UDK 621

MATHEMATICAL MODEL OF POWER TRANSMISSION SYSTEM OF A CAR EQUIPPED WITH AUTOMATIC CLUTCH

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The paper presents a short description of power transmission system of passenger car adapted for disabled people. The car was equipped, among other things, with an automatic clutch, which enables automatic coupling and uncoupling. The mathematical model of power transmission system was presented. The particular attention was paid on mathematical model of a friction clutch. The presented model was used to estimate the value of regulate parameter of automatic clutch in order to minimize a pick-up during start moving. Experimental verification of presented model was made. The run of moment acting on a drive shaft, getting from out-door experiment and computer simulation, ware compared in time domain.

car, transnission, automatic cluych, mathematical model

1. Introduction. Constructing the vehicle suitable for disabled people in majority of cases consists on equipping the cars, produced in series, in cover equipment enabling to drive the car by people with disability. The construction of this equipment should ensure a proper safety level and the drive comfort [4]. Such equipments should not make the driving properties of the car worse after being installed. One of the basic equipment installed in the vehicle for disabled is the automatically controlled clutch, working without of the necessity to press the clutch lever. This self acting clutch should make the starting the car and shifting the gear comfortable without the increased frictional wear of clutch plate. As showed the preliminary research, the speed of clutching in should depend, among others, on the speed of pressing on the acceleration lever. This paper presents the mathematic model of passenger car powertrain with frictional clutch. This model allows to investigate the response of the powertrain to a specify way of steering the clutch.

2. Model of powertrain with frictional clutch. The driveline is a fundamental part of a vehicle. In the field of dynamics modelling of vehicle powertrain a lot of research has been carried out recently, and different model are used, depending on the purpose of the work [1 — 4].

The present work will show the model of a passenger car powertrain enabling the analysis of the way of steering a dry clutch on longitudinal dynamics of a car, particularly in the process of start moving of a car. The model does not include the vibrations of high frequency and noise which appears in powertrain system. When modelling the power transmission system, the main attention should be paid to modelling the frictional clutch, whose task is not only to transmit the drive moment but also dumping of torsional vibrations which are transmitted from the engine to powertrain [2]. In order to adequately control the clutch, a model that can accurately predict the response of the driveline to a specified clutch input is needed [3]. For this model the various driveline components

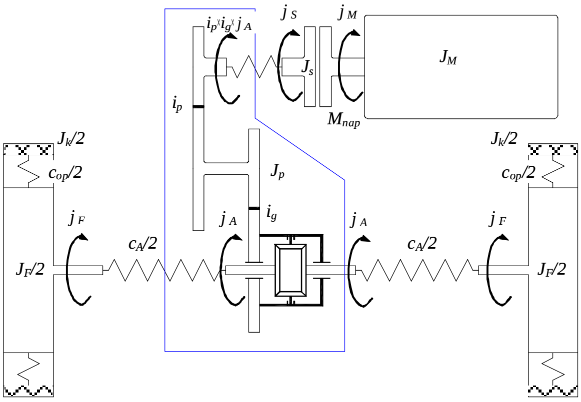

Fig. 1. Powertrain model of passenger car

need to be specified. These include: the engine, the clutch, gearbox, the differential gear, and the wheels. Also external forces acting on the vehicle and driveline need to be included in the analysis. First, a clear understanding of clutch operation is necessary.

The structure of the powertrain model is shown in fig. 1.

When the clutch works in slip, the equations of motion of powertrain can be derived as:

$$\begin{aligned} J_M \ddot{\varphi}_M &= M_{nap} - M_K; \\ J_S \ddot{\varphi}_S &= M_K - M_S; \\ J_p \ddot{\varphi}_A &= i_p i_g M_S - M_A; \\ J_F \ddot{\varphi}_F &= M_A - M_F; \\ J_K \ddot{\varphi}_K &= M_F - M_{op}, \end{aligned} \tag{1}$$

where: J_M — engine moment of inertia, J_S — friction plate moment of inertia, J_p — gear box and final drive moment of inertia, J_F — wheel rim moment of inertia, J_K — tyre moment of inertia, j_M — engine angular displacement, j_S — friction plate angular displacement, j_A — axle shaft angular displacement, j_F — wheel rim angular displacement, j_K — tyre angular displacement, i_p — gearbox ratio, i_g — final drive ratio, M_{nap} — engine torque (see), M_K — torque transferred by the slipping clutch, M_{op} — tyre moment, F_{x1} — tyre longitudinal force, F_{z1} — normal force of driving axle, R_d — tyre radius.

$$\begin{aligned} M_S &= c_S\left(\varphi_S - i_p i_g \varphi_A\right); \\ M_A &= c_A\left(\varphi_A - \varphi_F\right); \\ M_F &= c_{op}\left(\varphi_F - \varphi_K\right); \\ M_{op} &= F_{x1} R_d + M_t; \\ M_{t1} &= R_{d1} \cdot F_{z1} \cdot f_t. \end{aligned}$$

When the clutch is engaged the degrees of freedom of the system are reduced, and $\dot{\varphi}_M$ and $\dot{\varphi}_S$ are now equal. The equations of motion of powertrain, in this case, can be derived as:

$$\begin{aligned} \left(J_M + J_S\right)\ddot{\varphi}_M &= M_{nap} - M_S; \\ J_p \ddot{\varphi}_A &= i_p i_g M_S - M_A; \\ J_F \ddot{\varphi}_F &= M_A - M_F; \\ J_K \ddot{\varphi}_K &= M_F - M_{op}. \end{aligned}$$

Equations describing driving system were completed with the equation of car motion in longitudinal direction . The arrangement of forces affecting a car is shown on fig. 2.

$$m_N \ddot{x} = F_{x1} - F_{x2} - F_{AR},$$

where: m_N — vehicle mass, F_{AR} — air resistance force, F_{x2} — longitudinal force of rear wheels;

$$F_{AR} = c_x A_F \frac{\rho_p \dot{x}^2}{2};$$

c_x — drag coefficient, A_F — frontal reference area, r_p — air density.

To determine longitudinal force affecting the driving wheels the linear model of tyre was used. Dependence of friction coefficient in longitudinal slip is shown on fig. 3. The longitudinal force F_{x1} and F_{x2} may be determined through the dependence:

$$F_{x1} = \mu\left(s_1\right) F_{z1};\ \ F_{x2} = F_{z2} f_t$$

where: f_t — rolling resistance coefficient, s_1 — longitudinal wheel slip, for driven wheel slip may be evaluated through:

$$s_1 = 1 - \frac{\dot{x}}{R_d \dot{\varphi}_K}.$$

2.1. Engine. The engine torque is assumed to be a quadratic function of the rotational speed of the crank shaft and is proportional to the throttle plate position:

$$M_{nap} = \eta_p \left(M_{max} - \left(\frac{M_{max} - M_N}{\omega_N - \omega_M} \right)^2 \left(\dot{\varphi}_M - \omega_M\right)^2 \right),$$

where: h_p — throttle plate position signal $\eta_p \in \langle 0,1 \rangle$, M_{max} — maximum engine torque, M_N — engine torque for angular velocity corresponding to maximum power, ω_M — angular

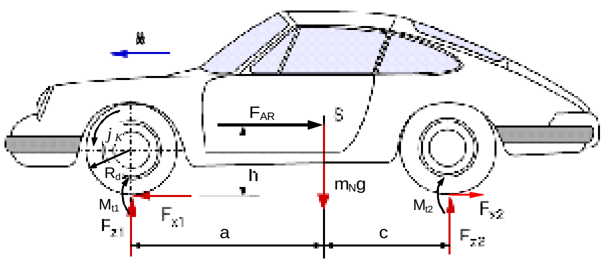

Fig. 2. Car with external forces

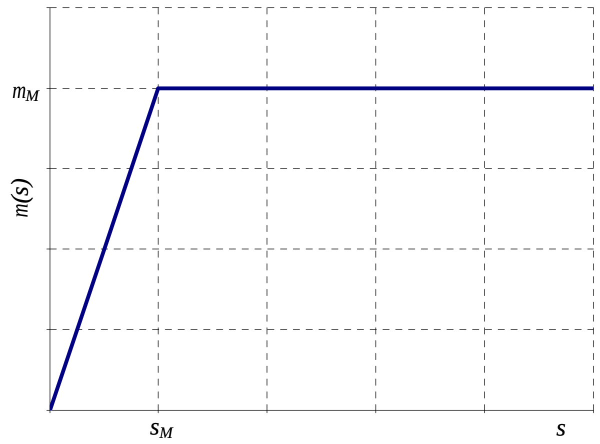

Fig. 3. Dependence of friction coefficient m in the function of longitudinal wheel slip *s*

velocity corresponding to maximum torque, w_N — angular velocity corresponding to maximum power.

2.2. Dry frictional clutch model. The clutch is modelled as a system of plates pressed together via a normal force F_n. The normal force is a function of the position of clutch fork. The dependence of normal force in the function of clutch fork position for a selected clutch may be determined by experiment on a special research stand.

When the clutch is sticking, the engine degree of freedom is rigidly coupled to the clutch disk at the friction interface. The two differential equations of the engine and the clutch, can he reduce to a single differential equation. The sticking of the clutch sustains as long as the torque transmitted through clutch remains below the maximally transmittable torque M_{Kmax}.

It is assumed that the moment transferred by the slipping clutch is a function of the normal force F_n, pressing together a clutch plates, according to the dependence:

$$M_K = sign(\omega_w)\, nF_n(\alpha_K) R_S \mu(\omega_w)$$

where: n — number of friction interfaces, R_S — active radius of the clutch plates:

$$R_S = \frac{2\left(R_z^3 - R_w^3\right)}{3\left(R_z^2 - R_w^2\right)};$$

w_w — relative angular velocity of clutch plates, is determined regarding:

$$\omega_w = (\dot{\phi}_M - \dot{\phi}_S);$$

$\mu(\omega_w)$ — velocity dependent friction coefficient, is determined regarding:

$$\mu(\omega_w) = \mu_0 e^{b|\omega_w|},$$

where: μ_0 — friction coefficient for $\omega_w = 0$, b — constant.

3. Experimental verification of presented model. In order to verification of the presented model the outdoor experiments was carried on, while vehicle start moving, for selected regulation parameters of automatically controlled clutch [5]. The following parameters were measured: throttle plate position angle (fi_op), clutch fork position angle (fi_s), drive shaft torque (MA_1 and MA_2), engine angular velocity (omega_M), driven wheel angular velocity (omega_k1 and omega_k2), longitudinal velocity (v_x), longitudinal acceleration (a_x).

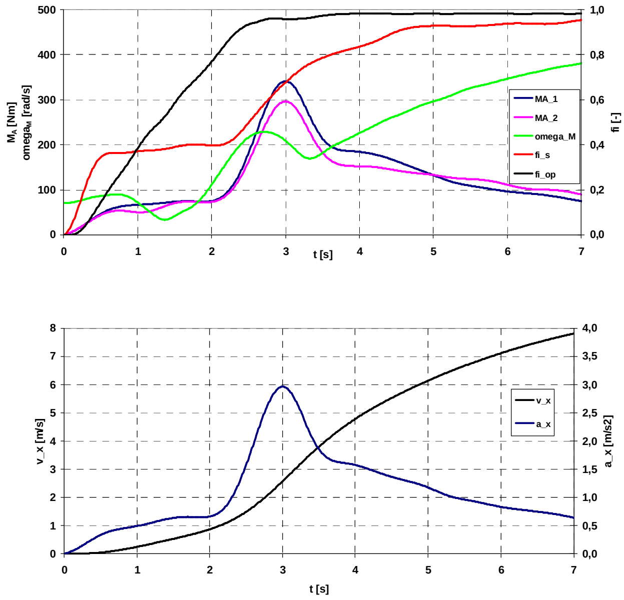

Fig. 4. The selected result of experimental investigation of start moving vehicle

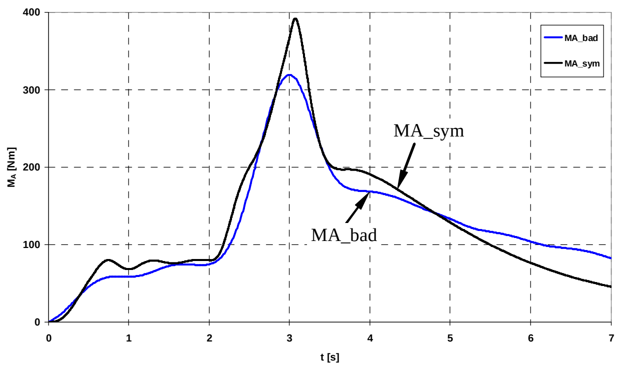

Fig. 5. Drive shaft torque obtained from computer simulations (MA_sym) and experiments (MA_bad)

In fig. 4 the results of experimental research for the selected test of start moving car can be seen.

The obtained results was used to experimental verification of prepared powertrain model of passenger car. As the input data to simulation, the throttle plate position angle and the clutch fork position angle obtained from outdoor experiment was apply. The average torque affecting the left and right drive shaft obtained from experiments and computer simulation was compared in time domain. The selected result of experimental verification of the model is shown on fig. 4. The obtained results allow to maintain that the prepared model describes with satisfactory precision the phenomena occurring in the driving system of a research car.

4. Conclusions. 1. The prepared model of a powertrain system provides good agreement of the computer simulation results and experimental result.

2. When vehicle starts to move there are significant resonances of drive shaft torque, which gives dynamic load of powertrain and deterioration of the driving comfort. Regarding this, it is necessary to have a suitable steering the clutch in order to eliminate these disadvantageous phenomena.

3. The prepared model may be applied to determine steering parameters of self acting clutch to provide optimum loads of the powertrain and driving comfort during vehicle start moving.

4. As the simulation results show, steering the work of clutch should depend, among other, on the run of engine driving torque.

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Математична модель силової трансмісії автомобіля, оснащеного автоматичним зчепленням

Краківська політехніка, Краків, Польща

Наведені характеристики трансмісії пасажирського авто для неповносправних. Овто оснащене автоматичним зчепленням. Розроблена математична модель такої трансмісії і досліджено динаміку автомобіля з метою мінімізації зривання під час рушання з місця. Проведена експериментальна перевірка математичної моделі і підтверджена її адекватність реальним процесам, що відбуваються у трансмісії.

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