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MODELING AND OPTIMIZATION OF THE PROCESSES OF MOVEMENT AND ACCELERATION OF THE OVERHEAD CRANE TROLLEY IN THE MODE OF DAMPING UNCONTROLLED LOAD OSCILLATIONS

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Abstract. The paper deals with the modeling and optimization of the processes of movement and acceleration of a bridge crane trolley in the mode of damping uncontrolled oscillations of the load. For the dynamic system of a flat pendulum with vibration damping, which describes the oscillations of a bridge crane load on a flexible rope suspension in a separate vertical plane, it is proposed to use third-order time splines that model the motion and acceleration of the load suspension point in the horizontal direction of the trolley's movement.

To determine the time dependence of the angle of deviation of the crane from the gravitational vertical, it is proposed to use the methods of classical calculus of variations (Euler-Poisson equation), which allow optimizing (minimizing) the value of this angle in the process of accelerating a trolley with a load suspended from the ropes of an overhead crane.

An analytical solution to the problem of damping residual uncontrollable oscillations of the overhead crane load, which usually occur after full acceleration or braking of the load suspension point on the trolley, is obtained. To derive the dependencies, an analytical approach was used to calculate the value of the angle of deviation of the overhead crane's cargo rope from the gravitational vertical, depending on the acceleration and displacement of the load suspension point.

The problem of loosening of a load moved by an overhead crane is considered and solved in a new way that allows to completely avoiding pendulum spatial oscillations of the load on a rope suspension. The mathematical apparatus of linear algebra (Kramer's rule, in particular) is used, which allows us to establish analytically the law of time motion of a rope with a load, the angle of deviation of which from the vertical takes minimum values in the process of acceleration of the cargo trolley.

Keywords: overhead crane, load trajectory, vibration damping, sway, optimization, movement, acceleration, trolley, rope.

Introduction. The movement of loads by overhead cranes with a non-rigid rope suspension of the load causes pendulum oscillations of the load, in which there is an uncontrollable component. Such oscillations significantly increase cycle time, reduce productivity and work safety [1-3]. The danger of collisions of cargo with objects present in the movement zone increases, and damage to the cargo itself, other objects and the overhead crane itself is also probable. In this context, it is advisable to fully extinguish the uncontrolled component of pendulum oscillations of the cargo both during its movement and when the cargo reaches the target point [4]. This is especially relevant when moving hazardous cargo: containers with liquid metal (in metallurgical shops), flammable liquids, etc.

It should be noted that the movement of goods on a flexible rope suspension, which is carried out, in particular, supported on one or two beams of a general-purpose overhead crane, it is advisable to conduct exactly in the mode of damping of the above uncontrolled oscillations of the cargo, and for complete damping of the residual component of the pendulum load oscillations during its movement in a separate plane it is possible to move the point of cargo suspension (cargo cart, or bridge of an overhead crane) by the given analytically time dependence (trajectory). Such a displacement, which is possible with the use of mechatronic control systems of the movement of the cargo cart by overhead cranes, will not only destroy the residual uncontrollable component of vibrations, but also provide a predetermined time dependence of the angle of deviation of the cargo rope by overhead cranes from the vertical.

Analysis of recent research sources and publications. Known methods for synthesizing the trajectory of the suspension point [3, 5-12] have a common disadvantage in the form of a relatively large error in the realization of both the angle of deviation of the cargo rope of the bridge crane from the vertical and the linear coordinates of the moved cargo. Typically, the angle of deviation of the load rope of the overhead crane from the vertical is not monitored and controlled, and the time of movement of the cargo increases.

Normalized controlled deviations of the overhead crane cargo rope from the gravitational vertical when moving loads are necessary. It is impossible to give the load horizontal acceleration without deviations of the load rope from the vertical! But it is advisable that these deviations are of short duration and do not exceed the specified limits.

Analytical dependences of optimal and quasi-optimal control modes of a pendulum system with a movable point of suspension for the problem of the fastest acceleration (braking) with damping of oscillations are known [4]. Restrictions are imposed on the velocity and acceleration of the suspension point. The disadvantage of the known method: only small oscillations of the pendulum around the equilibrium position are considered, the absence of a certain (limiting) value of the angle of deviation of the cargo rope of an overhead crane from the vertical. The optimal control has a relay character: the acceleration of the suspension point takes only marginal values. The dampening of oscillations of the pendulum does not occur during the entire time interval of the work cycle, but only at the end of acceleration or displacement of the system [4].

There are known works on the solution of the speed problem for a nonlinear pendulum (bringing a nonlinear pendulum to a stable equilibrium position) [13, 14]. However, a system with a fixed point of suspension and a moment applied along the angular coordinate of the pendulum are considered. In addition, the known works do not take into account energy dissipation. The equations of a pendulum with a fixed point of suspension are not suitable for describing the problem considered in this paper both in terms of structure and in terms of the parameters included in them.

When solving the problem of damping of oscillations of loads on a rope suspension with a movable upper point of suspension (with a movable base), such modern approaches as the use of proportional-plus-derivative controller and proportional plus derivative controller [5-9], the apparatus of fuzzy logic [3-10] and shaping algorithms [11, 12] find application. They are used to control the trajectory of the upper point of cargo suspension.

In [15, 16], an algorithm has been developed that, given the constraints in the form of maximum velocity and acceleration of the movable cargo suspension point on an overhead crane (cargo truck), synthesizes continuous (step less, non-relay) control of the suspension point by means of frequency-controlled drives for overhead cranes. A number of overhead cranes currently in production are equipped with such drives. The algorithm also takes into account the possibility of large angles of deviation of the load rope from the gravity vertical, which increases the speed of movement and performance of overhead cranes. On the limiting value of the deflection angle of the cargo rope during acceleration and braking of the overhead crane in this case are imposed tight restrictions in the form of its exact achievement. Fulfillment of this condition, according to the authors [15, 16], significantly increases the performance of overhead cranes, taking into account the achievement of maximum values of speed and acceleration of the moving point of suspension. However, this approach and its implementation involve significant difficulties and require complex electronic and mechanical equipment. In addition, in the above algorithm, according to the authors of this study, the cause-and-effect relationships in this technical system are broken, because the initial one is the time dependence of the deflection angle of the rope with the load on the gravity vertical, which is set in advance, and the movement of the cargo cart is regulated by this dependence (the deflection angle of the rope from the vertical in time), although by logic should be the opposite.

This incompatibility is eliminated in this study. In addition, the optimal trajectory of the cargo cart motion is found, by which the acceleration of this motion is minimized in the process of acceleration of the cargo cart to a stable, normative value of the speed of motion. Modeling, analysis and optimization of cargo cart movement by overhead cranes which in its turn causes minimal deviation of cargo from vertical and reduces essentially arising pendulum vibrations in overhead cranes are based on the results obtained in papers [17-20].

Goal and objectives. The aim of the work is to substantiate the physical-mechanical and mathematical models, which adequately describe and optimize the processes of displacement and acceleration of the cargo crane truck, which operates in the mode of damping of uncontrolled residual oscillations of the cargo. The mathematical apparatus of classical calculus of variations, mathematical physics and theory of ordinary differential equations was used to achieve the goal. Determination of regularities of a cargo cart movement is based on a spline approximation in time and a functional dependence of the deflection angle of an overhead crane rope with a cargo on linear algebra apparatus and methods of solving non-uniform differential equations of a high order of a linear type.

Research methods.

1. *Problem formulation*. In the given research the mathematical model of the plane pendulum with the moving point of suspension was accepted that was comprehensively analyzed by the authors [15, 16]. During computational (numerical) experiments on the PC it was established that for small values of deflection angles of ropes with a cargo ($<5^0$) spatial oscillations of the cargo can with a relatively small error represented as a superposition of vibrations in two mutually perpendicular planes. As in the works cited above, for the flat pendulum system, the following designations were adopted: m – weight of the cargo, kg; L – length of the cargo rope with bridge cranes from the movable point of suspension on the load cart (the center of the pulley block of the chain block) to the center of cargo mass, m; b – the viscous friction factor reduced to the angular coordinate, which sets the measure (level) of energy dissipation, N-m-s/rad; $q, \dot{q}, \ddot{q}$ – angle of deviation of a cargo rope by bridge cranes from a gravitational vertical and its first two derivatives in time, respectively, rad, rad/s^2; g=9.81 m/s^2 – free fall acceleration; $\ddot{x}$ – linear acceleration of a cargo suspension point in a horizontal direction of a cargo truck motion, m/s^2.

The system of a plane pendulum in "big" variables (deviations of a cargo rope over $(10...15)^o$ are admitted), is described by known non-linear differential equation of the second order [21-24]:

$$\ddot{q} + (2b/m)\cdot\dot{q} + (g/L)\cdot\sin q + (\ddot{x}/L)\cdot\cos q = 0. \quad (1)$$

During the research, assumptions were made about the constant length of the cargo rope L in the process of moving the cargo, about the step less nature of speed $\dot{x}$ and acceleration $\ddot{x}$ control of the acceleration and deceleration of the load suspension point (cargo bogie of the overhead crane, which provides mechatronic drive speed control system) in the horizontal direction and about too little influence of the mass of the moved cargo and moving links of the crane on the controlled parameters of the speed $\dot{x}$ and acceleration $\ddot{x}$ of the suspension point.

In the following an elementary cycle of movement is considered. The cargo from a resting state on a vertical rope suspension is moved by an overhead crane for some given distance for a certain time period. After moving (at the moment of time τ_p – duration of acceleration) the cargo is also in a state close to rest ($\ddot{x}\big|_{t=\tau_p} = 0; q\big|_{t=\tau_p} = 0; \dot{q}\big|_{t=\tau_p} = 0$), i.e., in the state of no residual oscillations. Thus, in this problem there are terminal conditions for $x(t)$ and $q(t)$.

This study uses a mathematical model of a flat pendulum for small deflection angles of the load rope ($sinq \approx q,\ cosq \approx 1$) to obtain analytical expressions $x(t),\ q(t)$, satisfying certain criteria for the quality of motion of the "cart – wire rope – load" system of an overhead crane. Therefore, after linearization, the second-order differential equation for the above flat pendulum takes the following form [21-24]:

$$\ddot{q}+\ddot{x}/L+(2b/m)\cdot\dot{q}+g\cdot q/L=0. \tag{2}$$

2. *Optimization of the "load bogie – rope – load" system movement mode of an overhead crane during its acceleration* $t\in[0,\tau_p]$.

From equation (2) it is easy to determine the following relation:

$$\ddot{x}=-L\cdot(\ddot{q}+2b\dot{q}/m+g\cdot q/L). \tag{3}$$

Let's set the conditions under which during the acceleration period of the system ($t\in[0,\tau_p]$) the following criterion for the quality of this movement is met:

$$I=\left\{\frac{1}{\tau_p}\cdot\int_0^{\tau_p}(\ddot{x})^2dt\right\}^{1/2}\Rightarrow\min, \tag{4}$$

i.e. minimizing of inertial forces acting in this system during its acceleration period. The necessary condition for realization of criterion (4) is the Euler-Poisson equation:

$$x^{(IV)}=0. \tag{5}$$

The solution of equation (5) is found in the following form:

$$x(t)=A_0+A_1\cdot t+A_2\cdot t^2+A_3\cdot t^3. \tag{6}$$

For finding the unknown constants A_0, A_1, A_2, A_3 we employ the following terminal conditions:

$$x|_{t=0}=0;\dot{x}|_{t=0}=0;\ddot{x}|_{t=0}=a;\ddot{x}|_{t=\tau_p}=0, \tag{7}$$

where: a is the acceleration at the beginning of overhead crane's load carriage acceleration. Substituting (6) into all conditions (7), we obtain:

$$A_0=0;\quad A_1=0;\quad A_2=a/2;\quad A_3=-a/(6\tau_p). \tag{8}$$

Therefore, expression (6) takes the form:

$$x(t)=\frac{a}{2}\cdot t^2-\frac{a}{6\tau_p}\cdot t^3. \tag{9}$$

Further, to analyze the law of motion of the rope with the load $q(t)$ we find:

$$\dddot{x}(t)=-a/\tau_p. \tag{10}$$

Using equation (2), determine the value of $q(t)$ for any point in time $t\in[0,\tau_p]$:

$$q=\frac{L}{g}\cdot\left\{-\ddot{x}\cdot L^{-1}-\ddot{q}-2b\dot{q}/m\right\}. \tag{11}$$

Let's determine, according to the known law $x(t)$ (9),subject to what conditions $q(t)$ the following criterion for the quality of motion of the system is fulfilled:

$$I_1=\left\{\frac{1}{\tau_p}\cdot\int_0^{\tau_p}q^2(t)dt\right\}^{1/2}\Rightarrow\min, \tag{12}$$

that is, the standard deviation of the angle $q(t)$ from the gravitational vertical during the acceleration period of the load carriage according to the law (9) acquires minimum values.

For the implementation of the movement quality criterion I_1 (12) the Euler-Poisson equation is a necessary condition (with $x(t)$ the given time function – (9)):

$$q^{(IV)}-(4b^2/m^2)\cdot\ddot{q}=-(2b\cdot a)/(mL\cdot\tau_p). \tag{13}$$

The characteristic equation for (13) with the right part equal to zero (i.e. for the homogeneous equation that follows from (13)) takes the form:

$$\lambda^4 - (4b^2)/(m^2)\cdot\lambda^2 = 0. \tag{14}$$

Consequently, the roots (14) acquire the following values:

$$\lambda_1 = \lambda_2 = 0;\ \ \lambda_3 = 2b/m;\ \ \lambda_4 = -2b/m. \tag{15}$$

Find the partial solution of (13) in the form:

$$q_{part.} = B\cdot t^2. \tag{16}$$

Substituting (16) into (13), we easily find:

$$B = (a\cdot m)/(4L\cdot b\cdot\tau_p). \tag{17}$$

Therefore, the general solution of (13) can be represented as follows:

$$q(t) = C_0 + C_1\cdot t + C_2\cdot\exp\left\{\frac{2b}{m}\cdot t\right\} + C_3\cdot\exp\left\{-\frac{2b}{m}\cdot t\right\} + \frac{a\cdot m\cdot t^2}{4L\cdot b\cdot\tau_p}. \tag{18}$$

To find the undefined constants C_0, C_1, C_2, C_3 use the following terminal conditions for $q(t)$:

$$q\big|_{t=0};\ \ \dot{q}\big|_{t=0} = 0;\ \ q\big|_{t=\tau_p} = 0;\ \ \dot{q}\big|_{t=\tau_p} = 0. \tag{19}$$

Substituting (18) into the conditions (19), we find a system of linear algebraic equations for finding the constants C_0, C_1, C_2, C_3 of the following kind:

$$\begin{cases} C_0 + C_2 + C_3 = 0; \\ C_1 + \dfrac{2b}{m}\cdot C_2 - \dfrac{2b}{m}\cdot C_3 = 0; \\ C_0 + C_1\cdot\tau_p + C_2\cdot\exp\left\{\dfrac{2b}{m}\cdot\tau_p\right\} + C_3\cdot\exp\left\{-\dfrac{2b}{m}\cdot\tau_p\right\} + \dfrac{a\cdot m\cdot\tau_p}{4L\cdot b} = 0; \\ C_1 + \dfrac{2b}{m}\cdot C_2\cdot\exp\left\{\dfrac{2b}{m}\cdot\tau_p\right\} - \dfrac{2b}{m}\cdot C_3\cdot\exp\left\{-\dfrac{2b}{m}\cdot\tau_p\right\} + \dfrac{a\cdot m}{2L\cdot b} = 0. \end{cases} \tag{20}$$

The linear inhomogeneous system of equations (20) for finding the coefficients C_0, C_1, C_2, C_3 is easily solved using a standard linear algebra procedure (Cramer's rule). The result for the coefficients C_0, C_1, C_2, C_3 in (18) will have:

$$\begin{cases} \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}; a_{11} = 1; a_{21} = 0; a_{31} = 1; a_{41} = 0; a_{12} = 0; a_{22} = 1; a_{32} = \tau_p; a_{42} = 1; a_{13} = 1; \\ a_{23} = \dfrac{2b}{m}; a_{33} = \exp\left\{\dfrac{2b}{m}\cdot\tau_p\right\}; a_{43} = \left(\dfrac{2b}{m}\cdot\tau_p\right); a_{14} = 1; a_{24} = -\dfrac{2b}{m}; a_{34} = \exp\left\{-\dfrac{2b}{m}\cdot\tau_p\right\}; a_{44} = \left(-\dfrac{2b}{m}\right)\cdot \\ \cdot\exp\left\{-\dfrac{2b}{m}\cdot\tau_p\right\}. \\ b_1 = 0; b_2 = 0; b_3 = -\dfrac{am\tau_p}{4bL}; b_4 = -\dfrac{am}{2Lb}. \end{cases} \tag{21}$$

$$C_0 = \frac{\Delta C_0}{\Delta}; C_1 = \frac{\Delta C_1}{\Delta}; C_2 = \frac{\Delta C_2}{\Delta}; C_3 = \frac{\Delta C_3}{\Delta}, \tag{22}$$

where the value $\Delta C_0, \Delta C_1, \Delta C_2, \Delta C_3$ can be found from the relations (23).

$$\begin{cases} \Delta C_0 = \begin{vmatrix} b_1 & a_{12} & a_{13} & a_{14} \\ b_2 & a_{22} & a_{23} & a_{24} \\ b_3 & a_{32} & a_{33} & a_{34} \\ b_4 & a_{42} & a_{43} & a_{44} \end{vmatrix}; \ \Delta C_1 = \begin{vmatrix} a_{11} & b_1 & a_{13} & a_{14} \\ a_{21} & b_2 & a_{23} & a_{24} \\ a_{31} & b_3 & a_{33} & a_{34} \\ a_{41} & b_4 & a_{43} & a_{44} \end{vmatrix}; \ \Delta C_2 = \begin{vmatrix} a_{11} & a_{12} & b_1 & a_{14} \\ a_{21} & a_{22} & b_2 & a_{24} \\ a_{31} & a_{32} & b_3 & a_{34} \\ a_{41} & a_{42} & b_4 & a_{44} \end{vmatrix}; \\ \Delta C_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \\ a_{41} & a_{42} & a_{43} & b_4 \end{vmatrix}. \end{cases} \quad (23)$$

Note that the found analytical expression for $q(t)$ from the relations (18)-(23) satisfies the quality criteria for the movement of the system I (4) and I_1 (12) and virtually eliminates residual uncontrolled pendulum oscillations of the load on the rope by overhead cranes.

Findings:

1. The physical-mechanical and mathematical models of the movement of the system "load bogie – rope – load" of an overhead crane are substantiated.

2. For the above model of motion the laws of motion of a cargo cart and the angle of deviation from the vertical of a rope with the cargo by overhead cranes have been established which minimize these deviations from the gravitational vertical, reduce undesirable uncontrolled residual pendulum oscillations of the cargo by overhead cranes. At the same time, the forces of inertia of the cargo bogie arising in it during its acceleration period are also minimized while it is following to the established parameters of motion (i.e. the movement of the cargo bogie with constant normative speed after the end of the acceleration process).

3. The results received in the work can serve further for specification and improvement of existing engineering methods of calculation of kinematic-force parameters of movement of mechanisms of rise and transportation of cargoes by overhead cranes, and also crane systems of other types as at the stage of designing of similar complex technical systems, and also in modes of their real operation.

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МОДЕЛЮВАННЯ ТА ОПТИМІЗАЦІЯ ПРОЦЕСІВ ПЕРЕМІЩЕННЯ І РОЗГОНУ ВАНТАЖНОГО ВІЗКА МОСТОВОГО КРАНА У РЕЖИМІ ГАСІННЯ НЕКЕРОВАНИХ КОЛИВАНЬ ВАНТАЖУ

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Анотація. У роботі проведене моделювання та оптимізація процесів переміщення і розгону вантажного візка мостового крана у режимі гасіння некерованих коливань вантажу. Для динамічної системи плоского маятника із затуханням коливань, яка описує коливання вантажу мостового крана на гнучкому канатному підвісі у окремій вертикальній площині, запропоновано використовувати сплайни по часу третього порядку, які моделюють рух та прискорення точки підвісу вантажу у горизонтальному напрямку руху вантажного візка.

Для з'ясування часової залежності кута відхилення вантажного крана від гравітаційної вертикалі запропоновано використати методи класичного варіаційного числення (рівняння Ейлера-Пуассона), котрі дозволяють оптимізувати (мінімізувати) величину вказаного кута у процесі розгону вантажного візка з вантажем, підвішеним на канаті мостового крана.

Отриманий аналітичний розв'язок задачі гасіння залишкових некерованих коливань вантажу мостового крана, які зазвичай виникають після повного розгону чи гальмування точки підвісу вантажу на вантажному візку. Для виведення залежностей використаний аналітичний підхід задля розрахунку величини кута відхилення вантажного канату мостового крана від гравітаційної вертикалі у залежності від прискорення і переміщення точки підвісу вантажу.

Розглянута проблема розхитування вантажу, який переміщується мостовим краном, вирішена новим способом, котрий дозволяє повністю уникнути маятникових просторових коливань вантажу на канатному підвісі. При цьому використаний математичний апарат лінійної алгебри (правило Крамера, зокрема), який дозволяє аналітичним шляхом встановити закон руху у часі канату з вантажем, кут відхилення котрого від вертикалі приймає мінімальні значення у процесі розгону вантажного візка.

Ключові слова: мостовий кран, траєкторія вантажу, гасіння коливань, розхитування, оптимізація, переміщення, розгін, вантажний візок, канат.

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