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**ABOUT OF SOME VARIANTS OF THE “PYTHAGOREAN”  
DECISIONS AND OTHER HIGHER-ORDER EQUATIONS  
(ELEMENTARY ASPECT)**

**О НЕКОТОРЫХ ВАРИАНТАХ РЕШЕНИЙ “ПИФАГОРОВЫХ”  
И ДРУГИХ БОЛЕЕ ВЫСОКОГО ПОРЯДКА УРАВНЕНИЙ  
(ЭЛЕМЕНТАРНЫЙ АСПЕКТ)**

**Annotation.** Are given in Section 1 the theorem and its proof, complementing the classical formulation of the ABC conjecture, and in Chapter 2 addressed the issue of communication with the elliptic curve Frey’s “Great” Fermat’s theorem.

**Аннотация.** Приведены в гл. 1 теорема и ее доказательство, дополняющая классическую формулировку ABC-гипотезы, и в гл. 2 рассмотрен вопрос о связи эллиптической кривой Фрея с ВТФ.

**Chapter 1**

**Theorem.** “With this  $\varepsilon = 1$ , and the constant  $K(\varepsilon) = 1$ , in which for any three relatively prime positive integers  $a, b, c$ , such that  $a^2 + b^2 = c^2$ , can be performed in an infinite the number of cases for the corresponding  $a = m^2 - n^2, b = 2mn, c = m^2 + n^2$  inequality  $c < rad(abc)^{1+\varepsilon}$  and  $c > rad(abc)^{1+\varepsilon}$ .” (1)

(The classic formulation:

“For every  $\varepsilon > 0$  there exists a constant  $K(\varepsilon) > 0$ , in which for any three mutually prime integers  $a, b, c$ , such that  $a + b = c$ , the inequality  $\max(|a|, |b|, |c|) \leq K(\varepsilon) \cdot rad(abc)^{1+\varepsilon}$ ”).

**Evidence**

**§ 1**

1.1. Obtained, for example, the following equations for:

- 1)  $c^2 > K(1) \cdot rad(abc)^{1+\varepsilon}$   
 $m=4 \ n=3 \ 7^2 + 24^2 = 5^4 \ 5^4 = 625 > K(1) \cdot rad(7 \cdot 24 \cdot 25)^{1+\varepsilon} = rad(2.3.5.7) = 210$
- $m=5 \ n=4 \ 9^2 + 40^2 = 41^2 = 1681 > rad(2.3.5.41) = 1230$
- $m=8 \ n=16 \ 3^2 + 16^2 = 65^2 = 4225 > rad(2.3.5.7.13) = 2730$
- $m=11 \ n=211 \ 7^2 + 44^2 = 125^2 = 15625 > rad(2.3.5.11.13) = 4290$
- $m=19 \ n=829 \ 7^2 + 304^2 = 42^2 = 180625 > rad(2.3.5.11.17.19) = 106590$

$m=24 \ n=752$

$7^2 + 336^2 = 5^8 = 625^2 = 390625 > rad(2.3.5.7.17.31) = 110670$

$2)c^2 < K(1) \cdot rad(abc)^{1+\varepsilon}$

$m=2 \ n=1 \ 3^2 + 4^2 = 5^2 < rad(2.3.5) = 30$

$m=3 \ n=2 \ 5^2 + 12^2 = 13^2 = 169 < rad(2.3.5.13) = 390$

$m=4 \ n=11 \ 5^2 + 8^2 = 17^2 = 289 < rad(2.3.5.17) = 510$

$m=5 \ n=22 \ 1^2 + 20^2 = 2$

$9^2 = 841 < rad(2.3.5.7.29) = 6090$

$m=6 \ n=13$

$5^2 + 12^2 = 37^2 = 1369 < rad(2.3.5.7.37) = 7770$

$m=7 \ n=24$

$5^2 + 28^2 = 53^2 = 2809 < rad(2.3.5.7.53) = 11130$

1.2. **Statement.** “Arbitrary Pythagoras’ equation

$a_1^2 + b_1^2 = c_1^2$ , where

$a_1 = m_1^2 - n_1^2, b_1 = 2m_1n_1, c_1 = m_1^2 + n_1^2, (a_1, b_1, c_1) = 1$  — relatively prime), a source receiving “exceptional” triples such that  $c_\infty^2 = c_1^{2^\infty} > rad(a_\infty b_\infty c_\infty)$  in equation  $a_\infty^2 + b_\infty^2 = c_\infty^2$  ( $\infty = 1, 2, 3, \dots$ ), since some “ $\infty$ ”, unless  $m_\infty = a_{\infty-1}, n_\infty = b_{\infty-1}$ , and so recursively to infinity”.

**Evidence.** 1) Always at some point in the infinite series  $2^1, 2^2, 2^3, \dots, 2^x, \dots$  there is " $\infty$ " "such that  $2^x > c_1$ , as  $b_x$  remains in any" " $\infty$ " "constant factor  $2^x \cdot (m_1^2 - n_1^2)$  ( $m_1, n_1$  — arbitrary coprime positive integers). Obviously,  $c_1^{2^{x-1}} > a_x$  and  $c_1^{2^{x-1}} > b_x$ . Hence  $c_x^2 = c_1^{2^x} > \text{rad}(a_x b_x)$ .

2) The number of "Pythagorean" numbers is infinite.

3) Thus, fully proved the theorem (1).

**Examples.**  $m_1=2, n_1=1$

$$3^2 + 4^2 = 5^2 < \text{rad}(3 \cdot 2 \cdot 5) = 30$$

$$m_2=3, n_2=4, 7^2 + 24^2 = 5^4 > \text{rad}(7 \cdot 2 \cdot 3 \cdot 5) = 210, \text{ Etc.}$$

1.3. Using the principle set out in the "Approval", it is applicable to arbitrary solvable in positive integers a three-term equations for countless "exceptional" triples, leading them to the "Pythagorean" mind.

Examples, 1

$$3^2 + 7^3 = 2^9, \text{ rad}(13 \cdot 7 \cdot 2) = 182 < 512, m=7, n=13$$

$$a=7^3 - 13^2 = 174, b=2 \cdot 7 \cdot \frac{3}{2} \cdot 13 = 26 \cdot 7 \cdot \frac{3}{2},$$

$$c = 7^3 + 13^2 = 2^9, 17 \cdot 4^2 + 26^2 \cdot 7^3 = 2^{18}$$

$$8 \cdot 7^2 + 13^2 \cdot 7^3 = 2^{16}, \text{ rad}(29 \cdot 3 \cdot 2 \cdot 13 \cdot 7) = 15834 < 65536,$$

Etc.

### Chapter 2

#### On the question of the relationship of the elliptic curve Frey with "Great" Fermat's theorem (Elementary aspect)

**Preface.** Interest in the title problem is caused by the following considerations:

1) Take, for example, "Pythagoras' equation, all of which are relatively prime solutions are defined by  $A = a^2 - b^2$  and  $B = 2ab$ . But if we choose  $A \neq a^2 - b^2$  and  $B \neq 2ab$  as a hypothetical "right" solution of this equation, then perhaps it will be possible to prove that, in this case, "Pythagoras' equation does not exist. But it really does not exist for the selected hypothetically "true" solutions.

2) The equation  $A^N + B^N = C^N$  and the equation of the elliptic curve Frey (as will be shown below for the proposed options to solve them) are not compatible.

3) Therefore, it seems, it does not look quite convincing relationship between the equation of an elliptic curve Frey Farm and the corresponding equation.

### § 1

#### Consider the following equation:

1)  $A^N + B^N = C^N$  (2), where  $A^N, B^N$  — theoretical-ly "correct" solution of equation (2) in natural numbers

( $A, B$ ) = 1,  $N$ , corresponding to the general the equation  $x^N + y^N = z^N$  (1).

2)  $y^2 + (x - A^N)(x + B^N) = y^2 + x^2 - (A^N - B^N)x - A^N \cdot B^N = 0$  (3). Hence, the proposed version of the solu-

tion of equation (3) obtained by  $A^N > B^N, x = A^N - B^N, y^2 = A^N \cdot B^N$ , ie. when  $N = 2k$  — even (option assumptions) and  $y = |A^k B^k|$ . If (3) — elliptical Frey, it exists.

3)  $y^2 = x^3 - (A^N - B^N)x^2 + A^N B^N$  (4).

If (4) — elliptic curve Frey, it exists at  $x = A^N - B^N, y = A^k B^k$  and  $N = 2k$  — even. Equations (4) and (3) are compatible.

4)  $y^2 = x^3 + (A^N - B^N)x^2 - A^N B^N$  (5). Clearly, (5) and (3), (4) is not compatible, but they are not compatible with (1).

5) Let  $a = x^3, b = (A^N - B^N)x^2 - A^N B^N$ . Then,  $a^2 + b^2 \pm 2ab = (a \pm b)^2 = \{x^3 \pm [(A^N - B^N)x^2 - A^N \cdot B^N]\}^2$  (6). Equations (4) and (5) are interconnected elements of the "Pythagorean" equation for arbitrary positive values of parameters contained in them.

6) The connection of these equations with equation (1) is not quite convincing.

### § 2

An identity:

$$[x(x^3 \pm 2y^3)]^3 \mp [y(2x^3 \pm y^3)]^3 = (x^3 \pm y^3)(x^3 \mp y^3)^3 \quad (7).$$

If we take into the equation  $a^n + b^n = c^n$  for  $n=3, x^3 + y^3 = z^3$  (8), (7) we get the recurrence equation, giving innumerable hypothetical "true" making (this can not be, since the identity is true for all  $0 < x < \infty, 0 < y < \infty$ ), then the equation (8) solutions in natural numbers for  $n = 3$  have, as you know, can not. It turns out that there is an equation that as if on the one hand, with a hypothetical "true" solutions can not exist, on the other hand, under the same "x" and "y" exists. It should be noted that the equations (7) and (8) are compatible.

• Since the solution of the equation (8) is among the natural

$0 < x < \infty, 0 < y < \infty$  the validation solutions will take longer than the decision itself. Reminds problem Cook-Levin — one of the challenges of the Millennium.

Generally, the identity (7) — the identity of a number of interesting properties. [1].

### Chapter 3

The above two chapters indicates that some established ideas in number theory requires, in our view, a more careful consideration and in some cases correct.

### Literature

1. R. Tint, "The identities of ordinary which is leading to the extraordinary consequences" (elementary aspect), p.2.6, pp 8 / 15–12 / 15. Asian Journal of mathematics and applications in 2013, IDama0031, ISSN2307–7743 <http://scienceasia.asia>.