## ASTROPHYSICS

# THE KINEMATICS PARAMETERS OF THE GALAXY USING DATA OF MODERN ASTROMETRIC CATALOGUES 

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#### Abstract

Based on the Ogorodnikov-Milne model, we analyze the proper motions of XPM2, UCAC4 and PPMXL stars. To estimate distances to the stars we used the method of statistical parallaxes herewith the random errors of the distance estimations do not exceed $10 \%$. The method of statistial parallaxes was used to estimate the distanes to stars with random errors no larger than $14 \%$. The linear solar veloity relative to the local standard of rest, which is well determined for the loal entroid ( d 150 p ), was used as a referene.

We have established that the model component that describes the rotation of all stars under consideration about the Galactic $Y$ axis differs from zero. For the distant ( $\mathrm{d}<1000 \mathrm{pc}$ ) PPMXL and UCAC4 stars, the mean rotation about the Galactic $Y$ axis has been found to be $M_{13}^{-}=-0.75 \pm 0.04 \mathrm{mas} \mathrm{yr}^{-1}$. As for distances greater than $1 \mathrm{kpc} M_{13}^{-}$derived from the data of only XPM2 catalogue becomes positive and exceeds 0.5 mas $\mathrm{yr}^{-1}$. We interpret this rotation found using the distant stars as a residual rotation of the ICRS/Tycho2 system relative to the inertial reference frame.


Keywords proper motions, reference systems, Galaxy: kinematics and dynamics: late-type.

## 1. Introduction

The method used in this work allows not only to determine kinematic parameters of the Galaxy but also to estimate the inertiality of the proper catalogues from the study of components of the solid-body rotation tensor, which describe the rotation about the $X, Y$ axes in the galactic coordinate system.

Since the Sun is located inside the giant stellargaseous complex known as the Gould belt ( $\approx 500 \mathrm{pc}$ in radius), the involvement of sufficiently distant stars, which would be free of the effects of both the near-solar flows and the local stellar system on the whole, is vital for the reliable application of the method.

The aim of this work is to study kinematic pa-
rameters of the large number of stars depending on their distances. To estimate the latter we applied the method of comparison of statistical parallaxes with the velocity of the Sun (Olling R. at all.: 2003), which is currently known quite reliably (Schönrich R. et al., 2010). $(U, V, W)=\left(\mathbf{1 1 . 1}_{-0.75}^{+0.69}, \mathbf{1 2 . 2 4}_{-0.47}^{+0.47}, \mathbf{7 . 2 5}_{-0.36}^{+0.37}\right)$ $\mathrm{km} \mathrm{s}^{-1}$

## 2. The model

In the present work, we use a rectangular galactic coordinate system with the axes directed in the following way: from the observer towards the galactic center $\left(l=0^{\circ}, b=0^{\circ}\right.$, axis $X$ or axis 1 ), along the galactic rotation $\left(l=90^{\circ}, b=0^{\circ}\right.$, axis $Y$ or axis 2$)$, and towards the Northern pole of the Galaxy ( $b=90^{\circ}$, axis $Z$ or axis 3). Within the Ogorodnikov-Milne model, we use the designations introduced by Clube S. (1972, 1973) and applied in the works by du Mont D. (1977, 1978). As is known, (Ogorodnikov, 1965), when using only the proper motions of stars, one of the diagonal elements of the local deformation tensor remains to be undefined. That is why we determine the differences of the form: $\left(M_{11}^{+}-M_{22}^{+}\right)$and $\left(M_{33}^{+}-M_{22}^{+}\right)$.
The conditional equations are written in the following form:

$$
\begin{align*}
\mu_{l} \cos b= & (1 / r)\left(X_{0} \sin l-Y_{0} \cos l\right) \\
& -M_{32}^{-} \cos l \sin b-M_{13}^{-} \sin l \sin b \\
& +M_{21}^{-} \cos b+M_{12}^{+} \cos 2 l \cos b \\
& -M_{13}^{+} \sin l \sin b+M_{23}^{+} \cos l \sin b \\
& -0.5\left(M_{11}^{+}-M_{22}^{+}\right) \sin 2 l \cos b,  \tag{1}\\
\mu_{b}=\quad & (1 / r)\left(X_{0} \cos l \sin b+Y_{0} \sin l \sin b-Z_{0} \cos b\right) \\
+ & M_{32}^{-} \sin l-M_{13}^{-} \cos l \\
- & 0.5 M_{12}^{+} \sin 2 l \sin 2 b+M_{13}^{+} \cos l \cos 2 b \\
+ & M_{23}^{+} \sin l \cos 2 b \\
- & 0.5\left(M_{11}^{+}-M_{22}^{+}\right) \cos ^{2} l \sin 2 b \\
+ & 0.5\left(M_{33}^{+}-M_{22}^{+}\right) \sin 2 b, \tag{2}
\end{align*}
$$

where $X_{0}, Y_{0}, Z_{0}$ are components of the peculiar velocity of the Sun, $M_{12}^{-}, M_{13}^{-}, M_{23}^{-}$- components of the vector of the solid-body rotation of the small circumsolar vincinity about the corresponding axes. According to the rectangular coordinate system we selected, the following rotations are positive: from axis 1 to axis 2, from axis 2 to axis 3 , and from axis 3 to axis 1 .
$M_{21}^{-}$and $M_{12}^{+}$values (mas yr ${ }^{-1}$ ) are connected with the Oort constants A and $\mathrm{B}\left(\mathrm{km} \mathrm{s}^{-1} \mathrm{kpc}^{-1}\right)$ respectively through the proportionality factor of 4.74. Each of the $M_{12}^{+}, M_{13}^{+}, M_{23}^{+}$values describes deformation of the velocity field in the corresponding plane.

The diagonal terms of the local deformation tensor $M_{11}^{+}, M_{22}^{+}, M_{23}^{+}$describe the general contraction or extension of the whole stellar system. The set of conditional equations (1), (2) involves eleven unknown variables which are found using the least squares method. As can be seen from the equation (1), two pairs of unknown variables $M_{13}^{-}$and $M_{13}^{+}$as well as $M_{32}^{-}$and $M_{23}^{+}$have the same coefficients $\sin l \sin b$ and $\cos l \sin b$, respectively. This has the result that the unknowns are resolved poorly. A quantity $1 / r$ is a parallactic factor, which is put to equal unity in solving the system of equations (1), (2). In this case stars are projected to the unit sphere. With this approach all parameters under determination are proportional to the heliocentric distance of the considered centroid and expressed in the same units as components of proper motions of stars namely in mas $\mathrm{yr}^{-1}$. Applying a such approach allow us to eliminate completely the effect of the distance errors in data analyzed. Indeed, when the method with known distances to the stars is applied left-hand side of (1) and (2) equations have to be multiplied by 4.74 r , while right-hand side - by r , and then the desired unknowns will be distorted by the errors in determining the stellar distances. At present, the reliable distances to individual stars (with errors of $<10 \%$ ) allow the circumsolar vicinity of 100 pc in radius to be analyzed, but this is insufficient for the purposes of the present work.

## 3. Used catalogues

XPM2. The XPM2 astrometric catalogue recently created in the Research Institute of Astronomy of the V. N. Karazin Kharkiv National University but until now unpublished is presently the only bulk one that contains highly accurate proper motions of about one billion stars. There is a full coverage of the sky in the declination range $90^{\circ} \leq \delta \leq 90^{\circ}$. The accuracy of proper motions of faint stars in the XPM2 catalogue is within 3 to 10 mas $\mathrm{yr}^{-1}$. As concerns its own proper motions, the XPM2 catalogue is a realization of the optical reference system independent on HCRF.

UCAC4. The UCAC4 astrometric catalogue (Zacharias, 2013), at present, is a catalogue, which con-
tains highly accurate proper motions of stars up to 16 mag. The sky coverage is full in the declination interval $90^{\circ} \leq \delta \leq 90^{\circ}$. The accuracy of proper motions for faint stars ranges from 3 to 10 mas $\mathrm{yr}^{-1}$. The catalogue expands the system of ICRS/Tycho-2 towards the faint stars.

PPMXL. The PPMXL catalogue contains $910,468,710$ entries, including stars, galaxies, and bogus entries. Of these, $412,410,368$ are in 2MASS, i.e., 2MASS is used to determine proper motions and the J,H,Ks magnitudes are given in the catalogue. The resulting typical individual mean errors of the proper motions range from 4 mas $\mathrm{yr}^{-1}$ to more than $10 \mathrm{mas} \mathrm{yr}^{-1}$ depending on observational history. We used only those stars from the catalogue, for which at least one of the stellar magnitudes $b 1$ or $b 2$ is given. If both of them are presented, the magnitude was accepted to equal $(b 1+b 2) / 2$. Ultimately, a number of stars we used from the PPMXL catalogue, made 750 millions.

## 4. Estimation of the group distances

To obtain the estimates of distances to the stars, we used a statistical method (Olling R. et al., 2003). For the value of the known velocity of the Sun's peculiar motion relative to the LSR values from the work (R. Schonrich R. et al., 2009) wewr adopted. $(U, V, W)=$ $\left(\mathbf{1 1 . 1}{ }_{-0.75}^{+0.69}, \mathbf{1 2 . 2 4} 4_{-0.47}^{+0.47}, \mathbf{7 . 2 5}_{-0.36}^{+0.37}\right) \mathrm{km} \mathrm{s}^{-1}$. The parallax has been calculated from the formulas:

$$
\begin{equation*}
\pi U=4.74 X / U ; \pi W=4.74 Z / W \tag{3}
\end{equation*}
$$

where $X$ and $Y$ are the are the components of the group velocity of stars (in mas $\mathrm{yr}^{-1}$ ) received from the solution of equations (1), (2). Since component $Y$ can be distorted by the asymmetric drift velocity (Dennen et. al, 1998), it was not used to determine the group parallaxes. The distance $d$ is found from the relationship: $d=1 / \pi$. In this case the error of distance can be estimated from the relationship

$$
\begin{equation*}
e_{d}=\left(\frac{e_{\pi}}{\pi}\right) d \tag{4}
\end{equation*}
$$

while the $e_{\pi}$ value for the motion along $Z$ coordinate was estimated in the following way:

$$
\begin{equation*}
e_{\pi}^{2}=4.74\left(e_{W} / W\right)^{2} Z^{2}+4.74\left(e_{Z} / W\right)^{2} \tag{5}
\end{equation*}
$$

A similar relationship can be derived for the motion along the $X$ coordinate.

At first we apply this method to the HIPPARCOS catalogue. Grouping was based on trigonometric parallaxes of the catalogue, herewith $e_{\pi} / \pi<1$. Derived statistical distances to the stars depending on trigonometric ones are presented in Fig. 1. It is seen from the figure that no significant systematic distortions in statistical distances are observed, as well as the fact that
for the distances larger that 200 pc , the errors in determining statistical distances are less that the errors of trigonometric distances. Further, we apply the method to the analysis of stars from the UCAC4, XPM2 and PPMXL catalogues.


Figure 1: Statistical distances of stars depending on trigonometric ones calculated by data of the HIPPARCOS catalogue.

## 5. Results of calculations

The several results of solving (1), (2) equation system derived on the basis of stars of mixed spectral composition from XPM2, UCAC4 and PPMXL catalogues depending on the distance are presented in Fig. 2. We have used virtually all stars from the XPM2 catalogue (about 1 billion), with a constraint imposed on the modulus of a star tangential velocity $\left|\mu_{t}\right|=\sqrt{\left(\mu_{\alpha} \cos \delta\right)^{2}+\left(\mu_{\delta}\right)^{2}}<150$ mas $\mathrm{yr}^{-1}$. All the stars have been grouped according to their magnitudes, with the magnitude interval width of 1 mag in each group. Each of these intervals, in turn, was divided into 1633 Sharlie areas. The resulting data (the mean proper motions) were used to solve the system of equation (1), (2). Thus, the average distance derived using the procedure described above was assigned to the each interval of magnitudes. Random errors of all parameters under determination are $0.05-0.10$ mas $\mathrm{yr}^{-1}$, while those of $\left(M_{11}^{+}-M_{22}^{+}\right)$and $\left(M_{33}^{+}-M_{22}^{+}\right)$are twice as much in the each interval of magnitudes. The procedures for obtaining the data needed to solve the system of equations (1), (2) for XPM2, UCAC4 and PPMXL catalogues are the same.

The distances found and utilized in the work show that the considered stars are located on average outside the boundaries of the local spiral arm (Orion arm), or the Local stellar system. This serves as a reason to consider the rotation about the galactic axis $Y$ found


Figure 2: Kinematic parameters inferred from the proper motions of XPM2, UCAC4 and PPMXL stars vs. distances.
with the use of relatively faint stars, as a residual rotation of the ICRS/Tycho-2 (Hög, 2000) system relative to the extragalactic system of coordinates.

As can be seen from the figures, the dependences of kinematic parameters derived from the data of XPM2 catalogue are provided from the distance of about 600700 pc . It is well seen that the components of rotation tensor for the stars of UCAC4 and PPMXL catalogues are almost coincided with each other within the interval of distances from 0 to 1 kpc and slightly different from those for XPM2 catalogue.

We also show a component of the deformation tensor in the $X Y$ plane for all the three catalogues. As can be seen from Fig. 2, the dependences of parameters $M_{12}^{+}$are very similar in the range from 0 to 1.5 kpc . The dependence is extended by the data from only the XPM2 catalogue at distances greater than 1.5 kpc and these data do not contradict the previous ones. We present in the Fig. 3 the dependence of the angular velocity of the Galaxy rotation obtained from the formula $\omega=\left(M_{21}^{-}-M_{12}^{+}\right)$on the heliocentric distance.


Figure 3: The dependence of the angular velocity of the Galaxy rotation on the heliocentric distance: $\omega=$ $\left(M_{21}^{-}-M_{12}^{+}\right)$

As can be seen from this figure, there are no differences in the behavior of the dependence of $\omega$ on the heliocentric distance to the stars from XPM2 and UCAC4 catalogues up to 1.25 kpc . As for larger distances (only XPM2 catalogue) the behavior of this dependence does not change. The angular rotation velocity derived from the data of the PPMXL catalogue differs from values derived from the data of XPM2 and UCAC4 catalogues by more than 0.5 mas $\mathrm{yr}^{-1}$ at the distance interval from 1 to 1.6 kpc . It was surprising for us that although the PPMXL catalogue contains virtually the same amount of stars as the XPM2 one, the distances derived from statistical parallaxes of PPMXL stars do not exceed 1.6 kpc .

## 6. Conclusions

As a result of the work performed, it was established that for relatively close stars $(d<1000 \mathrm{pc})$ one component of the Ogorodikov-Milne model which describes the rotation about the galactic axis $Y$ and two components which describe deformations in the $Y Z$ and $X Y$ planes, depend on the heliocentric distance. We associate it with the kinematic specialities of the Local group of stars. As for farther stars, in average, the existence of the deformations in $Y Z$ and $X Y$ planes is seen.
Besides, for the distant stars of the Tycho2/UCAC4 and Tycho2/PPMXL catalogue systems, located in the distance $>1000 \mathrm{pc}$ from the Sun, the mean value of the rotation about the galactic axes $Y$ is about +0.30 mas $\mathrm{yr}^{-1}$ that we interpret as the residual rotation of the ICRS/Tycho-2 system relative to the inertial coordinate system.

The angular rotation velocity of the Galaxy can be represented by the linear dependence on the heliocentric distance and change from -5 mas $\mathrm{yr}^{-1}$ to -3 mas $\mathrm{yr}^{-1}$ in the distance range from 0.5 kpc to 2 kpc .

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