previously cited result, we can write the following expression in the form:

$$\lambda = -\delta - \frac{1}{2} \omega_0 \gamma R \mathbf{F} \left(\frac{\omega_0 \gamma R}{\sigma^2}, \frac{4\Delta_{st}}{\sigma^2} \right) \quad \text{with probability 1.} \quad (39)$$

By using previous expression for the Lyapunov exponent λ in the forms of expressions (39) with probability 1 for evaluation of the stability or instability of the vibration process of the thermorheological oscillator, we must find the maximal value of the Lyapunov exponent, and determine kinetic parameters of the system such that this Lyapunov exponent is with negative values. Also, we can consider the case when $\Delta_{st} = \omega_{(s)k} - \Omega/2$ is equal to zero. For detail see [1].

7. Thermo-rheological coupled pendulums. In Fig. 5 a thermorheological system, containing two coupled pendulums (see Refs. [23], [2]), is presented. We take into consideration two coupled mathematical pendulums, both with material particles of mass m , with length \mathbf{l} and with two degrees of freedom defined by generalized coordinates φ_1 and φ_2 , and a standard light thermo-visco-elastic element termomodified by temperature $T(t)$, coupling pendulum at distance \mathbf{l} .

Now, we take into account that this standard light thermo-visco-elastic element termomodified by temperature $T(t)$ is in the dynamic state, and that we didn't neglect thermomodification of the element strain, then we can write that is $\Delta \mathbf{l} = \alpha_T T(t) (\mathbf{l}_0 + x)$, and that the constitutive relation of the thermo-visco-elastic stress-strain state is in the following form:

$$P(t) = c \left[\Delta \mathbf{l} + \mathbf{l} (\varphi_2 - \varphi_1) \right] + b \mathbf{l} (\mathfrak{C}_2 - \mathfrak{C}_1) = \\ = c \mathbf{l} (\varphi_2 - \varphi_1) \left[1 + \alpha_T T(t) \right] + \alpha_T \mathbf{l}_0 T(t) + b \mathbf{l} (\mathfrak{C}_2 - \mathfrak{C}_1) \quad (40)$$

same type as in previous part.

Differential equations of the thermo-rheological coupled pendulums presented in Figure 5. are:

$$m \mathbf{l}^2 \ddot{\varphi}_1 = c \mathbf{l}^2 (\varphi_2 - \varphi_1) \left[1 + \alpha_T T(t) \right] + \alpha_T \mathbf{l}_0 \mathbf{l} T(t) + \\ + b \mathbf{l}^2 (\mathfrak{C}_2 - \mathfrak{C}_1) - mg \mathbf{l} \varphi_1, \\ m \mathbf{l}^2 \ddot{\varphi}_2 = -c \mathbf{l}^2 (\varphi_2 - \varphi_1) \left[1 + \alpha_T T(t) \right] - \alpha_T \mathbf{l}_0 \mathbf{l} T(t) - \\ - b \mathbf{l}^2 (\mathfrak{C}_2 - \mathfrak{C}_1) - mg \mathbf{l} \varphi_2, \quad (40)$$

or in the form:

$$\mathfrak{C}_1 + \mathfrak{C}_0^2 \varphi_1 + \omega_0^2 (\varphi_1 - \varphi_2) \left[1 + \gamma \mathcal{P}(t) \right] + \\ + 2\delta (\mathfrak{C}_1 - \mathfrak{C}_2) = -h_0 \mathcal{P}(t), \\ \mathfrak{C}_2 + \mathfrak{C}_0^2 \varphi_2 + \omega_0^2 (\varphi_2 - \varphi_1) \left[1 + \gamma \mathcal{P}(t) \right] + \\ + 2\delta (\mathfrak{C}_2 - \mathfrak{C}_1) = h_0 \mathcal{P}(t), \quad (40)$$

where

$$\omega_0^2 = \frac{c}{m}, \quad \mathfrak{C}_0^2 = \frac{g}{\mathbf{l}}, \quad \gamma = \alpha_T T_0, \quad 2\delta = \frac{b}{m},$$

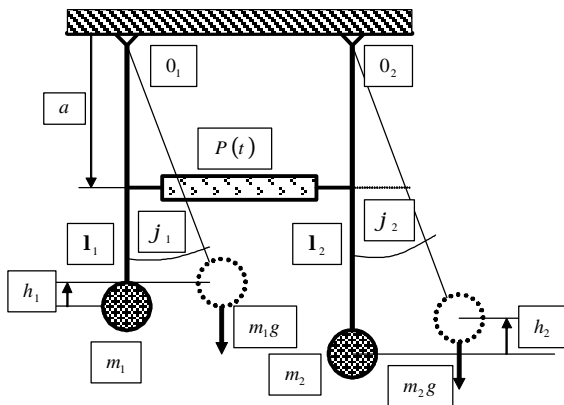


Fig. 5. System with two pendulums interconnected by standard light thermo-modified hereditary element

$$h_0 = \frac{\alpha_T \mathbf{l}_0 T_0}{m \mathbf{l}}, \quad P(t) = \frac{1}{T_0} T(t). \quad (41)$$

Basic linear equations of the previous system are

$$\begin{aligned} \ddot{\varphi}_1 + [\delta_0^2 + \omega_0^2] \varphi_1 - \omega_0^2 \varphi_2 &= 0; \\ \ddot{\varphi}_2 + [\delta_0^2 + \omega_0^2] \varphi_2 - \omega_0^2 \varphi_1 &= 0, \end{aligned} \quad (42)$$

with eigen frequencies:

$$\omega_{1,2}^2 = \delta_0^2 + \omega_0^2 \pm \omega_0^2 = \begin{cases} \delta_0^2 \\ \delta_0^2 + 2\omega_0^2 \end{cases} \quad (43)$$

and with solution:

$$\begin{aligned} \varphi_1(t) &= C_1 \cos(\omega_1 t + \alpha_1) + C_2 \cos(\omega_2 t + \alpha_2); \\ \varphi_2(t) &= C_1 \cos(\omega_1 t + \alpha_1) - C_2 \cos(\omega_2 t + \alpha_2), \end{aligned} \quad (44)$$

where C_1, C_2, α_1 and α_2 are constant.

If we make sum and difference between equations of the previous system (40) of differential equations of the thermo-rheological coupled pendulums presented in Figure 5. we obtain following:

$$\ddot{\varphi}_1 + \ddot{\varphi}_2 + \delta_0^2 (\varphi_1 + \varphi_2) = 0, \quad (45)$$

$$\begin{aligned} \ddot{\varphi}_1 - \ddot{\varphi}_2 + \delta_0^2 (\varphi_1 - \varphi_2) + 2\omega_0^2 (\varphi_1 - \varphi_2) [1 + \gamma P(t)] + \\ + 4\delta (\ddot{\varphi}_1 - \ddot{\varphi}_2) = -2h_0 P(t). \end{aligned} \quad (46)$$

By the change of coordinates in the following ways $\xi_1 = \varphi_1 + \varphi_2$ and $\xi_2 = \varphi_1 - \varphi_2$ previous system take form:

$$\ddot{\xi}_1 + \delta_0^2 \xi_1 = 0, \quad (45^*)$$

$$\ddot{\xi}_2 + \delta_0^2 \xi_2 + 2\omega_0^2 [1 + \gamma P(t)] \xi_2 + 4\delta \ddot{\xi}_2 = -2h_0 P(t). \quad (46^*)$$

First equation represent partial pure harmonic oscillator with frequency $\delta_0^2 = g/l$ of the free vibrations. This case is when both pendulum oscillate with same $\delta_0^2 = g/l$ as

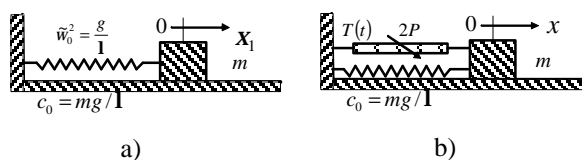


Fig. 6. Thermorheological oscillator 1 (a), thermorheological oscillator (b)

decoupled pendulums, as single mathematical pendulum, and then standard light thermo-visco-elastic element termomodified by temperature $T(t)$ haven't influence to this coordinate composed by sum $\xi_1 = \varphi_1 + \varphi_2$. On this coordinate oscillation are free, without temperature influence.

Second equation represent mathematical description of the thermo-rheological oscillator presented in Figure 6., with parallel coupled two light standard thermo-visco-elastic element termomodified by same temperature $T(t)$ and one elastic spring with rigidity $c_0 = mg/l$ in the dynamic state. By comparison equations (46*) and (23) we can see that these equation is in mathematical analogy, and that it is possible to use all results from part VI. And all conclusion, without calculations.

For this coordinate $\xi_2 = \varphi_1 - \varphi_2$, we can separate two main cases. For both cases, we take into consideration asymptotic approximation of the amplitude and phase of the dynamic process on this coordinate $\xi_2 = \varphi_1 - \varphi_2$ close around

I^* main resonance when $\Omega \approx \sqrt{\delta_0^2 + 2\omega_0^2}$ and II^* around

parametric resonance when $\Omega \approx \frac{1}{2} \sqrt{\delta_0^2 + 2\omega_0^2}$. Then, we can

conclude that on this coordinate is possible to appear under the corresponding kinetic parameters I^* regimes closest to main resonant state, as well as one main resonant state, and II^* regimes closest to parametric resonant state, as well as one resonant state under the thermo-viscoelastic temperature single frequency excitation.

Also we can conclude that basic system (linear-unperturbed system) corresponding to rheological – thermo-rheological perturbed system, have as a main normal coordinates ξ_1 and ξ_2 , and that for thermo-rheological perturbed system results in the forced and parametrically perturbation of the second mode, when first mode is unperturbed.

For solving system differential equations (45) and (46) we take into account the following:

$$\begin{aligned} \varphi_1(t) &= C_1(t) \cos \Phi_1(t) + C_2(t) \cos \Phi_2(t), \\ \varphi_2(t) &= C_1(t) \cos \Phi_1(t) - C_2(t) \cos \Phi_2(t) \end{aligned} \quad (47)$$

where $C_1(t), C_2(t), \Phi_1(t)$ and $\Phi_2(t)$ are unknown function of time t .

For solving system of the equation (40), and for investigation we can use Mthie-Hill function, and known Ince-Strutt stability cart. But we can use different way. We intend to investigate role of the temperature $T(t)$ for dynamic phenomena of the appearance of the main resonance regime, and parametric resonance regime. For that reason we take into account temperature excitation in the form:

I* $\mathcal{P}(t) = \sin(\Omega t + \beta)$ with deterministic constant value frequency Ω and constant deterministic phase β and II* we propose that random, bonded noise temperature excitation is taken in the following form

$$\mathcal{P}(t) = \sin\left(\frac{\Omega}{2}t + \sigma B(t) + \beta\right)$$

with deterministic constant value frequency $\Omega/2$ and: that $B(t)$ is the standard Wiener process, and β is a random uniformly distributed variable in interval $[0, 2\pi]$, then $\mathcal{P}(t)$ is a stationary process having autocorrelation function and spectral density function in the form (25) and (26).

I* For this first case with deterministic temperature function $\mathcal{P}(t) = \sin(\Omega t + \beta)$, we can introduce that:

$$\Delta_{det(i)} = \omega_i - \Omega, \text{ and } \Phi_i(t) = \Omega t + \phi_i, \quad i = 1, 2.$$

Now, we take into consideration that solutions are in the form (47) and that first derivatives of these solutions are same as in case that amplitudes $C_1(t)$, $C_2(t)$ and phases $\phi_1(t)$ and $\phi_2(t)$ are constant. By this condition and from equations (40) we obtain four equations with respect to the derivatives of the amplitudes $\mathcal{C}_1(t)$, $\mathcal{C}_2(t)$ and phases $\mathcal{C}_1(t)$ and $\mathcal{C}_2(t)$. After solving with respect to the derivatives of the amplitudes $\mathcal{C}_1(t)$, $\mathcal{C}_2(t)$ and phases $\mathcal{C}_1(t)$ and $\mathcal{C}_2(t)$ we obtain system of four differential equations first order. Now, we must apply the method of averaging to the right-hand sides of the four equations with respect to the full phases $\Phi_1(t)$ and $\Phi_2(t)$. After averaging the right-hand sides of all of the four equations with respect to the full phases $\Phi_1(t)$ and $\Phi_2(t)$, we obtain the system of averaged differential equations of the first approximation in the following form:

$$\begin{aligned} \mathcal{C}_1 &= 0, \quad \mathcal{C}_1 = \Delta_{st(1)}, \quad \Delta_{det(1)} = \omega_1 - \Omega, \\ \mathcal{C}_2 &= -2\delta C_2 + \frac{h_0}{\omega_0} \cos(\phi_2 - \beta), \\ \mathcal{C}_2 &= \Delta_{det(2)} - \frac{h_0}{\omega_0 C_2} \sin(\phi_2 - \beta), \\ \Delta_{det(2)} &= \omega_2 - \Omega. \end{aligned} \quad (48)$$

From averaged system equations (48) we can see that in first approximation parametric temperature excitation is small and neglectable, and in first approximation haven't influence to the vibrations closed to the one frequency vibrations with $\Omega \approx \omega_1$ and $\Omega \approx \omega_2$.

II* For second case with stochastic temperature function

$$\mathcal{P}(t) = \sin\left(\frac{\Omega}{2}t + \sigma B(t) + \beta\right), \text{ we can introduce that:}$$

$$\Delta_{st(i)} = \omega_i - \frac{\Omega}{2}, \text{ and } \Phi_i(t) = \frac{\Omega}{2}t + \phi_i.$$

Now, we take into consideration that solutions are in the form (47) and that first derivatives of these solutions are same as in case that amplitudes $C_1(t)$, $C_2(t)$ and phases $\phi_1(t)$ and $\phi_2(t)$ are constant. By this condition and from equations (40) we obtain four equations with respect to the

derivatives of the amplitudes $\mathcal{C}_1(t)$, $\mathcal{C}_2(t)$ and phases $\mathcal{C}_1(t)$ and $\mathcal{C}_2(t)$. After solving with respect to the derivatives of the amplitudes $\mathcal{C}_1(t)$, $\mathcal{C}_2(t)$ and phases $\mathcal{C}_1(t)$ and $\mathcal{C}_2(t)$ we obtain system of four differential equations first order. Now, we must apply the method of averaging to the right-hand sides of the four equations with respect to the full phases $\Phi_1(t)$ and $\Phi_2(t)$. After averaging the right-hand sides of all of the four equations with respect to the full phases $\Phi_1(t)$ and $\Phi_2(t)$, we obtain the system of averaged differential equations of the first approximation in the following form:

$$\begin{aligned} \mathcal{C}_1 &= 0, \quad \mathcal{C}_1 = \Delta_{st(1)}, \quad \Delta_{st(1)} = \omega_1 - \frac{\Omega}{2}, \\ \mathcal{C}_2 &= -2\delta C_2 + \omega_2 \gamma C_2 \cos(\phi_2 - \sigma B(t) - \beta), \\ \mathcal{C}_2 &= \Delta_{st(2)} - \omega_2 \gamma \sin(\phi_2 - \sigma B(t) - \beta), \\ \Delta_{st(1)} &= \omega_2 - \frac{\Omega}{2}. \end{aligned} \quad (50)$$

From averaged system equations (32) we can see that in first approximation parametric temperature excitation is main and not neglectable, and in first approximation have main role, and that this influence to the vibrations closed to the one

frequency vibrations with $\Omega \approx 2\omega_2$ or $\Omega \approx \frac{1}{2}\sqrt{\omega_0^2 + 2\omega_0^2}$

8. Concluding remarks. By using two examples: a thermorheological oscillator and a thermorheological coupled pendulums, we show and proof that standard light thermorheological element with temperature excitation is source of possible appearance not only forced oscillations, but also and parametric oscillations with possible unstable dynamics, and appearance forced resonante state, as well as parametric resonante state under the correspondnig kinetic parameters of the system, as well as of external temperature excitation frequency. Also in the system with more than one degree of freedom is possible appearance of dynamical absorbtion on one or more modes of basic linear (unperturbed system) when one of the modes is unperturbed under the temperature excitation.

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К. (Стевановић) Хедрих
Динаміка гібридних систем з терморелогічно спадковими елементами

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У доповіді наведені результати досліджень автора в області динаміки гібридних систем. Аналізуються п'єзо- та терморелогічні моделі. Подані приклади застосування розроблених методів для розв'язання інженерних задач.