

# DARK ENERGY AND KEY PHYSICAL PARAMETERS OF CLUSTERS OF GALAXIES

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**ABSTRACT.** We discuss the physics of clusters of galaxies embedded in the cosmic dark energy background and show that 1) the halo cut-off radius of a cluster like the Virgo cluster is practically, if not exactly, equal to the zero-gravity radius at which the dark matter gravity is balanced by the dark energy antigravity; 2) the halo averaged density is equal to two densities of dark energy; 3) the halo edge (cut-off) density is the dark energy density with a numerical factor of the unity order slightly depending on the halo profile.

## 1. Introduction

Dark energy treated as  $\Lambda$ -vacuum produces antigravity, and at the present cosmic epoch, the antigravity is stronger than the gravity of matter for the global universe considered as a whole. May the dynamical effects of dark energy be strong on smaller scales as well? Local dynamical effects of dark energy were first recognized by Chernin et al. (2000); the studies of the Local Group of galaxies and the expansion outflow of dwarf galaxies around it revealed that the antigravity may dominate over the gravity at distance of  $\simeq 1 - 3$  Mpc from the barycenter of the group (Chernin, 2001, 2008; Baryshev et al., 2001; Karachentsev et al., 2003; Byrd et al., 2007; Teerikorpi, 2008, 2010).

Further studies (Chernin et al., 2010) show that the nearest rich cluster of galaxies, the Virgo cluster and the Virgo-centric expansion outflow around form a system which is a scale-up version of the Local Group with its expanding environment. It proves that the matter gravity dominates in the volume of the cluster, while the dark energy antigravity is stronger than the matter gravity in the Virgo-centric outflow at the distances of  $\simeq 10 - 30$  Mpc from the cluster center. On both scales of 1 and 10 Mpc, the key physical parameter of the system is its "zero-gravity radius" which is the distance (from the system center) where the matter

gravity and the dark energy antigravity balance each other exactly. The gravitationally bound system can exist only within the sphere of this radius; outside the sphere the flow dynamics is controlled mostly by the dark energy antigravity.

The static solutions for polytropic configurations, and their dynamic stability, in presence of the cosmological constant, have been investigated numerically by Bisnovatyi-Kogan et al. (2011).

## 2. Dark energy on the cluster scale

Dark energy is a relativistic fluid and its description is based on General Relativity. Nevertheless it may be treated in terms of the Newtonian mechanics, if the force field it produces is weak in the ordinary accepted sense. The Newtonian treatment borrows from General Relativity the major result: the effective gravitating density of a uniform medium is given by the sum

$$\rho_{eff} = \rho + 3p. \quad (1)$$

With its equation of state  $p_\Lambda = -\rho_\Lambda$ , dark energy has the negative effective gravitating density:

$$\rho_{\Lambda eff} = \rho_\Lambda + 3p_\Lambda = -2\rho_\Lambda < 0. \quad (2)$$

It is because of this negative value that dark energy produces antigravity.

With this result, one may introduce "Einstein's law of universal antigravity" which says that two bodies imbedded in the dark energy background undergo repulsion from each other with the force which is proportional to the distance  $r$  between them:

$$F_E(r) = -\frac{4\pi G}{3}\rho_{\Lambda eff}r^3/r^2 = +\frac{8\pi G}{3}\rho_\Lambda r. \quad (3)$$

(This is the force for the unit mass of the body.) Let us consider a spherical mass  $M$  of non-relativistic matter embedded in the dark energy background. A test particle at the distance  $r$  from the mass center (and out of

the mass) has the radial acceleration in the reference frame related to the mass center:

$$F(r) = F_N(r) + F_E(r) = -G\frac{M}{r^2} + \frac{8\pi G}{3}\rho_\Lambda r. \quad (4)$$

(Note that (4) comes directly from the Schwazcschild-de Sitter spacetime in the weak field approximation (see, for instance, Chernin et al., 2006); (4) may also be used for the mass interior; in this case  $M = M(r)$  in (4), see Bisnovatyi-Kogan et al., 2011).

It is seen from (4) that the total force  $F$  and the acceleration are both zero at the distance

$$r = R_\Lambda = \left[\frac{M}{\frac{8\pi}{3}\rho_\Lambda}\right]^{1/3}. \quad (5)$$

Here  $R_\Lambda$  is the zero-gravity radius (Chernin et al., 2000; Chernin, 2001, 2008). The gravity dominates at distances  $r < R_\Lambda$ , the antigravity is stronger than the gravity at  $r > R_\Lambda$ . It implies that the gravitationally bound system with the mass  $M$  can exist only within the zero-gravity sphere of the radius  $R_\Lambda$ . Clusters of galaxies are known as the largest gravitationally bound systems. Thus, the zero-gravity radius is an absolute upper limit for the radial size  $R$  of a static cluster:

$$R < R_\Lambda = \left[\frac{M}{\frac{8\pi}{3}\rho_\Lambda}\right]^{1/3}. \quad (6)$$

The total mass of the Virgo cluster estimated by Karachentsev & Nasonova (2010) with the "zero-velocity" method is  $M = (6.3 \pm 2.0) \times 10^{14} M_\odot$ . This result agrees well with the earlier virial mass of the cluster  $M_{vir} = 6 \times 10^{14} M_\odot$  estimated by Hoffman and Salpeter (1980). Teerikorpi et al. (1992), Ekholm et al. (1999, 2000) found that the real cluster mass  $M$  might be from 1 to 2 the virial mass:  $M = (0.6 - 1.2) \times 10^{15} M_\odot$ . Tully and Mohayaee (2004) obtained the Virgo cluster mass  $M = 1.2 \times 10^{15} M_\odot$ . Taking for an estimate the total mass of the Virgo cluster (dark matter and baryons)  $M = (0.6 - 1.2) \times 10^{15} M_\odot$  and the cosmological dark energy density  $\rho_\Lambda$  (see Sec.1), one finds the zero-gravity radius of the Virgo cluster:

$$R_\Lambda = (9 - 11) Mpc \simeq 10 \text{ Mpc}. \quad (7)$$

### 3. Cluster overall parameters

The data of the Hubble diagram for the Virgo system (Nasonova & Karachentsev, 2010) enable us to obtain another approximate empirical equality:

$$\left[\frac{RV^2}{GM}\right]_{Virgo} \simeq 1. \quad (8)$$

This dimensionless combination of the overall physical parameters of the cluster resembles the traditional

virial relation. However its physical sense is different from that of the virial theorem, which has a form for the polytropic star with the polytropic index  $n$

$$\varepsilon_g = -\frac{n}{3}\varepsilon_g + \frac{2n}{3}\varepsilon_\Lambda,$$

see Bisnovatyi-Kogan et al. (2011).

The equation (8) does not assume any kind of equilibrium state of the system; it does not assume either any special relation between the kinetic and potential energies of the system. It assumes only that the system is embedded in the dark energy background and it is gravitationally bound.

The data on the Local Group (Karachentsev et al., 2009; Chernin et al., 2009) show in combination with (8) that

$$\left[\frac{RV^2}{GM}\right]_{Virgo} \simeq \left[\frac{RV^2}{GM}\right]_{LG} \simeq 1. \quad (9)$$

Here we use for the Local Group the following empirical data:  $R \simeq 1$  Mpc,  $M \simeq 10^{12} M_\odot$ ,  $V \simeq 70$  km/s which give the radius, the total mass and the velocity dispersion in the Local Group, correspondingly (Karachentsev et al., 2009).

Assuming that the bound inner component (the cluster) of the Virgo system has a zero-gravity radius  $R_\Lambda$  (6), we obtain from the empirical relation (9) that

$$V^2 \simeq \left(\frac{8\pi}{3}\right)^{1/3} GM^{2/3} \rho_\Lambda^{1/3}. \quad (10)$$

As we see, the velocity dispersion in the gravitationally bound system depends only on its mass and the universal dark energy density. The relation (10) enables one to estimate the matter mass of a cluster, if its velocity dispersion is measured in observations:

$$M \simeq G^{-3/2} \left[\frac{8\pi}{3}\rho_\Lambda\right]^{-1/2} V^3 \simeq 10^{15} \left[\frac{V}{700 \text{ km/s}}\right]^3 M_\odot. \quad (11)$$

On the other hand, the approximate empirical relation (9) may serve as an estimator of the local dark energy density,  $\rho_{loc}$ . Indeed, if the mass of a cluster and its velocity dispersion are independently measured, one has:

$$\rho_{loc} \simeq \frac{3}{8\pi G^3} M^{-2} V^6 \simeq \rho_\Lambda \left[\frac{M}{10^{15} M_\odot}\right]^{-2} \left[\frac{V}{700 \text{ km/s}}\right]^6, \quad (12)$$

what indicates that the observational data on the Local System and the Virgo System provide strong evidence in favor of the universal value of the dark energy density which is the same on both global and local scales.

### 4. Dark matter halos

Observations and computer simulations indicate – in acceptable agreement with each other – that the

spherically-averaged density profiles of the halos in various clusters are rather regular and reveal a simple dependence on the radial distance. In the simplest case, the halo density profile may be approximated by the isothermal power law:

$$\rho = \rho_1 \left(\frac{r_1}{r}\right)^2, \quad (13)$$

where  $\rho_1, r_1$  are two constants;  $\rho_1 = \rho(r_1)$ . It was demonstrated (for  $\Lambda = 0$ ) that the density profile of (13) may exist in a system of particles moving along circular orbits (Bisnovatyi-Kogan & Zeldovich, 1969) and in systems with almost radial orbits as well (Antonov & Chernin, 1975).

According to the considerations of the section above, the zero-gravity radius of a cluster like the Virgo cluster is roughly (or maybe exactly) equal to the total radius of the halo. If this is the case, we may identify  $r_1$  with  $R = R_\Lambda$ , and  $\rho_1$  is then the dark matter density at the halo's outer edge  $\rho_{edge}$ . With this, we find the total mass of the halo:

$$M = 4\pi \int_0^R \rho r^2 dr = 4\pi \rho_{edge} R_\Lambda^3. \quad (14)$$

On the other hand,  $M = \frac{8\pi}{3} \rho_\Lambda R_\Lambda^3$ ; then (14) gives:

$$\rho_{edge} = \frac{2}{3} \rho_\Lambda. \quad (15)$$

The cut-off density  $\rho_1 = \rho(R_\Lambda)$  proves to be a constant value which does not depend on the total mass or velocity dispersion of the isothermal halo; the density is just the universal dark energy density with the order-of-unity numerical factor.

It follows from (14) that the mean halo density,  $\langle \rho \rangle$ , is given again by the dark energy density, but with another numerical factor:

$$\langle \rho \rangle = 3\rho_{edge} = 2\rho_\Lambda. \quad (16)$$

The last relation is obviously valid for any halo's profile, not only for the isothermal one.

## 5. Conclusions

The antigravity produced by dark energy puts a clear limit to the extension of dark matter halos in clusters: the halo may exist only in the area  $r \leq R_\Lambda$  where the antigravity is weaker than the gravity produced by non-vacuum matter of the cluster. The dark energy density determines the mean matter density of the halo and its edge (cut-off) density. These are the three key physical parameters of clusters. More details see in Bisnovatyi-Kogan & Chernin (2012).

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