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Differential «stiff-string» model for drag and torque simulation in deviated bore-holes

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Поставлено проблему комп'ютерного моделювання поведінки бурильних колон у надглибоких, похилих і горизонтальних свердловинах і прогнозування можливого виникнення нештатних ситуацій під час буріння та спуско-підіймальних операцій. Показано, що з математичної точки зору всі вони, як правило, є сингулярно збуреними і тому важко піддаються теоретичному аналізу. Запропоновано алгоритми, що дають можливість подолати ці труднощі. Створене математичне забезпечення для аналізу цих явищ допомагає будувати траєкторію свердловини, що забезпечує найменші значення сил опору переміщенню бурильної колони, і підбирати найменш енергоємні та безпечні режими буріння.

Ключові слова: похила свердловина, “soft-string” модель, “stiff-string” модель, геометричні викривлення, спіральні недосконалості, безпечні режими, комп'ютерне моделювання.

Поставлена проблема компьютерного моделирования поведения бурильных колонн в сверхглубоких, наклонных и горизонтальных скважинах и прогнозирования возможного возникновения нештатных ситуаций при бурении и спуско-подъёмных операциях. Показано, что с математической точки зрения, все они, как правило, являются сингулярно возмущёнными и поэтому трудно поддаются теоретическому анализу. Предложены алгоритмы, позволяющие преодолеть эти трудности. Созданное математическое обеспечение для анализа этих явлений позволяет строить траекторию скважины, обеспечивающую наименьшие значения сил сопротивления перемещению бурильной колонны, и подбирать наименее энергоемкие и безопасные режимы бурения.

Ключевые слова: наклонная скважина, “soft-string” модель, “stiff-string” модель, геометрические искажения, спиральные несовершенства, безопасные режимы, компьютерное моделирование.

The problem of computer simulation behavior of drill strings in hyper deep vertical, inclined and horizontal wells is stated with the aim of forecasting the possible initiation of emergency situations during drilling operations. The questions of stability and post-critical non-linear deforming of the drill strings are considered. It is shown that all of them are singularly perturbed from the mathematical point of view and because of this, they are poorly amenable to theoretical analysis. The algorithms allowing surmounting these difficulties are proposed. The software for study of these phenomena is elaborated. The elaborated software permits one to construct its trajectory securing the smallest values of resistance forces and to choose the least energy-consuming and safe regimes of drilling.

Key words: deviated bore-hole, “soft-string” model, “stiff-string” model, geometrical distortions, spiral imperfections, safe regimes, computer simulation.

One of the most important technological components of hydrocarbon production is drilling of deep vertical and inclined wells. But their drivage is fraught with great technological difficulties caused by permanent balance change of the forces of gravity, resistance (friction), inertia, and elasticity acting on the drill string and its bit as well of the moments of thees forces [1, 2]. Therefore, in drilling curvilinear bore-holes the percentage of emergency situations and casualties continues to remain high. The above-listed factors make the problem of computer simulation of the processes of movement and elastic bending of drill strings in the channels of curvilinear bore-holes to be notably urgent.

Early in the development of techniques for deep curvilinear bore-hole drilling, as a rule, the bore-holes of the simplest configurations with small distortions of axial lines were consi-dered. In these bore-holes, the bending strains of drill strings are not essential and they can be neglected. In these cases the DS was simulated by an absolutely flexible thread and an approach called the minimum curvature method («soft-string» model) was adopted [3, 4].

With advances in techniques of the curvilinear bore-hole drilling, they became to acquire more complicated geometry, their depths enlarged, and their horizontal distances from vertical began to exceed 12 km. It is reasonable that the process of the well drilling, the energy consumption related to it, and the emergency situations attendant on it turn out to be by far most sensitive to the mistakes allowed at the stage of the well design and inevitable in its drivage [5].

In the studies [6, 7], a new approach based on the use of the theory of flexible curvilinear rods («stiff–string» model) was proposed for simulation of these processes. It was demonstrated that the resistance forces could be essentially reduced and even the sticking effects could be avoided through combination of the axial and rotary movements of the DS during performance of the raising-lowering operations.

In this paper, the problem of creation of techniques for computer simulation of technological means minimizing ener-gy consumption in movement of a DS in a curvilinear bore-hole with geometrical imperfections is stated. It can be used at the stage of the well geometry design and prescription of requirements on its accuracy, in the design of regimes of the DS drivage and their realization, as well as in the execution of the operation of the DS ridding of the sticking.

«Stiff– string» drag/torque model

To simulate the mechanical phenomena attending the drilling process and to select its most favorable characteristics, use the mathematical model based on the theory of curvilinear flexible rods [6, 7]. Assume that the DS axially moves with velocity $\dot{w}$ and rotates with angular velocity ω in a bore-hole channel with known geometry. In the *Oxyz* coordinate system its axial line *T* is prescribed in the form

$$\rho = \rho(s), \tag{1}$$

where ρ is the vector function $\rho = xi + yj + zk$ describing the axial line of the bore-hole; *i*, *j*, *k* are the unit vectors; *s* is the parameter measured by the length of the axial line of the DS.

Consider that the axial lines of the DS and bore-hole coincide. Then, it is possible to determine all geometrical characteristics of the bent DS including the curvature radius *R*, curvature k_R, and torsion k_T:

$$k_R = 1/R = |\rho''|, \; k_T = \frac{\rho'(\rho'' \times \rho''')}{\rho'' \rho''}. \tag{2}$$

With the use of correlations (1), (2), the equalities are deduced which determine the components of the gravity force vector

$$\begin{gathered} f_n^{gr} = -ag(\gamma_t - \gamma_l)n_z, \; f_b^{gr} = -ag(\gamma_t - \gamma_l)b_z, \\ f_\tau^{gr} = ag(\gamma_T - \gamma_l)\tau_z \end{gathered} \tag{3}$$

and components of contact force between the DS and the bore–hole

$$\begin{gathered} f_n^c = -k_R F_\tau + k_R k_T M_\tau - A k_R k_T^2 + A\frac{d^2 k_R}{ds^2} - f_n^{gr}, \\ f_b^c = k_R m_\tau^{fr} + 2A k_T \frac{dk_R}{ds} - M_\tau \frac{dk_R}{ds} + A k_R \frac{dk_T}{ds} - f_b^{gr}. \end{gathered} \tag{4}$$

Here F_τ and M_τ are the internal axial force and torque; *A* is the bending stiffness of the DS.

Assume that the DS is being dragged with velocity $\dot{w}$ and rotates with angular velocity ω simultaneously. Then, the conditions of the Coulombic friction are realized between the surfaces of the DS and bore-hole.

As a consequence, the stress-strain state of the DS in its axial movement with the $\dot{w}$ velocity and simultaneous rotation with angular velocity ω can be described by two first order differential equations

$$\frac{dF_\tau}{ds} = k_{Rn} - f_\tau^{gr} - f_\tau^{fr}, \; \frac{dM_\tau}{ds} = -m_\tau^{fr}, \tag{5}$$

where the functions k_R, f^{gr}_τ are known and the required friction functions f^{gr}_τ, m^{fr}_τ, and F_n can be determined with the help of Eqs. (3)–(5).

Solution of Eqs. (5) under given initial conditions allows one to construct functions $F_\tau(s)$, $M_\tau(s)$ and to find the values of axial force $F_\tau(S_i)$ and torque $M_\tau(S_i)$ which should be applied at the point of the DS suspension for performance of the required technologic regime for chosen ratio η between the velocities of rotary (ω*r*) and axial ($\dot{w}$) motions. As our theoretical results demonstrated, in doing so, the value of parameter η influenced essentially on the possibility to perform the designed operation [6, 7]. Thus, enlargement of the η value entails increase of $M_\tau(S_i)$ and decrease of $F_\tau(S_i)$ and vice versa, it is possible to enlarge $F_\tau(S_i)$ reducing η.

The marked opportunity of change $F_\tau(S_i)$, $M_\tau(S_i)$ and control over technological process through the η parameter varying at its every step permits not only to predict and avoid emergency situations and failures, but also to select the least energy consuming regimes of the drilling processes. Indeed, assume that the real geometry and admitted imperfections measured through the use of logging sonde are known. Then the work *dW* performed in the elementary segment *ds* can be represented in the form

$$dW = F_\tau(S)\,ds + M_\tau(S)\,d\varphi. \tag{6}$$

After solving system (5) under different η with allowance made for the measured geometrical imperfections, choose the η_i value providing minimal value dW_i for the considered regime and length S_i. At this case, the ω_i value is selected issuing from the technical data and possibilities of the engine device of the drilling rig.

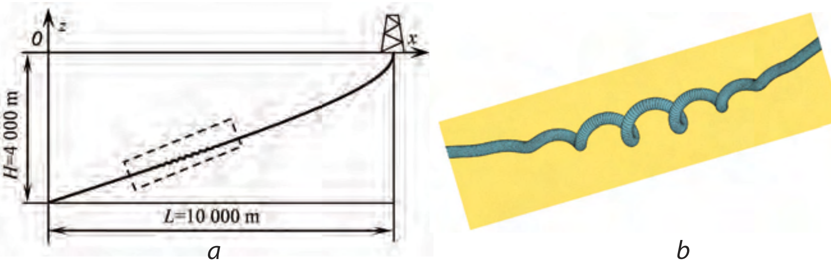
Notice that the proposed approach can be used both at the stage of the well design and its drilling. In the first case, the hypothetical parameters of the bore-hole trajectory and imperfections can be varied in wide limits reasoning from the technological possibilities of their realization. In the second case, the real magnitudes of these parameters are prescribed, which are found in the result of electrical logging analysis.

Energy saving regime of raising a drill string in a well with spiral imperfections

Let the well trajectory be originally designed as a part of an ideal hyperbolic curve, as shown in Fig.1, *a*. But in reality, it is not possible to ensure exactly the conceived outline of the bore-hole axis and usually some distortions are introduced into its geometry. The localized 3D spiral is one of the common shapes met in practice. Its pitch $\lambda = 2\pi/k$ is determined by the wave number *k* and is considered to be constant (Fig. 1, *b*), its radius $h(s)$ has maximal value h_c at the point $s = s_c$.

Through the use of the elaborated approach the energy sa-ving operation of a DS raising in a well with localized spiral imperfections is considered.

In design of a bore-hole geometry and techniques of its drivage, one is forced to take into account a great variety of determining factors including the horizontal distance from the vertical (exceeding 12 km), depth (down to 4 km), well outline (in our case, hyperbola), and possible occurrence of geometrical imperfections. Below, the case is treated when the design hyperbole is preset in the



a *b*

Fig. 1. Design scheme of an inclined well with geometric imperfections

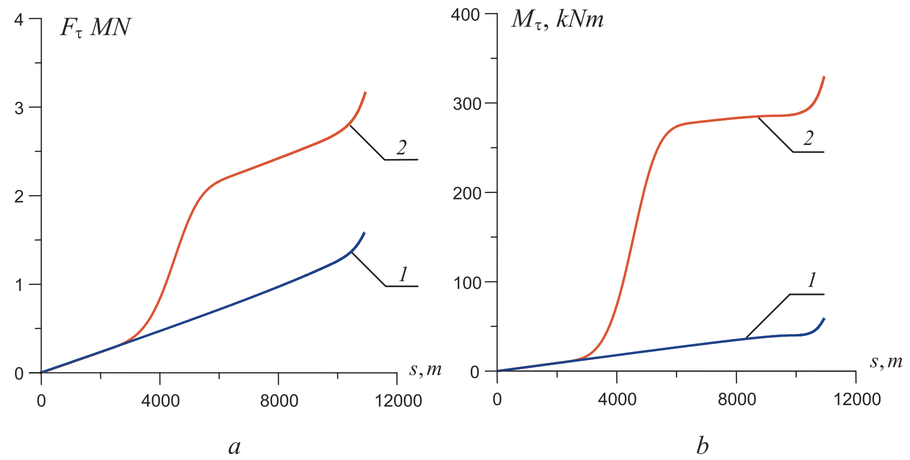


Fig. 2. The functions of axial force F_τ (*a*) and torque M_τ (*b*) along axial line of the drill string (curves *1* – for the ideal well, curves *2* – for the well with geometric imperfections)

domain $3\pi/2 \leq \vartheta \leq 2\pi$ and determined by the parameters $H = 4000$m, $L = 10000$m, and $\varepsilon = 3$. The set of spiral imperfections of amplitude $h_c = 2$m and pitch $\lambda = 109$m is superimposed on the hyperbola trajectory. Its center is assumed to be located at $S_c = 3S/8$. Here S is the well length.

It should be pointed out that these imperfections are not discernible at the real scale shown in Fig.1. So, their images are presented at larger scale in this figure.

In our analysis, the following typical factors were chosen: $r = 0{,}08415$m, thickness of the drill string tube $\delta = 0{,}01$m, $E = 2{,}1 \cdot 10^{11}$Pa, $\gamma_t = 7850$kg/m^3, $\gamma_l = 1500$kg/m^3, friction coefficient $\mu = 0{,}2$.

Firstly, consider the influence of the taken imperfections on the functions of axial force $F_\tau(s)$ (Fig. 2, *a*) and torque $M_\tau(s)$ (Fig. 2, *b*) at $\eta = 1$. In Fig. 2, the functions $F_\tau(s)$ and $M_\tau(s)$ are represented. Curves 1 in these diagrams correspond to the design well with ideal geometry, curves 2 represent the case when the bore-hole is distorted.

On the basis of the prescribed geometry, the problem for Eqs. (5) is solved for the chosen length S_i and different values of the ratio $\eta = \omega r/\dot{w}$ between the velocities of the rotary (ωr) and axial ($\dot{w}$) motions. Then, the value η_i is selected which minimizes the elementary work (6). To minimize energy consumption during execution of the raising operation the item-by-item simulation method was used at every stage of the optimization analysis. In order for this modeling to be made, the drill string length S was divided into ten equal segments $\Delta S_i = S/10$ ($i = 1, 2,\dots, 10$) which were detached one after another from the drill string length, imitating the drill string raising operation. In Fig. 3, the diagram of the η_i value change is shown.

It issues from this illustration, that the smallest resistance to the motion of the full length drill string is achieved when $\eta \approx 2{,}033$. But while the drill string is raising, the necessity to rotate the system decreases and when the well segment with imperfections is passed, it becomes possible to raise the drill string without rotation, because $\eta_i \approx 0$.

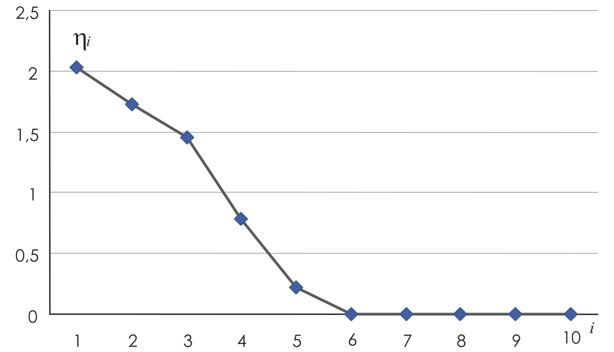


Fig. 3. The diagram of change of the parameter minimizing the energy consumption while raising the drill string

It is necessary to note also that the proposed software allows one to perform computer monitoring of tehnological operations of drilling.

Conclusions

1. The problem of an elastic drill string bending inside a curvilinear channel of a bore-hole is stated for evaluating the elastic, contact, and friction forces attending the drivage operations.

2. It is shown through the use of the elaborated software that the resistance forces impeding the drill string dragging inside the well can be regulated through the combination of its axial and rotary movements. The method for minimization of the energy consumption by the choice of a special ratio between the velocities of these movements is proposed.

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