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The paper presents a review of more than a century-long history of Kyiv school of the theory of structure, the foundation of which was laid by world-famous scientists V.L. Kirpichov and S.P. Tymoshenko. The birth of the Kyiv scientific school of the Theory of structures is associated in this paper with the establishment at the Kyiv Polytechnic Institute the Strength of Materials Department. It is noted that further formation and development of the theory of structures was facilitated by the creation in 1918 of the Ukrainian Academy of Sciences, the Institute of Mechanics of the NAS of Ukraine, expansion of relevant research in higher education institutions, creation of new academic and sectoral research institutions, most of which is located in Kyiv. The contribution of Kiev scientists to the development of methods for analyzing spatial structures of bar and shell type, their inelastic behavior, as well as dynamics and stability is reflected.

Particular attention is paid to the fundamentally new opportunities for the development of the theory of structures in the era of numerical analysis. The successes of Kiev mechanics in the field of development and improvement of structure analysis numerical methods, such as the finite difference method and various modifications of finite element methods, are emphasized. Kiev engineers and scientists are also known for their developments in the field of design and calculation of modern cable-stayed structures, as well as optimal design. The activities of the scientific school of structural mechanics of the Kyiv National University of Construction and Architecture are also covered in the review.

In the final part of the paper the new issues connected with the justification of calculation models and the analysis of reliability of constructions are considered. Some of this problems are dictated by the demands of practice, in particular those that arose in the process of Chernobyl New Safe Confinement designing.

The publication contains a wide bibliography.

**Keywords:** bar systems, stability, shells, structural mechanics, finite difference method, finite element method, calculation model.

**Foreword.** Theory of structures is complex discipline which largely lost its original essence of the “teachings about the life of structures”<sup>2</sup>. Now it is identified with a number of other disciplines such as strength of materials,

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<sup>1</sup> Dedicated to the 90<sup>th</sup> Anniversary of the Kyiv National University of Construction and Architecture

<sup>2</sup> Such a definition is given in the book by P.A. Velikhov "Theory of engineering structures" (Moscow: Gosstroyizdat, 1924).

structural mechanics, theory of elasticity and theory of plasticity (strength disciplines) and general sections of the engineering analysis of the bearing structures. Many of these disciplines are actively engaged in their own lives, naturally, serving the theory of structures, but often focused on some of the internal problems of general scientific orientation. So, to the theory of structures that is adjacent (but, in our opinion, are not included as part of) to the analysis of mechanical properties of structural materials, as well as such problems of mechanics of solids as the mathematical theory of elasticity, theories of plasticity, thermoelasticity, thermoplastic elasticity, etc. There is no clear boundary, but in this review we will not enter any seriously on “neighbouring territory”.

The emergence of the Kyiv scientific school of the Theory of structures is associated with the establishment in 1899 at the newly organized Kyiv Polytechnic Institute<sup>3</sup> (KPI) the Strength of Materials Department.

Further, the formation and development of the theory of structures was facilitated by the creation in 1918 of the Ukrainian Academy of Sciences, the Institute of Mechanics of the NAS of Ukraine, expansion of relevant research in higher education institutions, creation of new academic and sectoral research institutions. This review presents the substantive side of these studies. The bibliography is quite extensive.

When selecting quoted sources, monographic publications are indicated first. References to articles in periodicals, as a rule, represent specific examples and are not exhaustive lists of sources. The accents in the thematic selection, of course, are determined by the interests of the authors, are open to expansion and do not pretend to be ranked according to the importance of certain problems.

The authors hope that the book will be useful and will evoke pleasant memories for many students of the Kyiv School, who now work fruitfully in many countries of the world.

Studying the history of knowledge helps to familiarize the reader with historically objective evaluations of certain research results and priorities, to realize the sometimes long and extraordinarily thorny path of formation even small and obvious scientific achievements and truths. Equally important, it helps to reflect that atmosphere of deep mutual respect and goodwill which, despite the lengthy and intense discussions, has prevailed and should prevail in the scientific community of modern society.

The authors of the work are graduates of Kyiv Civil Engineering Institute (now Kyiv National University of Construction and Architecture), which celebrates its 90th anniversary in 2020. The authors dedicate this publication to this significant event, as well as the blessed memory of their dear teachers and colleagues.

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<sup>3</sup> Over the 120 years of its existence, the Institute has repeatedly changed its name, in (1934-1944) it was the Kyiv Industrial Institute, today it is the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

## 1. FIRST STEPS

An outstanding scientist and organizer of engineering education, the first Rector of the KPI Professor Emeritus V.L. Kirpichov clearly understood the need for radical reform of higher engineering education in accordance with urgent demands of industry, as well as the development of mechanics in general and structural theory in particular (and he was personally interested in this section of mechanics) in physics and technology direction.

In 1899, a year after the opening of the Institute, V.L. Kirpichov began to lecture at the Strength of materials Department on a wide range of strength disciplines [253, 256, 255] (strength of materials, graphical statics, etc.). His classic work [254] presented the theory of statically indeterminate structures in a very compact and transparent way and completed the period of the theory of structures formation in Russia.

*Kirpichov Viktor Lvovych (1845-1913) was a Privy Councillor and Professor Emeritus.*

*In 1868 he graduated from Mikhailovskaya artillery Academy. From 1869 he started lecturing on strength of materials. In 1873 he trained abroad. He listened the course of experimental and theoretical physics by G.R. Kirchhoff in Heidelberg. Then he got acquainted with the machine-building plants and hydraulic facilities in Germany, Belgium and Switzerland.*

*Later he worked under the direction of Thomson and J.C. Maxwell. In 1876 V.L. Kirpichov became a Professor of the Petersburg technological Institute.*

*In 1885 V.L. Kirpichov was instructed to organize practical technological Institute in Kharkiv. Viktor Lvovych brilliantly fulfilled the order. Under his leadership, the Kharkiv technological Institute quickly gained a high reputation. In early 1898 V.L. Kirpichov organized Kyiv Polytechnic Institute and headed it until 1902.*

*In 1902 he was appointed as a member of the Council of Minister of Finance, and in the spring of 1903 he became the Chairman of the Construction Commission of St. Petersburg Polytechnic Institute. Until the end of his days he lectured on applied mechanics and engineering*



On the initiative of V.L. Kirpichov many talented scientists from different universities of the former Russian Empire was invited to KPI. Among them were the bridge building specialist Professor Yevgeni Oscarovych Paton and specialist on the mechanics of materials and structures Professor Stepan Prokopovych Timoshenko. It was they, together with V.L. Kirpichov, who laid the Foundation of Kyiv scientific School of mechanics.

By the time of moving to Kyiv, Ye.O. Paton published the first volume of his four-volume course "Iron Bridges", as well as the work that played an important role in the theory of structures [318]. In this work, a study was carried out, revealing the conditions under which it was possible to use a hinged design scheme of a truss, nodes of which were not perfect hinges. A notable influence on the development of the theory of structures was made by the studies of Ye.O. Paton, dedicated to the "laws of weight" of bridge structures [320].

***Paton Yevgenii Oskarovich (1870-1953) was a famous scientist and engineer who worked in the field of welding, bridge engineering and structural mechanics. Hero of Socialist Labor (1943), laureate of Stalin Prize. He graduated from the Dresden Polytechnical Institute (1894), and the St.-Petersburg communications engineers institute (1896). He taught at the Moscow engineering school of communications (1899-1904), at the Kyiv Polytechnic Institute (1904-1938) and at the Kyiv construction College of railway transport.***



*In 1909-1911 the Mukhrani bridge across the Kura River in Tiflis was built according to his design.*

*In 1925 the chain bridge named after Eugenia Bosch was built in Kyiv according to the project of Paton. He was the author and project leader for more than 100 of the welded bridges. In 1929 he founded Welding lab and Welding Committee in Kyiv. E.O. Paton was head of Welding lab and Welding Committee (1929-1933), on the basis of which the Electric Welding Institute was created in 1934. Now the Electric Welding Institute bears his name.*

Further, Yevgenii Oskarovich led the department of bridge engineering in KPI for 25 years and many of his students have become famous scientists. Among them are the author of the Dneproges project, Academician I.G. Aleksandrov, Vice-President of the Academy of Sciences of the Ukrainian SSR K.K. Syminsky, Academicians of the Academy of Sciences of the Ukrainian SSR F.P. Belyankin, M.V. Kornoukhov and S.V. Serensen, corresponding member of the Ukrainian SSR Academy of Sciences B.M. Gorbunov, Doctor of Technical Sciences O.A. Umansky and others.

Combining teaching with practical engineering, E.O. Paton had already done a lot in the 1920s: according to his projects, bridges were built in Tiflis (Tbilisi), two bridges across the Ros River, a pedestrian bridge across the Petrivska alley in Kyiv. The third and fourth volumes of his fundamental work "Iron Bridges" were published, as well as a separate no less serious work "Wooden Bridges". In 1917, as many as two textbooks, one atlas of drawings, and nine scientific articles appeared from his pen. The Kyiv bridge named after Eugenia Bosch (Fig. 1) was restored under his leadership.



Fig. 1. Eugenia Bosch bridge

Yevgenii Oskarovych ran the Kyiv test station of the Central institute of structures of the People's Commissariat for Railways, and did a great job of summarizing the test results on operated and restored railway bridges.

In particular, an assessment of the errors that occur due to neglecting the rigidity of the nodes, when calculating bridge trusses, by comparing the data on the measured stresses with their calculated values was made [319]. Interesting data were also obtained while testing the bridge, specially designed and built by the Kyiv test station for experiments [324].

Together with his student B.M. Gorbunov, he continuously improved his multi-volume course of steel bridges, which was published in several editions and was a true encyclopedia of bridge building, in which the issues of structural resistance were presented in an exhaustively detailed form.

**Timoshenko Stepan Prokopovych** (1878-1972) - Professor, Academician of the Ukrainian Academy of Sciences (1918), Foreign Member of the Academy of Sciences of the USSR (1964, Corresponding Member, 1928). Honorary member of the Academies of Sciences, Scientific Societies, Honorary Doctor of the most famous universities in many countries of the world. In 1901 he graduated from the St. Petersburg communication engineers institute.



In 1906 he defended his dissertation. In 1906-1911, 1918-1920 - Professor of the Strength of Materials Department in the Kyiv Polytechnic Institute, 1912-1917 - Professor in the Polytechnic, Electrotechnical Institutes and Communication engineers institute in St. Petersburg.

In 1919 -1920 - the first director of the Institute of Technical Mechanics (now the S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine). In 1920-1921- Professor of the Zagreb Polytechnic Institute. From 1923 to 1927 - scientific consultant of the company "Westinghouse". He organized the mechanics section at the American Society of Mechanical Engineers (1927). In 1927-1936 - Professor of the University of Michigan; 1936 -1943 - Head of the Department of Mechanics, 1943 -1960 - Professor of the Department of Mechanics in Stanford University (California). From 1960 to 1972 he lived in Wuppertal, Germany.

The years of S.P. Timoshenko work in Kyiv became a bright stage in the development of the theory of structures in the KPI. He worked in the KPI from 1906 to 1920 with a break from 1911-1917. V.L. Kirpichov knew him very well from their joint work in Petersburg and offered Stepan Prokopovych to take part in the competition for the head of the strength of materials department.

Combining teaching with active research, S.P. Timoshenko received a number of outstanding results on various issues of strength analysis [451, 450, 452, 448], including those in a series of studies on the theory of the elastic systems equilibrium stability [453, 454, 455, 456]. The work [456] was awarded the D.I. Zhuravsky prize and medal. In 1908 S.P. Timoshenko published a textbook on the strength of materials [449, 458], which became a classic and was later reprinted many times in many languages of the world.

The energy approach was developed in the works of S.P. Timoshenko on the problem of equilibrium stability. Since his first work in 1907 [455], where

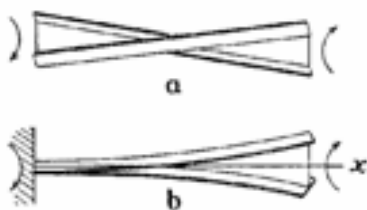


Fig. 2. Free and restrained torsion of a I-beam (picture is taken from [457])

the energy method was used, he has widely applied it to the most diverse problems of elastic systems stability [456], including the energy derivation of the Euler formula [454].

The problem of lateral buckling of beams with narrow rectangular cross-section was first considered by Prandtl. Further development of this problem belongs to Timoshenko, who in 1905-1906 obtained the basic differential

equation for the torsion of symmetrical I-beams and on this basis investigated the lateral buckling of transversely loaded high I-beams [457]. This study by S.P. Timoshenko was later represented at the Kyiv Polytechnic Institute for the adjunct in applied mechanics degree defense (the opponents were V.L. Kirpichov, A.A. Radtsig and N.B. Delone).

But the main thing is that S.P. Timoshenko turned the Prandtl academic problem into a problem of great importance for the practice of bridge engineering. In connection with the problem of spatial buckling, the issue of torsional loss of stability, which was also considered by S.P. Timoshenko [457], became very important.

It should be noted that in solving this problem S.P. Timoshenko found that the Saint-Venant principle is not applicable for torsion of an I-beam. The twist angle depends not only on the magnitude of the torque and the torsional rigidity of the beam, but also on the way of fixing its ends (Fig. 2).

In [455] S.P. Timoshenko considered the problem of buckling of rectangular plates, under various conditions of supporting the edges parallel to the acting compressive forces. He also considered the buckling of rectangular plate whose unloaded side is unrestrained.

## 2. ORGANIZATIONAL ACTIVITY. FOUNDATION OF THE UKRAINIAN ACADEMY OF SCIENCES. S.P. TIMOSHENKO INSTITUTE OF MECHANICS OF THE NAS OF UKRAINE

S.P. Timoshenko did a great deal of organizational work. From 1909 to 1911 he was dean of the mechanical and engineering departments. From the post of dean, he, along with two other deans of the KPI, was dismissed by order of the Minister of Education Kasso for refusing to dismiss Jewish students who had been accepted in excess of the so-called "percentage rate". Returning to Kyiv in 1918, S.P. Timoshenko took an active part in the work of the V.I. Vernadsky commission drafting a law on the establishment of the Ukrainian Academy of Sciences.

S.P. Timoshenko's participation in the creation of the Ukrainian Academy of Sciences should be especially noted.

After the collapse of the Russian Empire, the Ukrainian Scientific Society (USS), created in 1907 in Kyiv and headed by M.S. Grushevsky, at a joint meeting on July 8, 1917 formed the Commission for the establishment of the

Ukrainian Academy of Sciences. And on April 3, 1918, the USS addressed to the Ministry of Education of the Ukrainian People's Republic (UPR) with a proposal to consider the possibility of financing the work of reorganization the USS into the Ukrainian Academy of Sciences. National Academy of Sciences on the concept of M.S. Grushevskogo was to become a non-governmental institution without its own scientific institutions.

Almost at the same time, another concept of creating a national academic center emerged. In September-October 1917 in Petrograd, N.P. Vasilenko, who was a friend of the Minister of Public Education of Russia, together with V.I. Vernadsky, who was also a fellow of the Minister of Education of Russia, and others advocated the creation of state-owned research organizations in Ukraine, Georgia, and Siberia.

For the organization of the Ukrainian Academy of Sciences (UAS) M.P. Vasilenko invited V.I. Vernadsky, who was in Poltava during this period. The famous organizer of science V.I. Vernadsky was a supporter of the creation of a state network of research institutes. He, as a man of advanced views, openly believed that "the task is not a state organization of science, but state assistance to the scientific creativity of the nation." On June 7, V.I. Vernadsky discussed the creation of the UAS with M.S. Grushevsky, who stood on the principles of building the UAS as a free union of high scientific authorities. His position was against the concept of Vernadsky-Vasilenko. The same thing happened with respect to the staff and direction of the UAS. V.I. Vernadsky insisted on creating the "Academy of Ukrainian Studies", at least at the initial stage. M.S. Grushevsky himself categorically refused to participate in any activities proposed by the government of P.P. Skoropadsky.

The first meeting of the Commission for the development of the bill on the establishment of the Ukrainian Academy of Sciences was held in the office of Minister M.P. Vasilenko on July 9, 1918. It was attended by V.I. Vernadsky, N.F. Kashchenko,



Mykola  
Prokopovych  
Vasilenko  
(1866-1935)



Volodymyr  
Ivanovych  
Vernadsky  
(1863 - 1945)



Pavlo Petrovych  
Skoropadsky  
(1873 - 1945)

D.I. Bagaley, S.P. Timoshenko, P.A. Tutkovsky and others. Later A.E. Krymsky, M.I. Tugan-Baranovsky and others became members of the Commission.

The Commission identified the fundamental problems associated with the development of the structure of the Academy and the composition of its departments, a list of departments, scientific institutions, and the procedure for their formation. At this meeting, they came to the conclusion that the appointment of the first Academy staff by the highest authority is logical, given that the UAS was created by the state.

The initiative group, headed by V.I. Vernadsky instructed S.P. Timoshenko to make a report on the organization of the unit of applied sciences in the physical and mathematical department of the Ukrainian Academy of Sciences. The idea of bringing science closer to the demands of life has always been attractive to S.P. Timoshenko, and he joyfully and with great interest began to compose a note.

S.P. Timoshenko wrote in the introductory part, “A characteristic feature of the modern development of industry and technology is the widespread use of the scientific method and the facts gathered by science. The times when science and technology have taken different paths is over, and now they often use the powerful tool that mathematics and mechanics give us to solve purely technical problems. They use the methods of experimental sciences and widely adapt them to solve technical problems in the laboratory. ... The Academy of Sciences should take the initiative in combining science and technology. Due to its central position and scientific authority, it will be able to gather around itself the few scientific forces that currently exist in Ukraine and combine them in a common work where cooperation between people of technology and science will be possible.” S.P. Timoshenko further noted that “representatives of technical science will be able to use scientific methods and knowledge accumulated by pure science to a greater extent than now. On the other hand, representatives of pure science in the field of applied natural sciences will encounter a number of new, unexplored issues, the solution of which will not only enrich science, but will also contribute to the development of industry and the technical life of the region. In the field of experimental activity, people of science will be able to use those powerful tools that modern technology gives into the hands of the experimenter.”

S.P. Timoshenko believed that the newly formed Ukrainian Academy of Sciences should pay more attention to combining pure science with the solution of technical problems, in accordance with the needs of technology.

The commission completed its work on the draft law on the creation of the Ukrainian Academy of Sciences on September 17, and on October 12, the Minister of Education and Arts N.P. Vasilenko submitted a package of documents to the Council of Ministers. On November 14, 1918 Hetman of Ukraine P.P. Skoropadsky approved the “Law of the Ukrainian State on the Formation of the Ukrainian Academy of Sciences in Kyiv” adopted by the Council of Ministers, as well as the Charter and staff of the Academy and its institutions attached to it.

In the period from November 1918 to January 1919, the UAS carried out active scientific and organizational work. The Department of Physics and Mathematics of the UAS consisted of 14 departments of the main class and 16 departments of the class of applied natural sciences.

At this time, academic departments of the physics and mathematics subdivision were founded, including the Department of Applied Mechanics, which was headed by S.P. Timoshenko. At the same time, elections and approval in the posts of directors were held. On November 27, 1918, the first



General Meeting of the UAS was held, at which V.I. Vernadsky was elected Chairman-President of the Academy. And at the second General Meeting of the UAS, which held on November 30, the Institute of Technical Mechanics of the UAS was formed and S.P. Timoshenko was approved as its director. During the first decade of its existence, the institute occupied a leading place in the physical and mathematical subdivision.

In 1929 the Institute of Technical Mechanics was divided into the Institute of Structural Mechanics and the Cabinet of Transport Mechanics. In 1959, by the Decree of the Council of Ministers of the Ukrainian SSR, the Institute of Structural Mechanics was renamed the Institute of Mechanics of the Academy of Sciences of the Ukrainian SSR, and in 1993 by the Decree of the Presidium of the National Academy of Science of Ukraine (NASU), the Institute of Mechanics was named after S.P. Timoshenko.

Actually the entire history of the development of the institute, on the basis of which a number of other independent scientific institutions of the Academy of Sciences of the Ukrainian SSR was created, is based to some extent on the principles laid down by S.P. Timoshenko at the dawn of the emergence of Ukrainian academic science. It is worth recognizing that the ideas expressed by him almost a hundred years ago during the organization of the Ukrainian Academy of Sciences turned out to be quite progressive and embodied in practice. The experience of the first Ukrainian Academy of Sciences, where technical sciences were included in the number of academic sciences for the first time in world practice, was later spread in the practice of the USSR Academy of Sciences and in all other republican academies. Ideas of S.P. Timoshenko, laid down by him during the creation of the Ukrainian Academy of Sciences, over time having received further development, today contributes to the widespread implementation of the results of scientific achievements in practice.

The Institute was headed by well-known academicians of the NAS of Ukraine: S.P. Timoshenko (1918-1920), D.A. Grave (1921), K.K. Symynsky (1921-1932), S.V. Serensen (1932-1940), M.V. Kornoukhov (1940-1944), F.P. Belyankin (1944-1958), G.M. Savin (1958-1959), A.D. Kovalenko (1959-1965), V.O. Kononenko (1965-1975). Since 1976, the Institute has been headed by O.M. Guz.

Today, Ukraine has a high world level of development of mechanics and related sciences.

The level of science is determined, for example, by the existence of the Department of Mechanics and a number of scientific institutions at the National Academy of Sciences of Ukraine. These are such well-known in world science institutes as S.P. Timoshenko Institute of Mechanics, G.S. Pisarenko Institute for Problems of Strength, Institute of Hydromechanics, Ya.S. Pidstrihach Institute for Applied Problems of Mechanics and Mathematics, Karpenko Physico-Mechanical Institute, Institute of Applied Mathematics and Mechanics, M.S. Polyakov Institute of Geotechnical Mechanics, Institute of Technical Mechanics and a number of institutions close to mechanics: V. Bakul Institute for Superhard Materials,

E.O. Paton Electric Welding Institute, A. Pidgorny Institute of Mechanical Engineering Problems and others. It should also be noted that in Kyiv, Lviv, Odessa, Dnieper, Kharkiv there are world-famous scientific schools on mechanics and 4 scientific journals on mechanics are published, which are translated into English by the world's largest scientific publishing house Springer Group. Significant research has been carried out in departments and research institutes of leading universities, research institutes and other institutions.



Stepan  
Prokopovych  
Timoshenko  
(1878-1972)



Dmytro  
Olexandrovych  
Grave  
(1863-1939)



Kostiantyn  
Kostiantynovych  
Syminsky  
(1879-1932)



Serhii  
Volodymyro-  
vych Serensen  
(1905-1977)



Mykola  
Vasyliovych  
Kornoukhov  
(1903-1958)



Fedir Pavlovych  
Beliankin  
(1892-1972)



Gurii  
Mykolaiovych  
Savin  
(1907-1975)



Anatolii  
Dmytrovych  
Kovalenko  
(1905-1973)

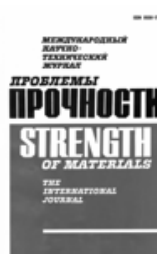


Viktor  
Olimpanovych  
Kononenko  
(1918-1975)



Olexandr  
Mykolaiovych  
Guz

The National Committee of Ukraine for Theoretical and Applied Mechanics (NCU) was formed in accordance with the Decree of the Presidium of the Academy of Sciences of Ukraine No. 191 dated July 3, 1992. The same Decree entrusted the Institute of Mechanics of the Academy of Sciences of Ukraine (IM) with the functions of the base institution of the NCU.



The constituent meeting of the committee, which was attended by 202 leading scientists of Ukraine, working in the field of mechanics and related sciences and representing various scientific centers of Ukraine, was held on June 7, 1993.

The main tasks of the NCU are: preparation and holding of scientific forums on theoretical and applied mechanics and related sciences; facilitating the coordination of scientific research on selected issues of mechanics conducted by scientists in various institutions, departments and industries; strengthening the relations of Ukrainian mechanics with foreign scientists, as well as organizations and international societies with the aim of developing mechanics; spread of scientific and technical information on mechanics; representation of Ukrainian mechanics at the International Union of Theoretical and Applied Mechanics (IUTAM) and other international organizations in mechanics and related sciences.

IUTAM was organized on September 22, 1946 at the constituent assembly of the world's leading mechanics at the Sorbonne University in Paris and is currently the most prestigious scientific union in the field of mechanics. It is appropriate to indicate that world physics is united into the International Union of Pure and Applied Physics - IUPAP.

Second most important after IUTAM is the European Mechanics Society, this society assumes only the personal membership of scientists. The third is the Society for Applied Mathematics and Mechanics (GAMM - Gesellschaft für angewandte Mathematik und Mechanik), where Ukrainian scientists have their representatives along with other scientists from Eastern Europe.

The last at the time of writing this work, the Presidium of the NCU was elected at the General Reporting and Election Meeting on February 10, 2014. It consists of 5 people: the chairman - Guz O.M., the deputy chairman - Bazhenov V.A., Matveiev V.V., Martynyuk A.A., Shevchenko V.P., Scientific Secretary - Ruschitsky Ya.Ya. Chairman of the Committee O.M. Guz is an academician of the National Academy of Sciences of Ukraine, member of European Academy of Sciences (Brussels), Member of Academia Europaea (London), member of the World Innovation Foundation (London).

NCU is an affiliate of IUTAM (in total, 55 countries in which mechanics have reached a certain level of development are affiliated members). In 2000 at the session of the IUTAM General Assembly in Chicago (USA), following a vote, Ukraine was admitted to IUTAM. O.M.Guz was elected representative of Ukraine in the General Assembly.

NCU promotes the development of all important and relevant areas of basic research in the field of mechanics and related sciences, using various means (support for publications in the international scientific journal *International Applied Mechanics*, discussion at scientific seminars and scientific conferences, etc.).

In total, the NCU currently consists of 282 members - doctors of sciences in mechanics and related branches of science, who represent approximately equally academic and university science.

NCU is the only all-Ukrainian public organization that brings together scientists from all areas of Ukraine working in the field of mechanics and related sciences, and which creates a platform for discussing all the important problems of the development of mechanics and related sciences. The influence

of NCU members on scientific research in the field of mechanics is dominant and determining.

The activity of the NCU is aimed both at maintaining the world level of development of mechanics achieved by previous and modern generations of Ukrainian scientists, and at developing new areas of mechanics (for example, nanomechanics, biomechanics, tribomechanics, etc.)

### 3. FURTHER DEVELOPMENT AND FORMATION OF THE THEORY OF STRUCTURES

The further formation and development of the Kyiv School of Theory of Structures is associated with graduates of Kyiv University and the Kyiv Polytechnic Institute.

So, the practical classes in mechanics were supervised by Olexandr Mykolaiovych Dynnyk, who graduated from Kyiv University in 1899 and became a laboratory assistant in physics at the newly organized Kyiv Polytechnic Institute. In 1909, he wrote a paper [87], in which he first gave a solution to the shock problem for the case of linear contact, and also determined the maximum tangential stresses and the points where they are observed.

KPI graduate of 1907 K.K. Syminsky began his pedagogical activity, which lasted 25 years at the same institute.

Thesis for the title of Adjunct of Structural Mechanics K.K. Syminsky defended in 1914, after which he was elected as a professor of the strength of materials department and head of the mechanical laboratory of the KPI. While working in the KPI, he lectured on the strength of materials course, as well as on graphic statics, and led the practice of students. K.K. Syminsky was the first to study the problems of fatigue and strength of steels, spatial trusses for bridges [439], created a number of instruments for testing bridges and lattice structures, and developed the theory of granite strength.

***Syminsky Kostiantyn Kostiantynovych (1879-1932) - Professor (1914), full member of the Academy of Sciences of the Ukrainian SSR (1925).***

*He graduated from the Kyiv Polytechnic Institute in 1907, and in 1907-1932 he worked as a teacher at the Kyiv Polytechnic Institute. In 1920-1921 - Dean of the Faculty of Civil Engineering, 1924-1926 - Vice-Rector for Academic Affairs.*

*At the same time, he worked in 1921-1932 as director of the Institute of Technical Mechanics of the Academy of Sciences of the Ukrainian SSR (now S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine), in 1929-1932 - Director of the Kyiv department of the Scientific and Research Institute of Structures.*



He led the strength of materials department and mechanical laboratory from 1914 to 1932. Beginning in 1921, in connection with the appointment of K.K. Syminsky to the post of Director of the Institute of Technical Mechanics of the Academy of Sciences of the Ukrainian SSR, the scientific work at the department was closely associated with the scientific activities of this institute.

Most of the department staff worked at the same time at the institute. These were such apprentices of K.K. Syminsky as famous scientists in the field of mechanics of materials and engineering structures M.M. Davydenkov and F.P. Belyankin.

Own work of K.K. Syminsky on structural mechanics [442, 443, 444, 445], especially on the analysis of spatial systems [438, 440], contained a number of new interesting results. In particular, in the textbook [443], a method for the gradual introduction of increasingly complex basic determinate structure has been developed. It should also be noted that in his article [441] the problem of prestressing was apparently considered for the first time as a design technique.

The works of K.K. Syminsky had a great influence on the direction of research that was carried out at the KPI and at the Institute of Technical Mechanics. On his initiative, the focus of the Institute's work was clarified, and the Institute itself was reorganized in 1929 into the Institute of Structural Mechanics of the Academy of Sciences of Ukraine.

The graduate of the department was S.V. Serensen, the world-famous founder of the scientific direction in the cyclic and thermocyclic strength of modern mechanical engineering, in particular aircraft engine building, the fracture and durability of engineering structures. He was a graduate of 1926 and passed the way from a laboratory assistant to a professor, head of the department (1931). From 1932 to 1940 S.V. Serensen worked as director of the Institute of Structural Mechanics of the Academy of Sciences of the Ukrainian SSR and head of the department of aircraft engineering at the Kyiv Aviation Institute (1933-1941). In 1939 he was elected an academician of the Academy of Sciences of the Ukrainian SSR.

In the prewar years, KPI conducted research on strength in power engineering, the results of which were defended in the form of a number of candidate dissertations. Among them were dissertations of graduate students A.D. Kovalenko "Study of stresses in the wheels of turbomachines" (1938) and G.S. Pisarenko "Determination of deflections and stresses in detachable elements of steam turbines" (February 1941). Both of them later became academicians of the Academy of Sciences of the Ukrainian SSR.

Due to the occupation of Kyiv, many teachers of the department were evacuated together with the Academy of Sciences of Ukraine to Ufa. On returning after evacuation, the strength of materials department of KPI was headed by a corresponding member of the Ukrainian Academy of Sciences F.P. Belyankin (full member since 1948). Simultaneously with the leadership of the department (1944-1952), he was the director of the Institute of Mechanics (1944-1958), and gave lectures on "Strength of Materials" and "Theory of Elasticity".

In 1952-1959 and 1961-1984 the department was headed by Professor G.S. Pisarenko. Out of 32 years of heading Department, 26 he worked part-time, having the main job in the Academy of Sciences of the Ukrainian SSR - as director of the Institute for Problems of Strength (1966-1988), Chief Scientific Secretary of the Presidium (1962-1966), First Vice-President of the

Academy of Sciences of the Ukrainian SSR (1970-1978). From 1959 to 1961 the strength of materials department was headed by prof. V.V. Khilchevsky.



Mykola  
Mykolaiovych  
Davydenkov  
(1879 - 1962)



Fedir Pavlovych  
Beliankin  
(1892 -1972)



Anatolii Dmytrovych  
Kovalenko  
(1905-1973)



Serhii  
Volodymyrovych  
Serensen  
(1905 -1977)

Managing the department, prof. G.S. Pisarenko did a lot for the development of creative cooperation of the department with scientific institutes of the Academy of Sciences of the Ukrainian SSR, attracting leading scientists from the Academy to work simultaneously in the department, conducting scientific seminars with the participation of scientists from other universities. For the preparation of scientific personnel not only the experimental base of the department was exploited, but also the one of Academy of Sciences.

G.S. Pisarenko considered the training of highly qualified scientific and pedagogical personnel as one of his main targets. He took care of improving the teaching of the course as well as the preparation of textbooks and tutorials on the strength of materials. He considered laboratory studies as a necessary condition for increasing the level of assimilation of educational material by students and made significant efforts to attract talented youth to work at the department.

In the sixties, actual researches on the strength and durability of heat-resistant materials received further development at the department. The results of these researches were introduced into the design practice of a number of enterprises and organizations of the aerospace complex. These studies were led by a student of academician G.S. Pisarenko prof. M.S. Mozharovsky, who headed the department in 1984.

The staff of the department G.S. Pisarenko, V.A. Agarev, O.L. Kvitka, V.G. Popkov and E.S. Umansky wrote several textbooks, including a full course for mechanical specialties, first published in 1963 under the title "Strength of Materials". This textbook was reprinted in 1967, 1973, 1979, 1986, 1993 and 2004.

An important stage in the development of the school of mechanics of the KPI was the opening of specialty "Dynamics and strength of machines" at the strength of materials department in 1970. The necessity to introduce such a specialty was caused by the needs of both the institutes of the Academy of Sciences of the Ukrainian SSR (such as Institute for Problems of Strength, Institute of Mechanics, Institute for Superhard Materials, Institute for Problems

in Materials Science, Electric Welding Institute), and large machinebuilding, aircraftbuilding and shipbuilding enterprises of Ukraine.

The most famous scientists in this field are academicians: O.M. Dynnyk, S.P. Timoshenko, F.P. Belyankin, M.M. Davidenkov, V.V. Kharchenko, A.D. Kovalenko, M.V. Kornoukhov, A.O. Lebedev, V.V. Matveyev, M.V. Novikov, Ye.O. Paton, S.V. Serensen, K.K. Syminsky, G.I. Sukhomel, V.T. Troshchenko, corresponding members: B.M. Gorbunov, I.Ya. Shtaierman, V.A. Strizhalo, A.Ya. Krasovsky, M.I. Bobyr.

In 1989 Doctor of Technical Sciences, professor Ye.O. Antypov was elected the head of the department of dynamics and strength of machines and strength of materials.

Today, the graduating department of dynamics and strength of machines and strength of materials has become the leading department in terms of strength and reliability of machines and structures. It provides at the same time teaching of the general engineering course of strength of materials to students of many faculties and institutes of KPI. Nowadays it is headed by Professor S.O. Pyskunov.



Yevgenii Oscarovych  
Paton  
(1870-1953)



Oleksandr  
Mykolaiovych  
Dynnyk (1876  
-1950)



Georgii Yosypovych  
Sukhomel  
(1888 - 1966)



Illia Yakovych  
Shtaierman  
(1891-1962)



Borys Mykolaiovych  
Gorbunov  
(1901-1944)



Mykola Vasyliovych  
Kornoukhov  
(1903 -1958)



Georgii Stepanovych  
Pisarenko  
(1910 -2001)



Anatolii Oleksiiovych  
Lebediev  
(1931—2012)

A significant role in the development of structural mechanics and theory of structures was played by G.S. Pisarenko, who headed the Strength of Materials Department of KPI for many years. He was the founder and first director of the Institute for Problems of Strength of the Academy of Sciences of Ukraine (now G.S. Pisarenko Institute for Problems of Strength of the National Academy of Sciences of Ukraine). From 1989 to 2011, the Institute was headed by Academician of NAS of Ukraine V.T. Troshchenko, and from 2011 to the present - Academician of NAS of Ukraine V.V. Kharchenko

In October 2016, the institute celebrated its 50th anniversary and during that time it became a recognizable scientific center for fundamental and applied research on the strength of materials and structural elements under extreme thermomechanical loading. The main activities of the institute are: ultimate state and strength criteria of materials and structures; computational and experimental methods for studying the stress-strain state; fracture mechanics and survivability of structures with cracks; vibration of non-conservative mechanical systems.

Well-known scientific schools have been founded and are successfully developing:

- strength of materials and structural elements under extreme conditions of thermomechanical loading (Academician of NASU G.S. Pisarenko);
- criteria of strength and patterns of deformation and fracture of materials and structures in a complex stress-strain state (Academician of NASU A.O. Lebediev);
- material fatigue and fracture criteria and statistical theories of the strength of materials under cyclic loading (Academician of NASU V.T. Troshchenko)
- Theory of vibration of non-conservative mechanical systems (Academician of NASU V.V. Matveiev).

#### 4. PATHS OF DEVELOPMENT

The intensive development of the Kyiv School of the Theory of Structures took place mainly in the following research centers: Kyiv Polytechnic Institute, Institute of Structural Mechanics, Institute for Problems of Strength, and Kyiv Civil Engineering Institute (KCEI). The studies dealt with a wide range of issues, both general theoretical and applied ones.

##### 4.1. Planar and spatial bar systems

The main objects of study in the first third of the twentieth century were truss and frame models of structures. The method of their analysis was perfected by solving a number of specific problems, not least the bridges, whose restoration problems in the 20s were dealt with by Ye.O. Paton.



Borys  
Mykolaiovych  
Gorbunov  
(1901–1944)



Olexandr  
Azariiovych  
Umansky  
(1900–1973)

The Kyiv School of the Theory of Structures paid much attention to the computation of strength of spatial bar systems. This important area of research began with the works of V.L. Kirpichov [252] and K.K. Syminsky [438], and was brilliantly continued in a series of studies by B.M. Gorbunov and O.A. Umansky [144, 145, 151, 160, 161, 469, 470, 479], the first of which were published when the authors were

still students.

They graduated from the Kyiv Polytechnic Institute in 1925, graduation projects were carried out under the guidance of Ye.O. Paton.



Until 1930, both were engaged in the design of bridges of various types and carried out research work at the Institute of Structural Mechanics. At the same time, they taught at the Kyiv Polytechnic Institute and at the Kyiv Civil Engineering Institute of (KCEI), which was separated from the KPI in 1930.

Here O.A. Umansky headed the Department of Structural Mechanics created by him (1930-1933), and B.M. Gorbunov was a professor at the Department of Metal Structures (1930-1941).

In the monograph [161] published in 1932, new methods and ideas were introduced into the structural mechanics of bar systems (the “image method” of B. Mayor, the “method of the motors” of R.v. Mises, the method of traces, etc.), thoroughly revised and supplemented by the authors.

Whereas a plane system of forces can always be reduced to one force or to one pair of forces, in the spatial case, the system of forces is brought to a motor, which is two crossing vectors, namely force and moment (fig. 3). The fundamental importance of the operation of scalar multiplication of motors (screws), which unites all operations of vectoral algebra, essential for the structural mechanics, was clarified.

In addition, the approach of von Mises, based on the consideration of the displacements of the nodes of the pin-jointed truss system, was supplemented by the consideration of the displacements of the bars or disks, which made it possible to consider the bar systems of the frame type. The appearance of this book transferred the analysis of spatial trusses to a new, higher level. Its ideas also had a noticeable influence on the development of descriptive geometry. B.M. Gorbunov used this apparatus not only for spatial trusses but also for constructing theory of spatial frames analysis using the displacement method [156].

In [479] O.A. Umansky and B.M. Gorbunov showed that the virial of external forces (the sum of the moments of these forces, rotated 90 degrees, about an arbitrary point) is equal to the sum of the products of the forces in the bars and their lengths. Later, the concept of virial was used to determine the theoretical weight of trusses.

In a work published by O.A. Umansky in 1932 [476] the method of initial parameters was substantially developed and, apparently, for the first time, concentrated dislocation effects such as a fracture of the beam axis and local shear were taken into account. In those same years, he published a number of studies on the analysis of the work and the method of computing beams on elastic supports [475, 476, 479]. These studies later served as the basis for his monograph “Floating Bridges” [471] and was included in his two-volume “Special Course of Structural Mechanics” [474].

O.A. Umansky continued his studies of the kinematic properties of spatial frame structures [477] and established a theorem on the identity of the

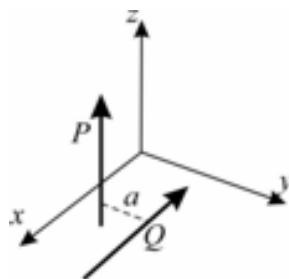


Fig. 3. Example of a motor

equilibrium conditions of a rigid body and the closure conditions of a spatial bar contour. He also proved that the computation of any bar system by the force method coincides with the calculation of some (mutual) system by the displacement method. He also developed the method of initial parameters as applied to structures with bars of varying cross section [478].

Studies related to the analysis of the work and the method of computation of the spatial bar systems were continued by D.V. Vainberg and V.G. Chudnovsky [485, 486, 498], who, among other things, considered the peculiarities of the behavior of cyclically symmetric frames in the calculation of strength [485, 486, 496, 498].



David Veniaminovich  
Vainberg  
(1905-1973)



Volf Grygorovych  
Chudnovsky  
(1908-?)

The use of cyclic symmetry in order to simplify calculations was continued in the later works of V.G. Chudnovsky [61, 64, 65].

Here, as in previous works, the basis of the approach was an implicit decomposition into a finite trigonometric series.

A detailed analysis of the work of spatial systems with any type of point symmetry based on the theory of group representation was

subsequently performed by M.L. Buryshkin and V.M. Gordeiev and presented in the monograph [55].

The calculation of spatial trusses was often reduced to decomposition into flat faces. Difficulties in using such a technique arose in the calculation of torsion. Studies in this direction were carried out by M.D. Shmulsky [403] and A.M. Vasilenko [513].

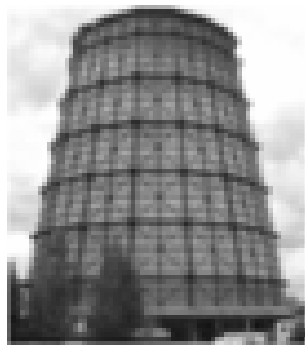


Fig. 4. Cyclically symmetric frame  
of a cooling tower

Another series of works was devoted to the analysis of structures composed of thin-walled bars. As a rule, thin-walled bars do not work alone, but as part of flat or spatial systems. In this regard, there were works where an attempt was made to solve the corresponding problems. The main problem here was the issue of compatibility of deformations in the joints of thin-walled elements. This question is easily solved only for a continuous thin-walled bar, in which the displacements of the end cross sections of the joined elements on the intermediate support coincide.

Apparently, the first who tried to solve the problem of calculating a frame composed of thin-walled bars was B.M. Gorbunov, who proposed a method for calculating plane frames [148] under spatial loading directed perpendicular to the plane (Fig. 5(a)). Here, as in the subsequent works of B.M. Gorbunov with O.I. Strelbitska [152, 157, 158, 159] the issues of calculating thin-walled wagon frames were covered. The hypothesis of absolute stiffness of the joint gusset in its plane (Fig. 5(b)) was used, which ensured the equality of the end cross-sections deplanations of all the bars converging at the node.



Oleksandra Ivanivna  
Strelbitska  
(1905-1980)

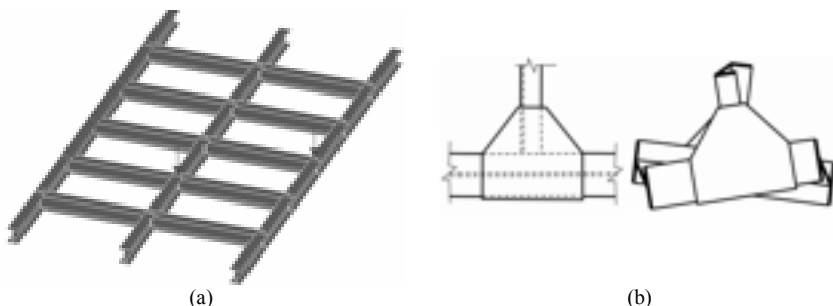


Fig. 5. Frame of thin-walled profiles: (a) - scheme, (b) - node deformation

Many attempts have been made to build a sufficiently universal algorithm for calculating arbitrary thin-walled bar systems, and here the main problem was the formulation of boundary conditions at the ends of the thin-walled bar. In some works, it was assumed that at the end of the bar the deplanation is either completely absent or does not encounter any obstacles. One of the first works of this direction was the article [427]. Spatial (in particular, cyclically symmetric) systems of thin-walled bars were considered in this work under the assumption that the bar nodes are either infinitely rigid, and the deplanation of the end sections of all the bars converging at the node is zero, or the design of the node is such that deplanation is not restricted.

In other studies, the hypothesis of equal deplanations at the ends of all thin-walled bars converging at a node was used. In the general case, the failure of the hypothesis about the equality of the deplanations of all end sections adjacent to the node was demonstrated in [351]. It was demonstrated using simple examples that the deplanations of the end sections of all elements converging in a node do not coincide, and their values depend on the structure of the node, the deformation of which has a noticeable effect on the behavior of the structure.

Much attention was paid to various kinds of improvements in classical methods of structural mechanics. An example here is the work of M.V. Kornoukhov [272], where the displacement method was generalized

taking into account shifts in wide bars (Timoshenko bars). Other improvements to the classical displacement method have been proposed [166, 346].

The issues of developing effective methods and algorithms for solving problems of structural mechanics were considered. Before the computer era the problems of this kind, aimed at reducing the volume and increased convenience of calculations, were at the center of attention of specialists in structural mechanics. So, in the works of M.V. Kornoukhov [274] and O.A. Umansky [473] the problem of solving systems of three-term equations related to problems of structural mechanics was analyzed in detail. Starting from the research of I.Ya. Shtaierman [408], various schemes for an iterative method for solving problems were considered as an option for simplifying calculations. A series of works of this kind was later published by P.M. Sosis [423, 424, 426]. Later, the problem of choosing computational algorithms became acute in relation to nonlinear problems of structural mechanics, where both iterative and step methods were used [129, 257, 326].

#### 4.2. Stability and dynamics of structures

The scientific tradition of stability analysis, established by S.P. Timoshenko, found its successors in the walls of the same Polytechnic Institute in the person of I.Ya. Shtaierman and O.M. Dynnyk. At the beginning of their research, they dealt with the stability analysis of a single bar (straight or curved) [411]. A typical example is the stability of the arches.



Oleksandr  
Mykolaiovych  
Dynnyk  
(1876 -1950)

Illia Yakovych  
Shtaierman  
(1891-1962)

A large amount of research here is made by O.M. Dynnyk, who published the results of his work performed in the 1930s and 1940s, in a monograph [88]. He calculated arches of circular, parabolic, sinusoidal and chain shape under various loadings. He also

considered loss of stability of a circular ring and an arch beyond the elastic limit.

In particular, O.M. Dynnyk drew attention to one significant circumstance that escaped Timoshenko's attention, who was investigating the behavior of a shallow sinusoidal arch under the vertical load. The fact is that in a certain range of parameters of a shallow arch, the loss of stability of its equilibrium occurs not after reaching the limit point on the equilibrium state curve, but somewhat earlier by the bifurcation criterion.

The out-of-plane loss of stability of the arches was studied by I.Ya. Shtaierman [410]. He also began to study the stability of pipes and shells [405]. But all these works dealt with problems of a "local type" where important, but still isolated elements of a complex constructive system were considered. Methods for testing the stability of such systems began to be studied later.

A brilliant cycle of works devoted to the stability of bar systems was performed by M.V. Kornoukhov and his team [268, 269, 277, 278]. It should

be noted that the formulation of similar problems was stimulated by the demands of practice, in particular, by the research of the high-rise construction of the Palace of Soviets of the USSR, which was designed in the prewar years (Fig. 6).

The idea of applying the method of displacements to the problem of the stability of bar systems was proposed by M.V. Kornoukhov in 1937 [276]. Its meaning was that the coefficients of the canonical equations of the displacement method were calculated using formulas that reflected the dependence of the reactions on the value of the longitudinal force in the bar under consideration. This approach became subsequently generally accepted and is used today.

M.V. Kornoukhov actively promoted the method of calculating "stable strength", which consisted, in fact, in the calculation of the system according to the deformed scheme [269, 275], but it was not widely adopted. In the modern formulation, the computation according to the deformed scheme, like the "P- $\Delta$  method", began to be advanced in the Eurocode.

Kornoukhov's works were comprehensively presented in his monograph [276] (it was awarded the State Prize of the USSR in 1950), where, in addition to description of the stability analysis method, a number of general principles of the theory of stability was pointed out. It was indicated how to replace one load with another, equivalent to the first, what simplifications of the design scheme are permissible, how to compare the safety factor at combined loading with those at separate components of the loading. One of the results arising from qualitative analysis is the fact that a "culprit" in the loss of stability of the entire structure is often one element or a small group of elements. In this connection, M.V. Kornoukhov introduced the concept of states of stability. But only 50 years later in the work of A.V. Perelmuter and V.I. Slivker [350] the criterion for determining the type of bifurcation (constrained or forced) of the bar or any part of the structure and the method for calculation based on the energy approach were presented.

In [270] M.V. Kornoukhov studied the mechanism for the loss of stability of the two-bar truss (von Mises truss) in detail, and pointed out the possibility of loss of stability without passing through the bifurcation point.

The complex of important practical problems of structural stability is presented in the monograph by I.Ya. Shtaierman and A.A. Pikovsky [413], intended more for practicing engineers than for researchers. The authors analyze in detail the ideology of calculating systems with compressed elements, paying particular attention to



Fig. 6. The skeleton of the Palace of Soviets (draft)



Mykola  
Vasyliovych  
Kornoukhov

the limiting value of the carrying capacity of such elements. Here, by the way, it was proposed to divide the total safety factor into two parts: “load dependent” and “structure dependent”, i.e. ideas that later formed the basis of the method of limit state design.

Peculiar problems were solved when calculating the stability of thin-walled bars. Here one can point to theoretical and experimental studies of M.I. Dlugach [89, 91] and L.N. Stavradi [427, 428, 429], the latter also investigated the stability of frames made of thin-walled profiles [429]. He obtained the value of the torsional stiffness coefficient of a compressed thin-walled bar and used it to construct a method for calculating the spatial framework of such bars. In this case, the stability of cyclic frameworks and dome like structure consisting of hinged bars was investigated. The latter problem was studied in detail in the work of P.S. Polyakov [366]. A detailed analysis of possible buckling modes of single-tier spatial frames was carried out by L.N. Padun-Lukyanova [314]. Multiple buckling modes of cyclically symmetric frames were investigated by M.Ya. Borodiansky [52].

Studies on the theory of stability have always been the focus of the Kyiv school of the theory of structures. They concerned both the fundamental questions of the theory and specific classes of structures.

In particular, in the works of O.M. Guz and his students (see, for example, [227, 228, 230]) stability problems of bars, plates, and shells were considered in a three-dimensional formulation. It was proved that stability theories based on the Kirchhoff–Love hypothesis and assumption of sectional rigidity are asymptotically exact.

The three-volume monograph [347, 349] is devoted to the presentation of stability theory based on the general equations of mechanics of a deformable solid, when the transition to applied theories that use hypotheses of a static or kinematic nature is realized by an accurate formal transformation of the corresponding general equations. At the same time, effects were discovered that were skipped by the usual approach based on some obvious static-geometric representations.

Among the stability problems that consider specific types of structures, one can point to I.Ya. Amiro and V.O. Zarutsky cycle of works devoted to the stability of ribbed cylindrical shells [2, 4, 503]. Nowadays the works of G.D. Gavrilenko [126, 127] can be considered as the continuation of these studies. The studies of the stability problems of systems with unilateral constraints [18] as well as frame systems with elements of varying cross-section are also worth mentioning here.

The use of the approach, that was based on the dynamic stability criterion, was proposed in the works of



Igor Yakovych  
Amiro  
(1921-1998)



Volodymyr  
Oleksandrovykh  
Zarutsky  
(1932 – 2011)

V.G. Chudnovsky [60]. Nevertheless, this approach turned out to be more in demand when considering the dynamic problems of systems with compressed bars.

Dynamic problems were considered by S.P. Timoshenko in connection with studies of bridge vibrations [450]. Experimental studies of bridge dynamics were carried out by the Kyiv Bridge Testing Station, which was organized by Ye.O. Paton in 1920.

The application of the displacements method to the dynamic problems was proposed by M.V. Kornoukhov [273] and V.G. Chudnovsky. The latter focused on the analysis of vibrations of compressed systems. His studies on this problem were presented in detail in the monograph [62]. In addition, V.G. Chudnovsky noted that the well-known Bubnov problem of the critical stiffness of an intermediate elastic support, which translates its work into a class of absolutely rigid, is also generalized to the oscillation frequencies [59]. Vibrations of circular arches were considered by D.V. Vainberg [483, 495]. S.V. Serensen dedicated his work [391] to the dynamic analysis of multi-storey frames.

The dynamics of bars and bar systems has been the subject of research by G.S. Pisarenko, whose fundamental research concerned the theory of vibrations of elastic systems accounting energy dissipation in the material [355, 357] (awarded the M.M. Krylov Prize of the Academy of Sciences of the Ukrainian SSR).

His further work is related to the study of the strength of materials and structural elements in extreme conditions. The same subject became the main one in the Institute for Problems of Strength of the Academy of Sciences of the Ukrainian SSR created on the initiative of G.S. Pisarenko. He headed this Institute for over twenty years.

The problem of the dynamic action of a shock load has been the subject of many studies, beginning with the solution of S.P. Timoshenko, based on the accounting of local shock deformations in accordance with the approach of Hertz. Yu.V. Blagoveshchensky and D.V. Vainberg suggested using the series method to solve the basic equation of S.P. Timoshenko [51]. In the monograph by M.O. Kilchevsky [246] the dependencies obtained by I.Ya. Shtaierman were used instead of Hertz's relations. These dependencies related to cases of tighter touch of bodies. In addition, M.O. Kilchevsky considered the process of not only elastic, but also elastoplastic deformation of the contact zone [243].

### 4.3. Elastic plates and shells

The further development of fundamental works in the field of analysis of the structures behavior was mainly connected with the development of the theory and methods for calculating plates and shells. The beginning of such research was laid by the works of I.Ya. Shtaierman performed at the Kyiv Polytechnic Institute. As early as 1924, he considered an axisymmetric problem in the theory of calculating a spherical shell [409]; he was the first to



Georgii  
Stepanovich  
Pisarenko  
(1910-2001)

suggest a method of asymptotic integration of the equations for a shell of revolution [407]. The practically important problem of calculating a cylindrical tank with a wall of variable thickness [406] was also solved, and a simple to use method of analogy between the dome and the arch on an elastic foundation [404] was developed.

One of the first works carried out at the Institute of Structural Mechanics of the Academy of Sciences of the Ukrainian SSR and devoted to the calculation of shells was the study of A.L. Goldenveizer [134] carried out in 1935. This famous scientist continued his work later in Moscow.

The approach V.G. Chudnovsky, presented in [58], was based on the synthesis of structural mechanics methods for bar systems and the theory of elasticity. It treated the ribs as a frame lying on an elastic foundation created by the shell itself. Later this method was transferred to the calculation of the ribbed domes according to the moment theory [65].

In a series of works of M.O. Kilchevsky [244, 245, 248] a generalized method of reducing the three-dimensional problem of the theory of elasticity to a two-dimensional one was proposed and developed. A number of variants of the theory of shells were proposed, in relation to which the ordinary theory can be considered as a particular case. The formulation of boundary value problems of the theory of shells in the form of integral and integro-differential equations was given and various ways of composing these equations are developed as well. In particular, for the first time it was proposed to use the potential method for solving problems of shell statics [242], which was later developed in the works of D.V. Vainberg and O.L. Sinyavsky [489, 490].



Mykola  
Oleksandrovych  
Kilchevsky  
(1909 - 1979)

A number of studies by D.V. Vainberg [482, 484, 491, 493, 494] was devoted to the plate calculation. In the monograph [484], awarded the Academician Galerkin Prize, the plane problem and the problem of plate bending are considered as one generalized boundary problem of the theory of analytic functions.

This made it possible to obtain a number of new results for plates consisting of concentric rings, as well as for the strip problem.

The theory of calculating circular plates of constant and variable thickness was developed by A.D. Kovalenko, Ya.M. Grigorenko and their students, who obtained exact solutions to a number of problems on the stressed state of circular plates and shells of rotation of variable thickness [196, 200, 280, 281, 282].

The method of calculating plates and shells with discontinuous parameters (ribs, step change in thickness, kinks, etc.), based on the use of impulse functions, was proposed by D.V. Vainberg and I.Z. Roitfarb [487].

The probability of the shells loss of stability were the subject of research in a series of works performed by M.V. Goncharenko at the Taras Shevchenko Kyiv State University [136, 137, 138, 139, 140].



The solution of a large number of problems on the calculation of plates of various shapes and with various boundary conditions was obtained by the grid method, the development of which was carried out in the works of P.M. Varvak and his followers [506, 507, 508, 510, 511].

But the particularly rapid development of research on the calculation of plates and shells began when computers became a research tool, since the transition to computer analysis required a number of improvements in the theory related to formalization and algorithmization, and also revealed new possibilities in formulating solved problems (see below).

#### 4.4. Inelastic behavior of structures

At the Institute of Structural Mechanics, beginning in the 1930s, new scientific directions for that time began to develop - an analysis of the behavior of structures beyond the elastic limit and the assessment of their ultimate bearing capacity. This analysis was based on a number of experimental and theoretical studies carried out under the guidance of M.D. Zhudin [530, 532, 533]. A distinctive feature of this cycle of work was its focus on the study of the work of specific constructive forms. For example, the behavior of continuous steel beams beyond the elastic limit under the action of repeated and moving loads was studied [531, 535]. The continuation of these works was an analysis of the behavior of the trussed beams, made by V.V. Trofimovych [459, 460] and the research of L.P. Kunitsky [288] on the assessment of the influence of shear force on the reduction of the maximum load acting on the statically determinate and statically indeterminate beams. Experimental work [534], adjacent to the same cycle, was aimed at assessing the bearing capacity of columns of the high-rise structure of the Palace of Soviets of the USSR, which was designed in the pre-war years.

The oblique (biaxial) bending of rectangular beams beyond the elastic limit for the case of perfectly plastic material was considered by B.M. Gorbunov and V.G. Chudnovsky [155], and somewhat later by O.I. Strelbitska. She also investigated the limiting state of a thin-walled profile under constrained torsion, including the dependence between torque and bimoment [431].

The study of the behavior of elastoplastic structures under alternating loading revealed the need for an analysis of adaptability. The works [287, 291] were devoted to this problem.

The work begun by M.D. Zhudin was continued by O.I. Strelbitska, who studied the inelastic behavior and the limiting state of thin-walled bars and plates [430, 433, 434]. Thus, the previously initiated studies of frames made of



Petro Markovych  
Varvak  
(1907 - 1979)



Mykola  
Dmytrovych  
Zhudin  
(1891 - 1968)

thin-walled bars were continued, but already concerned their behavior beyond the elastic limit. In this case, the main attention was paid to the development of models for the limiting equilibrium of thin-walled bars and their experimental testing. In particular, the ultimate load for a complex stress state (bending, complicated by torsion, etc.) was investigated. Adjacent to the same problems are the studies of S.A. Palchevsky [315, 316].

These studies, like the works of M.D. Zhudin, were focused on identifying the limits of the bearing capacity of steel structures for which the assessment of the reserve of strength and stability is very important [533]. An approximate method of calculation based on the use of an idealized Prandtl diagram and the hypothesis of the instantaneous appearance of plastic hinges has proven useful here. The validity of this approach was confirmed by a number of experimental studies performed by Ye.O. Paton and B.M. Gorbunov [321], B.M. Gorbunov [154], M.D. Zhudin [532].

But later a similar problem arose acutely in relation to structures made of reinforced concrete. Such studies were carried out in the Research Institute of Building Structures of the State Building Committee of the USSR (Naukovodoslidny instytut budivel'nykh konstruktsiy, NDIBK) by A.M. Dubinsky [102] as applied to the limit equilibrium of flexural plates, and successfully continued in a series of works by A.S. Dekhtyar and O.O. Rasskazov [73, 371] as applied to the assessment of the carrying capacity of shells.

But for the calculation of concrete and reinforced concrete structures the most significant is the consideration of the possible formation of cracks and the assessment of the role of creep. Here, the fundamentally important results belong to I.I. Ulytsky, who in a series of experimental studies, analysed a number of features of the creep process (the influence of environmental humidity, the role of thinness, etc.), which made it possible to justify proposals for development a creep measure [466, 467, 468].



Yosyp  
Ioakhymovych  
Ulytsky  
(1912-1965)



Yakiv  
Davvdovych  
Livshvts  
(1907 – 1984)

Accounting for the effects of shrinkage and creep on redistribution of forces in statically indeterminate systems has been the subject of research by Ya.D. Livshyts [296] and many scientists (see, for example, [7, 104, 135, 285]). Correct accounting for the formation of cracks was performed in numerical studies [298, 380, 529]. A detailed analysis of the elastoplastic behavior of

reinforced concrete structures has become the topic of recent work [117].

## 5. THE ERA OF NUMERICAL ANALYSIS

The problem of solving complex design problems has always been a "sore spot" of the theory of structures. Among the attempts to solve it, one can name the replacement of the solution of the corresponding resolving equations by a model experiment. Here, first of all, one should mention the polarization-

optical method for studying stresses (the method of photoelasticity) - an experimental method for studying the stress-strain state on transparent models of optically sensitive materials. At the Taras Shevchenko Kyiv State University such researches conducted V.I. Savchenko and his students [387, 388, 389], whereas in NDIBK they were performed by B.M. Barishpolsky [9, 497]. In the early 60s of the twentieth century, an energetic attempt was made to solve the aforementioned problem through the use of electrical modeling systems EMSS, developed under the direction of G.Ye. Pukhov [367, 368].

However, in mass practice, such approaches have not been rooted; they have been driven out by rapidly developing numerical methods, which were based on the use of an electronic computer.

### 5.1. Computer technology and numerical methods

The transition to machine-based problem solving technology was preceded by a relatively short period when the main computational tool for engineering calculations was desktop electric arithmometers of the Rheinmetall type serviced by the operators of computing stations. To work with such computing stations, a special technology and language of communication was created, allowing formalizing the sequence of computational procedures to which the problem solving algorithm came down. Here, the initiator was P.M. Sosis [421].

Already in the first steps it was found out that it was necessary to algorithmize some stages of calculation. In particular, there was a problem of a formalization of description of a complex flat or spatial system, its topology, reflected structure composition, and the basic operations of static computation of discrete systems.

For a hinged trussed framework with an arbitrary topology A.V. Perelmuter suggested, apparently, for the first time to use the "node-bar" incidence matrix [334], and it turned out that the difference and summing operators of L.G. Dmitriev [100] were realized.

And it is quite clear that the finite difference method has become the natural way to solve the problems of calculating plates, shells and systems made up of them. A complication in its use was the need to move from the central differences to the "left" or right "differences when formulating the boundary conditions. One of the means of circumventing this complexity was the creation of bar models for which the equilibrium equations were equivalent to finite difference equations [99, 95, 305]. The authors of these papers justified the discrete model by the coincidence of its equilibrium equations with finite-difference representations of differential equations of a continuous medium.

So, for the biharmonic equation of the plane problem written through the Erie function  $\varphi(x,y)$ , when using the grid showed in fig. 7(a), a typical difference equation is as follows:

$$20\varphi_{0,0} - 8(\varphi_{1,0} + \varphi_{-1,0} + \varphi_{0,1} + \varphi_{0,-1}) + 2(\varphi_{1,1} + \varphi_{-1,1} + \varphi_{1,-1} + \varphi_{-1,-1}) + \varphi_{2,0} +$$



Petro Moiseiovych  
Sosis  
(1918-1967)

$$+\varphi_{-2,0}+\varphi_{0,2}+\varphi_{0,-2}=0.$$

This equation can be interpreted as the canonical equation of the force method, written for a statically indeterminate truss, in which two variants of the structure are presented in Fig. 7, and sections of elements are selected in a special way. The advantage of such a simulation was that the models did not require the use of fringe points.

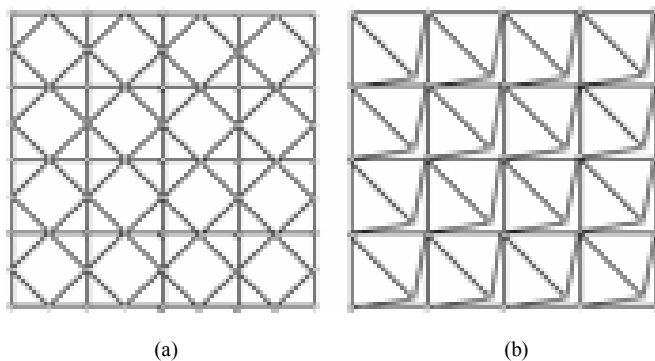


Fig. 7. Simulation of a finite-difference equation with a bar lattice

In the works of the team of the Problematic Research Laboratory of Thin-Walled Spatial Structures (Problemna Naukovo-Doslidna Laboratoriya Tonkostimnykh Prostorovykh Konstruktsiy, PNDL TPK) of Kyiv Civil Engineering Institute (KCEI), conducted under the direction of V.I. Guliaev, the finite-difference method was successfully used to study shells of a non-canonical shape. For example, studies of the spiral shell with a variable radius of curvature shown in Fig. 8 were conducted [207], as well as shells with rapidly changing geometric parameters, shells of variable thickness, etc. [210, 212].

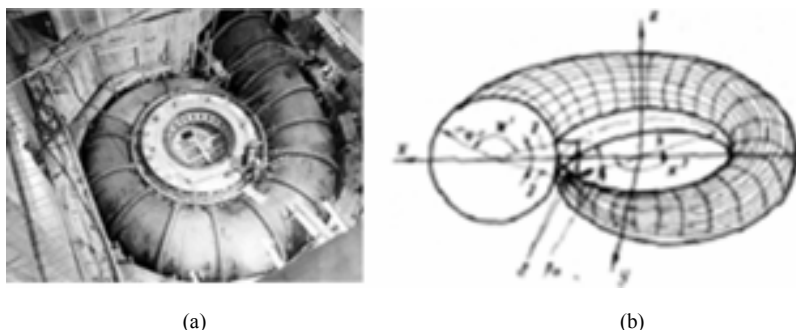


Fig. 8. The spiral chamber of a hydraulic turbine:  
(a) - general view, (b) – computational structural model

A radical tool turned out to be the transition to a variational formulation of the difference method, proposed for the first time in the studies of the team of the PNDL TPK of KCEI [501]. The usual way to obtain grid equations is to

apply the finite difference method to differential equations and to boundary conditions. However, such a path in some cases (corners, contact conditions, mixed boundary conditions) leads to the fact that the matrix of systems of equations does not always turn out to be symmetric, which is inconsistent with the principle of reciprocity and complicates the problem computationally.

If a variational method of deriving grid equations is used, based on replacing the expression for the total potential energy of a system with a certain algebraic form according to certain rules of numerical differentiation and integration, the resolving function turns into a collection of its discrete values at the nodes of the region, and the functional reduces to a quadratic form of this values.

The grid equations are obtained from the expressions for the components of the gradient of the elastic potential at each grid point, while the symmetry of the coefficient matrix is guaranteed.

Many practically important problems were solved using the finite difference method in a variational formulation, (see, for example, [84, 93, 94, 502]).

With the active participation of O.A. Kyrychuk, studies were conducted aimed at ensuring the reliability of a fuel tank with a protective capacity at the Ukrainian Antarctic station Academician Vernadsky [289, 290]. Obtained results made it possible to formulate the basic requirements for the further safe operation of the tank in extreme Antarctic conditions.

In the 70-80s of 20th century in Kyiv a variant of the finite difference method was developed under the guidance of Professor Ye.O. Gotsuliak. It was the so-called curvilinear grid method, in which the problem of accounting rigid displacements was solved [195]. The method was effectively used to solve nonlinear problems of deformation and stability of complex shells [194].



Oleksandr  
Adrianovych  
Kyrychuk  
(1948 – 2018)



Yevgenii  
Oleksandrovych  
Gotsuliak  
(1942–2011)

## 5.2. Finite element method

But the true revolution in the problem of numerical calculations came with the introduction of the finite element method (FEM). This method was used and developed by many scientific teams of Kyiv (KCEI, Institute of Mechanics of the Academy of Sciences of the Ukrainian SSR, UkrRDIsteelconstruction, Kyiv Zonal Researche Innstitute for Experimental Design (Kyivsky zonalny naukovo-doslidny instytut eksperymentalnoho proektuvannya, KyivZNDIEP), etc.).

In the PNDL TPK of KCEI O.S. Sakharov developed the moment scheme of the method (MSFE), which formed the basis of a system of software for a computer, and was actively used to solve a wide variety of problems including plates, shells and three-dimensional bodies computation [38, 384]. This

scheme was actively developed and adapted to solving problems of statics and dynamics of systems of various types, including non-linear problems [38, 39, 261, 379].

One of the advantages of the MSFE is the zero response to the rigid displacement of the element. Some of the other variants of the finite element method did not always correctly take into account the problem of rigid displacement when calculating shells. In addition, for many of them, another negative property of the stiffness matrix, called “shear locking,” was observed, when bending thin plates and shells modeled with three-dimensional elements. In this case, errors associated with the manifestation of fictitious shear deformations significantly increase.

In particular, these problems become significant when curvilinear coordinate systems are used, which are introduced in order to better describe the geometry of bodies of complex shape. Then the components of the deformations depend not only on the derived displacements, but also on the displacements and turns of the element as a whole.

In order to eliminate these deficiencies of the MSFE, techniques [261, 377, 379, 385] were developed, allowing to take into account the basic properties of rigid displacements for isoparametric and curvilinear finite elements. Their essence lies in the fact that when recording the conditions for the connection of deformations with displacements, those expansion terms of deformations are rejected in a series that react to rigid displacements and to fictitious shear deformations. In this case, the exact equations of the connection of deformations and displacements are replaced by approximate ones.

The moment scheme was used mainly in the work of the PNDL TPK of KCEI for solving problems, the design scheme of which was represented by three-dimensional finite elements. At the same time, in the works of V.M. Kyslooky and V.K. Tsykhanovsky the nonlinear deformation of cable-stayed systems, as well as soft and hanging shells was investigated using the moment scheme (Fig. 9) [43, 258, 263].



Volodymyr  
Mykytovych  
Kyslooky  
(1940-2017)



Valentyn  
Kostiantynovych  
Tsykhanovsky  
(1937 – 2012)



Viktor  
Volodymyrovych  
Kyrychevsky  
(1945 – 2006)

In the works of V.V. Kyrychevsky, this approach was used to solve problems related to nonlinear deformation and fracture of structures made of elastomers [249, 250].

Other teams preferred to use the classical FEM scheme, where the library of finite elements contained one-dimensional (bar), two-

dimensional (plates and shells) and three-dimensional elements (see, for example, [187, 188, 190, 306]).

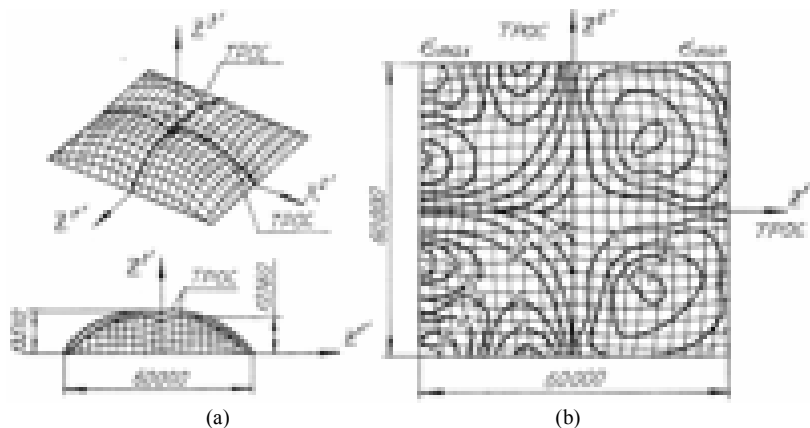


Fig. 9. The results of the calculation of the air-supported soft shell with cables [43]:  
(a) – deformed discrete model of the middle surface, (b) – countours of principal stress

The finite element method in various modifications was used as the basis for the development of software systems and application packages that were developed at KCEI (the STRENGTH-85 system), KyivZNDIEP (GAMMA-2), UkrNDIproekt, Giprokhimmash, Research Institute for Automation Systems in Construction (Naukovo-doslidny instytut avtomatyzovanykh system u budivnytstvi, NDIASB) (programs of the LIRA family), UkrRDIsteelconstruction (PARADOX, PARSEK, SUDM), Research and Production LLC SCAD Soft (SCAD Office) and in other Kyiv organizations. In the 60s of the twentieth century, Kyiv became the de facto “capital of the FEM” of the Soviet Union, it remains the main center of development of this type even today, when the SCAD and LIRA software systems are the main tool for designers in the CIS countries.

The development of the finite element method took place in different directions: the library of finite elements was expanded, including at the expense of incompatible elements, for which convergence was proved [110, 187, 240], elements of multilayer plates and shells [360], and others. An attempt was made to create a finite element system [181, 182] that has a number of attractive features (the interpolating function realizes the minimum potential energy of the element deformations, all the major minors of the stiffness matrix and all its eigenvalues have a minimum value, etc.). A finite element model was implemented based on the moment theory of elasticity (Cosserat's body), which, according to the authors, made it possible to better represent the behavior of the structure in stress concentration zones [310].

The original scheme of the finite element method was proposed by Ye.O. Gotsuliak. The peculiarity of this scheme was the vector representation of the displacement function in the general curvilinear coordinate system. When constructing the stiffness matrix of a curvilinear finite element, the vector approximation of the displacement function was represented by Maclaurin's series, whose coefficients were the values of the desired vector

function and its derivatives in the center of the finite element. The proposed approach was later extended to geometrically nonlinear problems of deformation and stability of thin shells with shape imperfections (Fig. 10) [300]. Figure 10(a) shows a finite element model of a fuel tank with real shape imperfections, and figure 10(b) shows a picture of a deformed state in the cross section of the shell corresponding to the stationary point of the equilibrium curve.

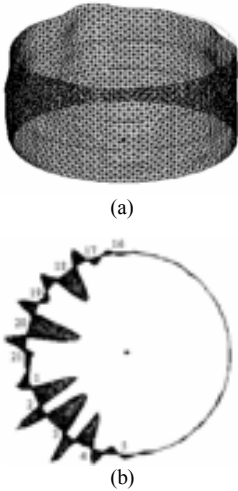


Fig. 10

Certain difficulties in justifying the FEM arose for the case of using incompatible finite elements. The shape functions for these elements do not belong to the energy space and the standard method of proving “convergence in energy” does not work here. A rather general method for studying the convergence of incompatible finite-element systems, as well as a method for constructing convergent incompatible elements, was indicated only in 1981 by I.D. Yevzerov [108, 109].

Many developments do not use curvilinear finite elements of shell theory instead of which flat elements are used (the shell is modeled by a polyhedron). The refusal to use curvilinear elements, which gave a number of simplifications, was justified by the evidence of their convergence [111, 112].

A definite alternative to the finite element method is combined approaches to solving multidimensional problems, in which the dimension of the original equations is reduced analytically at the first stage, and numerical methods are used at the second stage to solve the reduced equations. One of such combined methods is the potential method used by D.V. Vainberg and O.L. Syniavsky [489] for shells calculations, and in the works of Yu.V. Veriuzhsky [514, 515] for solving of a fairly wide class of problems.

The investigations of G.B. Kovnerystov [283, 437] were devoted to the contact problems, which were solved numerically by means of integral equations.



Oleksandr Leonidovich Syniavsky  
(1937-2018)



Yurii Vasyliovych Veriuzhsky  
(1937-2017)



Pavlo Pavlovych Voroshko  
(1940-2014)



Georgii Borysovych Kovnerystov  
(1929 – 1992)



To determine the stress-strain state of various structural elements, as well as solving problems of fracture mechanics, P.P. Voroshko applied both the finite element method [526] and the method of boundary integral equations [525].

In the works of V.K. Chybiryakov the method of finite integral transformations was modified for the first stage, while different effective numerical methods were used to solve the reduced equations at the second stage. This method was widely used by the author and his students for solving static and dynamic problems [56, 57].

Other methods for solving spatial problems were also developed [293, 294, 517].

Instead of the transition from the system of differential equations to algebraic equations, which is characteristic of the finite element method, a transition to a system of ordinary differential equations is possible. Close to this idea is the method of lines (differential-difference method) – this is a very effective method of reducing the dimension of the original boundary value problems.

L.T. Shkeliiov [401, 402] extended the method of lines to the stress-strain analysis of plates of arbitrary shape.

It can be noted that a significant role in the development of methods for calculating spatial structures of the shell type was played by problems connected with the analysis of a number of original engineering objects that were developed and put into practice by the team of researchers of the KyivZNDIEP Institute (Fig. 11).

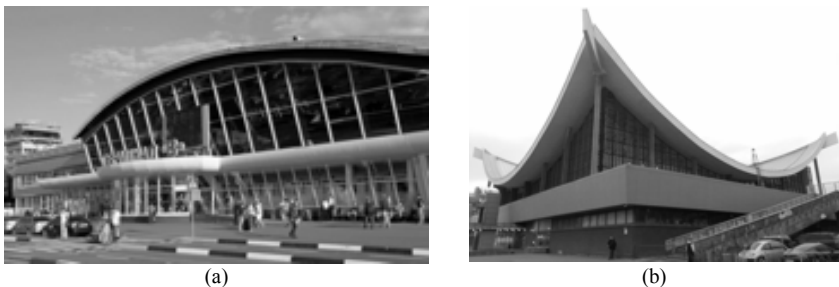


Fig. 11. Shell roofs: (a) - airport terminal in Boryspil, (b) - furniture house in Kyiv

### 5.3. Semi-analytical finite element method

Significantly increase the efficiency of the FEM allows its integration with the method of separation of variables. This approach is called the semi-analytic finite element method (SAFEM). Its essence consists in decomposition of the unknown quantities along one coordinate in a certain system of continuous smooth basis functions in combination with finite element discretization along the other two.

For a long time, the Research Institute of Structural Mechanics of the Kyiv National University of Construction and Architecture conducted research aimed at expanding the scope of SAFEM to simulate the processes of

deformation of spatial inhomogeneous bodies of complex shape for cases of static and dynamic loading [21, 24], which required a number of important issues. Thus, the issues of combining SAFEM with a moment scheme of finite element (MSFE) [222] were considered which made it possible to create a unified approach to modeling the deformation processes of massive, thin-walled and combined structures. In contrast to the well-known approaches, where SAFEM was used only for homogeneous rotation bodies or prismatic bodies with hinged boundary conditions at the ends, the method is adapted for calculating non-uniform non-canonical bodies: cyclically symmetric [219, 220, 381, 383] and prismatic curvilinear bodies [222, 224]. In addition, modeling of arbitrary boundary conditions at the ends of bodies is provided [223].

On the basis of the new types of created finite elements, the techniques for physically and geometrically nonlinear problems analysis [21, 221, 222, 224] as well as dynamic processes [23, 26, 29, 226] have been developed. Also the techniques for modeling crack development from the point of view of fracture mechanics [28, 225] were obtained.

Some examples of solved problems are illustrated in Figures 12, 13.

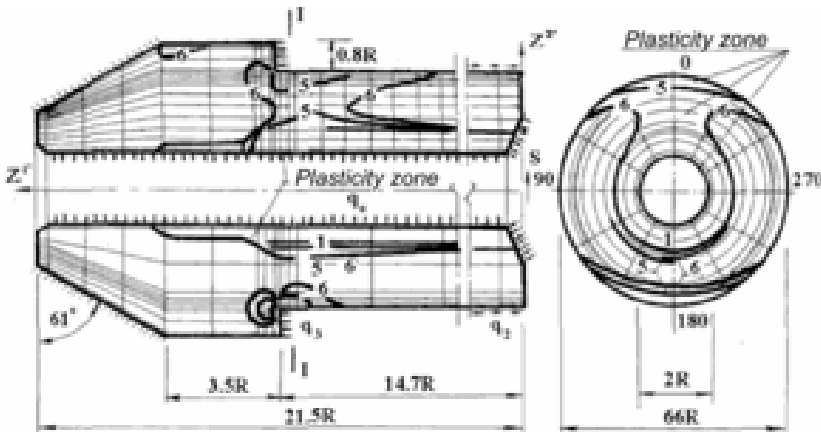


Fig. 12. Nonaxisymmetric nonlinear deformation of the high-pressure valve

#### 5.4. Numerical analysis of shells

The application of FEM has significantly expanded the class of researchable problems. The subject of research was the non-classical problems of the theory of plates and shells, where non-canonical-shaped structures, complex boundary conditions, multilayer shells, etc. were considered. Great attention was paid to shells with holes and cutouts, as well as shells with different types of reinforcements [30, 33, 198, 259]. Tissue shells were also considered [167, 172]. A large complex of studies was associated with the analysis of the behavior of multilayer shells [19, 37, 372, 375].

The stress-strain analysis of layered shallow shells, the material of which is orthotropic, is described by a system of differential equations in partial derivatives of the tenth order with variable coefficients and corresponding

boundary conditions. The solution of such a problem involves considerable computational difficulties. Therefore, to solve it, a numerical-analytical approach based on reducing the two-dimensional boundary-value problem to systems of ordinary differential equations using the spline-approximation method in one of the coordinate directions was proposed. The obtained one-dimensional boundary-value problem was solved by a stable method of discrete orthogonalization [204, 205, 524]. In this way, a large number of static and dynamic problems were solved [196, 202, 203].

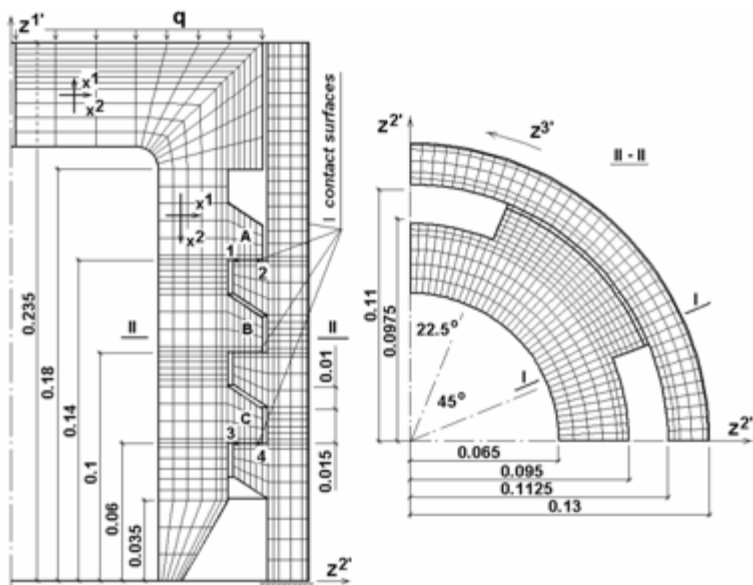


Fig. 13. Dynamic interaction of parts, heterogeneous in the annular direction

A team of authors from the Kyiv Automobile and Highway Institute (V.G. Piskunov, V.Ye. Veryzhenko, A.F. Ryabov and others) developed the theory of multilayer plates and shells, which was based on the principle of using kinematic hypotheses common to the entire multilayer package [357, 358, 359, 370, 519].

Modern trends in the development of structural mechanics and the practice of designing thin-walled shell structures lead to the development of refined numerical methods for the study of nonlinear deformation and stability of shells of various types.

Under the guidance of Professor M.O. Solovei at the Kyiv National University of Construction and Architecture numerous studies of nonlinear deformation, stability and post-critical behavior of a wide class of thin elastic inhomogeneous shells of various shapes and structures under the action of thermomechanical fields were conducted [41, 42, 259].

The two-dimensional theory of plates and shells, as well as the one-dimensional theory of bars, was not used, and the entire analysis was carried



Vadym  
Georgiiovych  
Piskunov  
(1934–2016)



Mykola  
Oleksandrovyeh  
Solovei  
(1946–2014)

out from the standpoint of the three-dimensional theory of elasticity. The need for a three-dimensional approach to the computation of thin shells is caused by the fact that real shell structures are often designed as heterogeneous systems: smooth and step-variable thickness, with kinks, reinforced ribs and pads, weakened holes, grooves and channels, faceted, multi-layered. The problem of choosing a design model (or a combination of several) for a fairly accurate approximation of design sections with different geometric and physical features was solved as follows.

Two types of its features were considered: a) geometric features in the form of continuous and step-variable thickness; b) structural inhomogeneities of the material along the thickness and in the plan in the form of a combination of multilayer packets. Each layer of material may be anisotropic and different. Thus, thin multilayer shells of variable thickness and of complex geometric shape are considered as three-dimensional bodies that can be supported by ribs and overlays, weakened by grooves, channels and holes, and have breaks in the middle surface (Fig. 14) while Fig. 15 shows examples of calculated structures of complex

shape.

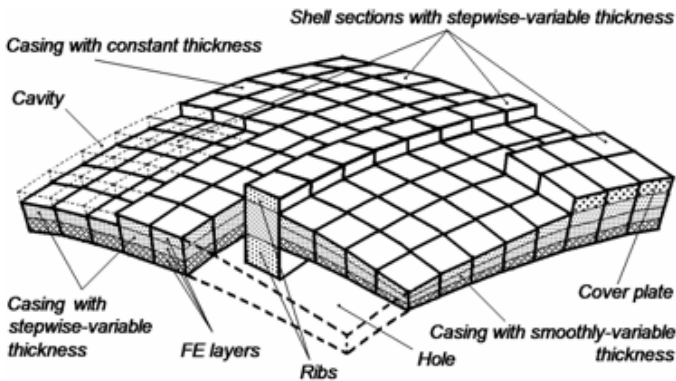


Fig. 14. Shells heterogeneity

A non-classical kinematic hypothesis of a deformable straight line is used - a straight line in the thickness direction, which contracts or elongates, remains straight after the deformation of the shell. This line is not necessarily the normal to the midsurface of the shell. The layers of the shell are rigidly interconnected in a monolithic package and are jointly deformed without slipping and tearing along the surfaces of contact on which the condition of equality of the components of the displacement vector is satisfied. A model of

an elastic nonlinearly deformable continuous media is used. More specifically, the components of large displacements and small deformations are linear functions of displacements.

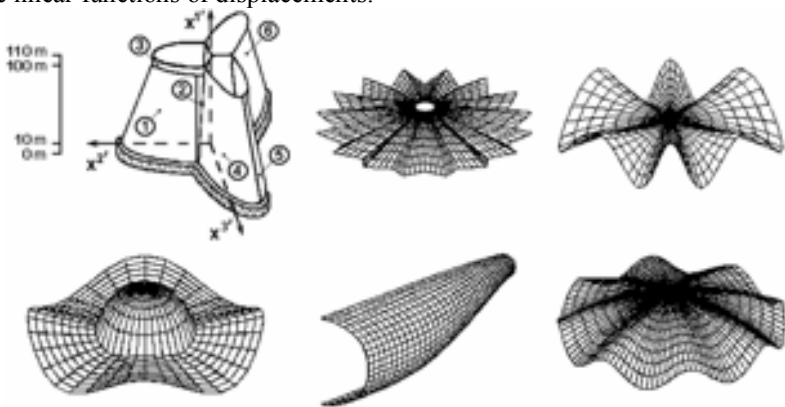


Fig. 15. Examples of thin-walled shell structures of complex shape

The difficulties of taking into account the joint work of structural elements of various dimensions in a heterogeneous shell are overcome by modeling sections of stepwise variable thickness with the same type of spatial finite element. In the formation of resolving equations, the procedures for determining the necessary values of the element parameters are used, which allow to accurately describe the geometry of structural elements and take into account their joint work as three-dimensional bodies.

An assessment of the curvature  $K = 2a^2/(Rh)$  effect on the stability of a smooth, hinged panel of constant thickness is presented in Fig. 16 as an example.

The shell under consideration was heated linearly over thickness (the outer surface cooled and the inner heated by the same value  $T$ , °C). Equilibrium modes are stable up to the top and after the bottom critical points of the diagrams. The panels at  $K \leq 16$  do not lose their stability. It was established that the “shallow” panels ( $K \leq 30$ ) deform and lose their stability according to the “P” mode. The modes of deformation of “non- shallow” panels ( $K > 30$ ) during the loading process change and become more complex (modes “a - e” for  $K = 48$ ).

Currently, this topic has been extended to the problems of natural vibrations of inhomogeneous shells, taking into account the prestressed state, when the modal characteristics of the object are calculated at each step of thermo-force loading [10].

Note that not only the stability of the shells was developed in the framework of numerical studies using computers. Some characteristic tasks were investigated, the solution of which without the use of computers was previously inaccessible. These included such essentially nonlinear problems as the stability analysis of systems with one-way connections [16, 32], the spatial

stability problem of the skeleton of high-rise buildings [364], a subtle analysis of the role of ribs [3], and many other problems that arise in design practice.

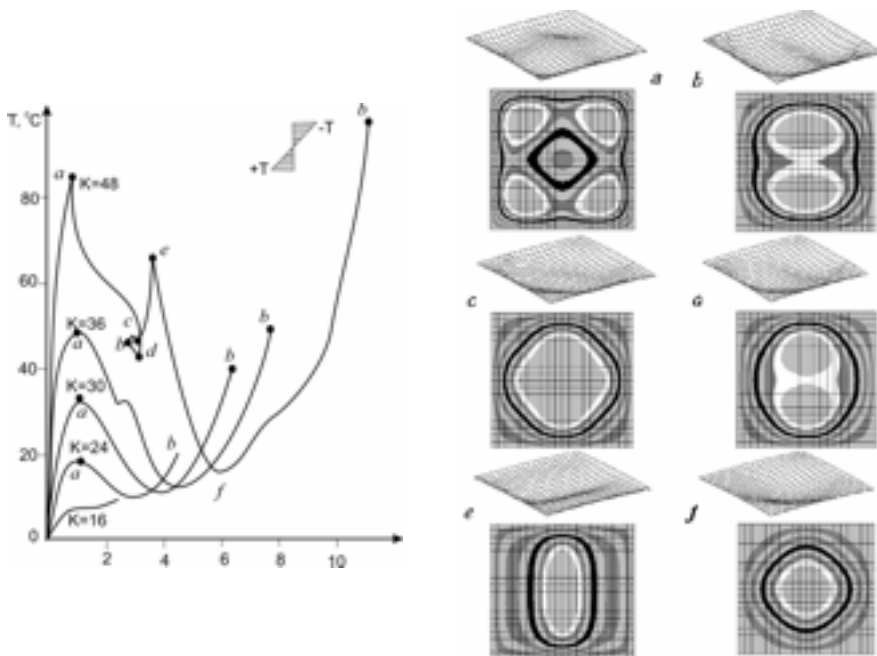


Fig. 16. Shell stability under thermal loading

### 5.5. Computational problems

One of the most laborious tasks arising in the process of calculating structures is the solution of large systems of linear and nonlinear equations. The limitations of the random-access memory and the low speed of exchanges with external memory, characteristic of the first computers available for use, made it necessary to focus on the use of iterative methods for solving the problem [264, 499, 500]. Various variants of the descent method were tested and developed: the gradient method, the ravine method, the upper relaxation method, etc. The gradient descent method was used to find the minimum eigenvalue in the stability problem for rectangular plates with notches [376].

Only gradually, around the end of the sixties, were the first successful results of applying direct methods for solving large systems of linear equations demonstrated. For example, it was pointed out that the Gauss block method was effectively used with a minimized number of references to external memory [232]. It was especially effective in solving problems with a large number of loading options or in combination with iterative algorithms that require multiple solutions of equations with a constant matrix and variable right-hand sides (for example, in the method of elastic solutions).

Two circumstances played an important role in promoting direct methods: taking into account the tape structure of the coefficient matrix, which is realized with a certain numbering of unknowns, and effective discipline of exchanges with external memory [18, 422, 423]. Naturally, the development of computational algorithms, leading to the appearance of a successful structure of the coefficient matrix, was the subject of special studies [5, 186].

Starting from the 90s, direct methods, which take into account the sparse structure of the matrix, have gained great popularity. Due to efficient ordering algorithms, which significantly reduce the fillings in the factorization process, it is possible to significantly reduce the size of the factorized matrix and the duration of calculations [121, 122].

In modern commercial and scientific FEM programs when solving large-scale problems, the most widespread are the subtle implementations of the subspace iteration method and Lanczos method. The method of iterating a subspace has established itself as a very reliable, although being not the fastest method. The block version of this method is especially effective when using shifts [5].

The generalized conjugate gradient method based on aggregate multilevel preconditioning [113] combines the advantages of the conjugate gradient method, aggregate preconditioning, and the use of a shift strategy.

Serious computational difficulties were overcome in solving the problem of the inelastic behavior of reinforced concrete structures. Two different methods of solving it were considered. In the first method, the load parameter is the leading one. The algorithm [118] uses the implicit Newmark method when balancing nonlinear iterations by the Newton-Raphson method. In it, the leading parameter is the load. When solving many problems of statics, this approach leads to the fact that when “slipping” the limit point of the equilibrium state curve (the “load-characteristic displacement” curve), the Newton-Raphson method diverges. Therefore, an algorithm for solving a nonlinear static problem was applied, in which the leading parameter is the length of the perpendicular to the tangent to the equilibrium curve.

The procedures for the formation of a matrix of tangential stiffness, its factorization, direct and inverse substitutions, and finally, calculations of the vector of internal forces are performed by more than 99% of the computational work, so they demanded maximum acceleration. The stiffness matrix factorization and direct / inverse substitutions are determined by the choice of the solver [115, 116, 118]. The peculiarity of these tasks is that the duration of the aggregation procedure of the tangential stiffness matrix, as well as the procedure for determining the vector of internal forces are values of the same order as the duration of solving a system of linearized algebraic equations, since the calculation of finite element stiffness matrices is associated with the calculation of a large number of integrals. Therefore, schemes were developed for parallelizing these algorithms.

The appeal of specialists in the theory of structures to the construction of computational algorithms proved to be useful and fruitful. As an example, we can refer to the idea of using superelements, proceeding from mechanics, which then received independent development in computational mathematics in the form of a block multi-frontal method for solving equations [118]. At the same time, the very idea of the simultaneous formation and solution of resolving equations (frontal method) was also proposed by specialists in structural mechanics. Other issues of the computational plan were also solved, in particular, methods for improving conditionality were found [118, 373], a system for monitoring results was created. The discovered mechanical interpretation of the computational operations turned out to be useful, in particular, the fact that the step of Jordan exceptions corresponds to the installation / removal of the connection [170].

However, not only the problem of solving resolving equations was the subject of research. For example, the algorithmization of stability problems with the massive use of complex spatial design schemes required more precise answers to a number of fundamental questions. And here some important results were obtained: it was shown in [348] that the habitual use of the concept of computational lengths cannot be applied in some spatial problems, and in problems solved taking into account the sequence of construction creation it is necessary to clarify the very issue of stability [331, 340]. As applied to spatial problems, it was necessary to more strictly substantiate the possibility of using the well-known method of checking the location of a given loading parameter in the spectrum of critical values [419].

### **5.6. Numerical solution of nonlinear problems**

Computer modeling has become practically the main method for solving nonlinear problems, for which complicated performances have become real, both from the point of view of the geometry and structure of the analyzed objects, and taking into account the complicated loading conditions and the influence of the external environment.

Naturally, the transition to non-linear problems required clarification, specification and improvement of the usual approaches. It was necessary to develop algorithmic numerical methods for nonlinear analysis, which can be implemented on a computer, and allow for a global study of resolving nonlinear differential equations, identification of singular points, construction of branching solutions and analysis of their stability.

To overcome the basic mathematical difficulties in solving static problems, in [208] it is proposed to use the method of continuation with respect to the loading intensity parameter, based on the transition to the Cauchy problem, and the numerical analysis of the obtained problem. And in the neighborhoods of singular points where the problem degenerates and vanishes the Jacobian of a system of nonlinear algebraic equations (critical value), an algorithm for approximate branching is used. In the study of the stability of T-periodic processes in [209], a synthesis of the procedure for the continuation of the solution by the leading parameter, the Floquet theory, and the methods of the



theory of branching was proposed. An example of such an analysis from [213] in the form of the dependence of the amplitude of vertical oscillations of the node of the von Mises truss on the loading intensity is presented in Fig. 17.

When passing through the limiting points, the method of changing the leading parameter was used, later a method was used that applies the length of the arc of the state curve as the continuation parameter [35]. This markedly simplified the passage of a turning point in the study of equilibrium states or regular periodic motions. The continuation technique makes it possible to find regions of stable and unstable periodic solutions using the eigenvalues of the monodromy matrix on the basis of the Floquet theory.

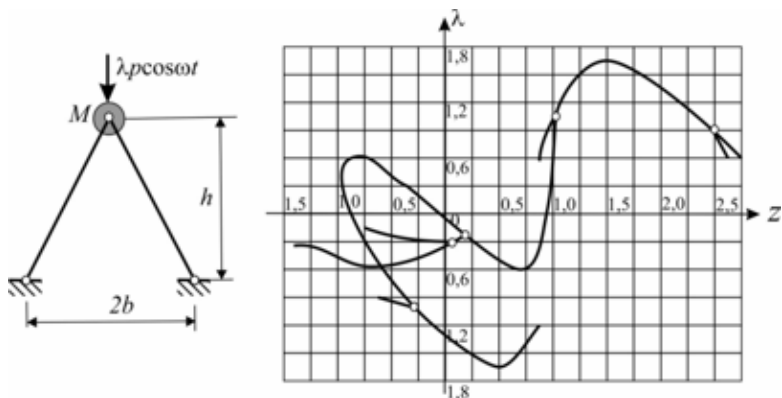


Fig. 17. Dynamic loading of the von Mises truss

The methods developed in [208, 209] were widely used to solve a number of specific problems that were performed at the PNDL TPK of the KCEI (see, for example, [213] or [216]).

In the monograph by V.I. Guliaev, V.V. Gaidachuk and V.L. Koshkin [214] nonlinear problems of statics and dynamics of flexible bars were considered. The behavior of flexible rectilinear and curvilinear bars, subjected to arbitrary static and dynamic disturbances caused by force and kinematic excitation of harmonic oscillations, portable, relative and Coriolis inertial forces, gyroscopic forces of rotating rotors, nonconservative forces of interaction with external and internal flows of liquid and gas was studied. An example is the analysis of the behavior of helical coil springs under static loading (Fig. 18).

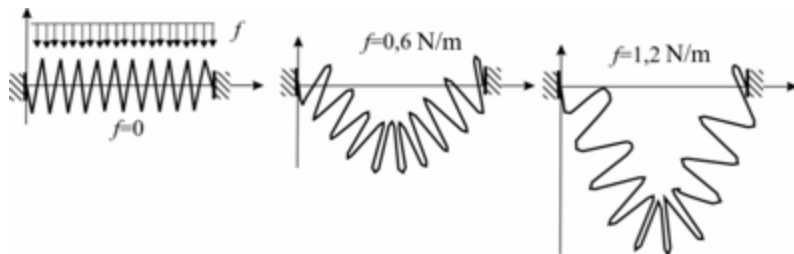


Fig. 18. The deformation of the coil spring

Later V.I. Guliaev, V.V. Gaidachuk and their students considered a number of practical important and complex tasks related to the blades of helicopters and wind turbines, as well as the operation of drill strings for ultra-deep oil and gas production (see, for example, [215, 217, 218]). In this case, some effects and phenomena that had not been studied before were discovered.

When performing the stress-strain analysis of plates and shells, first of all, attention was paid to their work during large deflections (see, for example, [179, 199, 201, 295]). Nonlinear problems with physical nonlinearity of various origins were also studied (see, for example, [22, 103, 176, 180, 191, 252, 284, 435]).

Speaking about the achievements of the Kyiv school in solving nonlinear problems of the theory of structures, it should be noted that it was in the Kyiv Institute of Hydromechanics of the Ukrainian Academy of Sciences where E.Y. Rashba opened a new class of nonlinear problems that later became called genetically nonlinear ones [369]. This work was the first study in which the mechanical problem of building up a solid in the field of mass forces was solved. Quasistatic elastic stresses in an infinitely extended wedge, continuously increased by not strained but heavy layers were determined in it. The resulting solution is markedly different from the classic one, which is clearly shown in Fig. 19.



Emmanuil  
Yosynovych Rashba

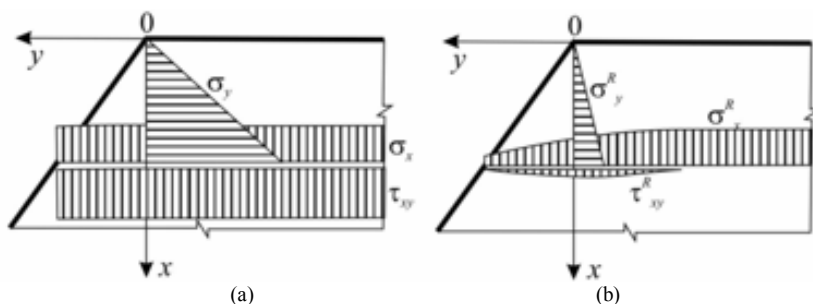


Fig. 19. Comparison of solutions:

(a) - with instant erection, (b) - taking into account gradual build-up

Nonlinearity of this type is especially pronounced in massive structures such as dams. Later another employee of the Institute of Hydromechanics L.I. Dyatlovtsky actively engaged in the computation of such objects after the professor E.I. Rashba [84, 85, 86].

It should be noted that in recent years, genetic nonlinearity is almost always taken into account when designing high-rise buildings, bridges and other complex structures, where the effect of changes in the calculation structural model is noticeable [342].

### 5.7. Numerical methods of probabilistic calculation

Only a few problems of probabilistic analysis and statistical dynamics of structures can be solved analytically. Numerical studies of such problems were performed in the department of statistical methods in structural mechanics at the PNDL TPK of KCEI (later - Institute of Structural Mechanics of Kyiv National University of Construction and Architecture, KNUCA) under the guidance of Ye.S. Dekhtiariuk and dealt with various problems.

Statistical processing of the results of field tests was the subject of research in the works [48, 123, 518]. On the basis of the developed algorithms, a software system was created that focused on processing experimental data concerning the parameters of the mechanical behavior of aircraft.

The development of a generalized statistical testS method (Monte Carlo method) and its use for solving problems in the theory of elasticity are presented in [79, 303, 304]. Selective realizations of random external loads were modeled on the basis of the fast Fourier transform [302]. In this way, problems of stationary random vibrations of mechanical systems were solved in linear and geometrically nonlinear formulations, and the probability of failures was also studied [304].

In problems of statistical dynamics [14], the main attention was paid to the analysis of dynamic stability and the study of simple and combination resonances [12, 13, 77, 80, 81, 82, 233].

### 5.8. Cable-stayed structures

A peculiar class of nonlinear problems is associated with the calculation of cable-stayed and cable-stayed-bar systems, which began to be very actively studied in the sixties of the twentieth century. Interest in this problem was initiated by the translation of the book by F. Otto "The Suspended Roof", which appeared in 1960, and was stimulated by the real problems of cable-stayed systems design that were solved in UkrRD steel construction, KyivZNDIEP and NDIBK.

The first works performed in Kyiv belonged to V.M. Gordeiev [168, 171, 173] and L.G. Dmitriev [96, 97, 98]. The basic resolving equations of statics and dynamics for cable-suspended roofs having the form of a grid located on the surface of negative Gaussian curvature (Fig. 20) were obtained. In the work [167], the mesh was so thick that it was already a tissue membrane.



Yevgenii  
Semenovich  
Dekhtiariuk  
(1935-2012)

Similar was the formulation of the problem in works performed in the PNDL TPK of KCEI [257, 262, 377], where the main resolving equations were obtained and analyzed for statics and dynamics of cable-stayed networks. The approach to static calculation as a special case of dynamic analysis made it possible to develop an effective scheme for solving nonlinear problems of statics using a modified dynamic relaxation, called the method of discrete breaking [257].

The problem of calculating the cable system should be solved in a geometrically nonlinear formulation. An exception can only be a calculation for the equilibrium load, and then only if the elastic displacements it causes are sufficiently small. With regard to cable-stayed networks, the possibility of linear formulation was examined and justified by V.M. Gordeiev [171, 172], who indicated the limits of applicability of such an approach and solved a number of important practical problems.

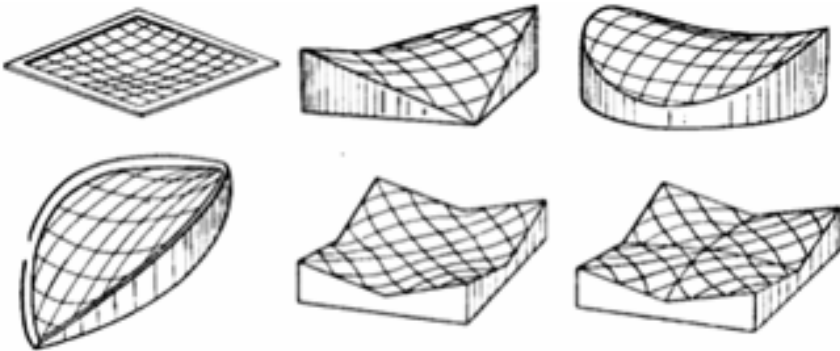


Fig. 20. Cable-stayed networks

Static-kinematic analysis of cable-stayed systems was investigated by L.G. Dmytriiev [96], while the fundamentally important role of the possibility of creating a prestress for instant-rigid systems to which cable networks belong, was established, and also the ranking of properties associated with the possibility of creating a prestress was indicated. It is indicated that the prestress corresponds to the constraints that prevents the skewing of the bars.

In another work, L.G. Dmytriiev presents cable-stayed networks of not only a simple rectangular structure, but also systems of a more general form (triangular, hexagonal, etc.) [97]. Extensive information on the structural solutions of cable-stayed coatings, including the author's development, is presented in a monograph written by him together with O.V. Kasilov [99]. The work of the team, headed by



Leonid  
Georgiiiovych  
Dmytriiev  
(1931–2018)



Oleksandr  
Vasyliiovych  
Kasilov  
(1933 - 1988)

L.G. Dmytriiev and O.V. Kasilov, on the development and implementation of cable-stayed structures in the construction were awarded the State Prize of Ukraine.

L.G. Dmytriiev proposed an iterative process for calculating cable-stayed networks, at each stage of which the magnitudes of displacements are refined until the unbalanced part of the load found from the static conditions is sufficiently small [101]. Subsequently, L.G. Dmitriiev applied another fast converging process to the calculation of networks, based on the separation of two types of geometric nonlinearity), which make it possible to use the computer efficiently [100].

A special case is associated with the calculation of vertically arranged flat cable-stayed networks, which served as a supporting structure for a short-wave antenna (Fig. 21). The tension of the elements of such a network was provided by a system of counterweights and were known.

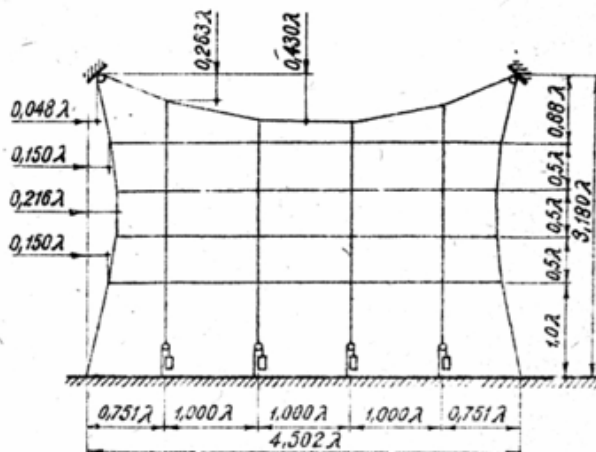


Fig. 21. Layout of antenna network for radio waves of  $\lambda$  length

For such networks, a kind of structural mechanics has been built, where the relative cable tension - the projections of the force in the thread onto the network plane related to the length of the thread section - play the role of stiffness. This structural mechanics is linear with respect to the displacement of network nodes out-of-plane and with respect to the angle of inclination of cables to the network plane. These values are linearly dependent on the load. But equilibrium equations alone, as a rule, are not enough for constructing these diagrams. As a result, statically indeterminate problems arise that can be solved by the force method and the displacement method.

An independent and difficult task is the problem of forming spatial cable-stayed networks. In such a network, the length of the cables must be chosen so that the network, even in the absence of a load, is taut and has a functionally determined shape.

Most cable-stayed networks are designed so that the cables in the nodes are not interrupted, and during the installation period they only touch the crossing points and slide one over the other. The network acquires its form at this particular time.

To solve the corresponding problem, a kind of technique was proposed [165, 1884, 185], in which a nodal insert was assigned to the crossing point of two cables (Fig. 22) without interfering with the mutual displacement of the cables.

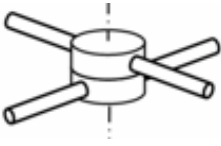


Fig. 22. Nodal insert

Each insert is characterized by three spatial and two cable coordinates, which are unknown when performing the calculation. The spatial coordinate is the Cartesian coordinate of the center point of the insert in the general coordinate system, the cable

coordinate is the distance from the beginning of the cable to the axis of the insert. The system of nonlinear equations for the mentioned unknowns was solved by the step iteration method.

Fig. 23 shows the result of the choice of the shape of a radial cable network with a rigid external and flexible internal contour, to which the radial cables are attached so that their ends can freely slide along the contour. The shape of the inner contour and the coordinates of attachment of the radial cables to it are defined flawlessly.

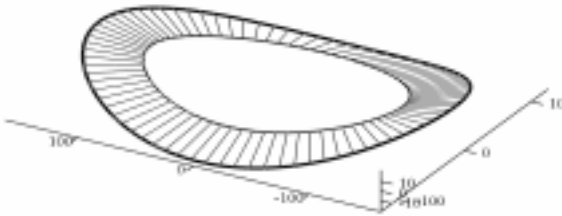


Fig. 23. Radial cable network with flexible internal contour

Suspended roofs have often been considered from the point of calculation of very flexible membrane-type shells (see, for example, [263]). It should be noted here that when it comes to awning structures where the fabric is used, it is necessary to take into account that the fabric shell not only does not perceive bending moments (and this is similar to a membrane), but also does not perceive a shear force. This effect was taken into account in [167, 172], and is not taken into account, for example, in [43]. Thus, tent constructions were excluded from the consideration, and examples of calculations of soft shells belonged to constructions made of rubber-material.

For such structures, as well as for thin membranes, the problem of transition to a uniaxial stressed state in the zones of fold formation is important (Fig. 24). To solve this problem, two main methods were used: modeling the zone of fold formation by a constructive anisotropic material [43, 395], or

modeling with a material that did not perceive compressive stresses, i.e. peculiar unilateral constraint [179].

The methods of static calculation and stability testing were developed for the cable-bar systems, to which the masts, bridges, etc. belonged (Fig. 25). The canonical equations of the force method, the displacement method and the mixed method were generalized for such structures [169, 328, 333]. The dynamics of such systems is discussed in [258]. At the same time, the possibility of disconnecting from the work of the cable element, which turned out to be compressed at some point in time, and its incorporation into work at restoration of tension, was taken into account.

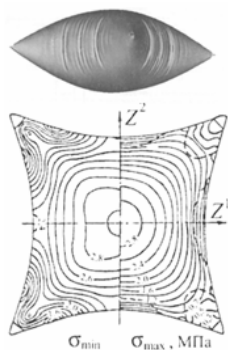
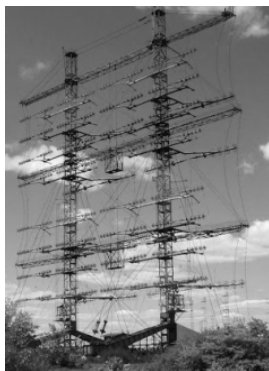


Fig. 24. Results of study of pneumatic pillow (the folded area is limited by the dotted line)



(a)



(b)

Fig. 25. Cable-bar systems: (a) - transmitting antenna, (b) - cable-stayed bridge

An important engineering problem related to the regulation of forces in such structures as applied to cable-stayed bridges was solved in the works of G.B. Fuks and M.M. Korneiev [125, 267], who considered regulation as a process of creating prestressing in a weightless design scheme by forced deformation of the design model being manufactured.

In parallel with the works on the theory of cable and cable-bar systems, studies have been developed on the methods for calculating flexible ropes of finite stiffness which were conducted by V.M. Shymanovsky [393, 396, 400].



Geogii Borysovykh  
Fuks  
(1927-2008)



Vitalii  
Mykolaiovych  
Shymanovsky  
(1928-2000)

A natural generalization of the design model of the cable-bar system, in which there are non-perceiving compression cable elements, is a system with unilateral constraints, to the analysis of which was devoted a series of works of a pioneer nature. Thus, in [183] it was proved that the problem of static calculation of systems with unilateral constraints can be represented as a quadratic programming problem. The ways of formulation of such a problem in forces and in displacements were indicated in this paper. Methods of static calculation and verification of stability, as well as properties of possible solutions, are analyzed in detail in [330]. Algorithms for solving the

problems of stability and vibrations of deformable systems with unilateral constraints, and the solution of a number of such problems are presented in the book [18].

In order to reduce the dimension of the original nonlinear equations and to decrease the complexity of the nonlinear problems of shell statics and dynamics, a reduction of the basis was proposed. This reduction was based on the assumption that the desired form of equilibrium of the entire construction with a sufficient degree of accuracy can be represented as a linear combination of several specially selected functions [78, 192, 260, 286].

### 5.9. Optimal design

The desire to optimize design has always been present in project practice and served as a motive for a number of research work. One of the first in this series is the work dedicated to the search for the regularities, which the constructed structures of different types possess. Such a systematic comparison of various schemes of bridge structures with the aim of choosing the best solution was carried out by many researchers, including Ye.O. Paton [320].

This direction of research to some extent is not theoretical but experimental, since it can be assumed that every real construction is a unit experiment, although not specifically designed to investigate the regularity of the behavior of the whole set of similar structures. However, the multiplicity of this kind of experimental data allows them to be used to identify hidden patterns or to formulate a problem for conducting special investigations [92, 318, 323].

The problem of optimization is often considered as part of a work devoted to the analysis of behavior and recommendations for the design of structures of a particular type or destination. As an example, it can be noted that a significant part of the book by Ye.O. Paton and B.M. Gorbunov [321] is devoted to the problems of choosing the best constructional solution (optimal height and magnitude of spans, the best grid scheme, etc.)

The simplest problems associated with the optimal layout of the cross section of the beam, or with choosing the shape of the beam of equal resistance were considered. Here, the work of I.Ya. Shtaierman [412], can be marked. He, unlike other authors, as a beam of equal resistance chose one, for which



the greatest tangential stresses were identical along the entire length (the third strength theory).

Starting from the middle of the 20th century mathematical methods of optimization began to be used to solve such problems: linear and nonlinear mathematical programming, theory of optimal control, nonclassical variational calculus, and so on. Most often, tasks of optimizing objects of a certain type were considered. Thus, for the tower structures using the Pontryagin maximum principle, the optimum shape of the tower structure belts was found [178]. For the spatial structural roofs the optimal parameters of the design solution were found with all costs included, accounting heating costs and other current costs [49, 178, 398, 523]. With the use of computer technology and in a refined formulation, optimal design of crane beams [139] was considered, when not only the cost of materials was taken into account, but also the laboriousness of constructions production. Interesting results were obtained here, showing the differences in the design, depending on the ratio of the value of materials and labor resources.

Optimization problems were solved, as a rule, in relation to structures of a particular type. The problems of finding the optimal configuration of the shell surfaces [20, 68, 70, 505, 509], optimal reinforcement of constructions from reinforced concrete [74, 76, 133, 527] and others were considered. Searching for an optimal shape was most often considered in the form of searching for optimal parameters when choosing from a certain family of functions (for example, surfaces of a transfer or surfaces developed from a certain initial form by two-component geometric transformations [71]).

It is known that when calculating for single load, the construction of the minimum weight is statically determinate. However, most of the real problems are related to the case of multicomponent loads and there are significant complications. In the series of works by I.D. Glikin and A.I. Kozachevsky [130, 131, 132, 133], an iterative step-by-step approach to the optimum with small changes in the parameters of the problem (sections of the elements or their reinforcement) was proposed for optimization at many loads. It leads to the necessity of solving the problem of linear programming at each step of the iteration. In the general case, such a process leads to a local minimum. An alternative approach is to find the optimal solution taking into account the possibility of creating a pre-stress. The search for such a solution, as the task of finding the Chebyshev point of the system of inequalities describing the limitations of strength, was proposed in [335].

The practice of solving specific optimization problems has shown that search for an exact solution, especially for multi-extreme problems, is associated with large computational and fundamental difficulties. Therefore, in such cases, the problem was coarsened and some approximate solution was searched for.

It should be noted that into the concept of an approximate solution to the optimization problem mathematicians and engineers often put not one and the same meaning. From the mathematical point of view, the discrepancy with the exact solution is estimated by the difference in the coordinates of the points of the approximate and exact solutions, whereas from the engineer's point of

view, the deviation from the global minimum of the objective function should be considered as a solution assessment. With a small deviation, the acceptability for the practice of such an approximate result is not in doubt.

This idea apparently was first expressed and broadly considered by V.M. Gordeiev, who suggested instead of finding the point of an extremum to seek out, and then analyze in more detail all the set of solutions adjacent to this point [162, 174, 397, 398]. The fact is that most real optimization problems have a "smooth extremum", i.e. even a noticeable deviation from the ideal solution does not change the value of the objective function much. So it gives the opportunity to take into account additional difficult to formalize conditions (for example, the discreteness of certain parameters) without leaving the set of solutions close to the optimal in the space of design parameters.

To solve the problem in such a statement, a specially developed method of uniform reserves proved to be well-adapted. This new method is the development of known methods of the interior point and the method of centers for solving mathematical programming problems [398]. In addition, approximations of the solution region close to the optimal by n-dimensional ellipsoid (n is the number of design variables) were used.

The cycle of studies related to the optimization of metal structures, including metal cable-stayed bridges, was performed by V.V. Trofymovych and V.O. Permiakov [352, 353, 461, 463, 464]. Among others, problems were solved, in which the prestress value of the system and the cross-section of its elements were chosen, as well as the problems of finding optimal configuration of the lattice structure. Linear programming was used as a research tool.

More complicated formulations of optimization problems were solved by methods of nonlinear mathematical programming, while using the genetic algorithm in combination with descent methods. For example, the construction of a wind power plant was optimized. Here, based on the minimizing the cost of kilowatts of the output power, the parameters of the bearing structure as well as the diameter of the wind wheel were optimized [325].

As for reinforced concrete structures, here the problem formulation often involved optimizing only of structural reinforcement. It is also assumed that the formwork dimensions of the structure are specified and are not subject to change (i.e. stiffness parameters are practically known). When optimizing shell structures, additional assumption was used according to [68, 69, 70, 74, 527]. This assumption is that reinforcement should correspond to the maximum load, which



Viktor  
Volodymyrovych  
Trofymovych  
(1926–2004)



Volodymyr  
Oleksandrovych  
Permiakov  
(1938–2007)

is determined on the basis of the kinematic model of the method of limiting

equilibrium. The account of redistribution of internal forces due to the inelastic behavior of the material in the optimization problem of reinforcement was carried out account in [40].

In view of the relatively low consumption of reinforcement in shell structures, the optimization effect is relatively small. More interesting results are related to finding the optimal surface of the shells. Studies of this type of were performed for the shells of revolution [67, 371] and shells whose shape was a transfer surface [505].

The description of optimization methods applied in structural mechanics, including problems with incomplete load information or multicriteria optimization problems, was presented in the monograph by V.I. Guliaiev, V.A. Bazhenov and V.L. Koshkin [211]. In contrast to the canonical optimization problems formulated in finite-dimensional spaces and solved by methods of mathematical programming, the main attention was paid to optimal design problems in functional spaces. As illustrative examples, the problems of minimizing the mass of thin conical, spherical (Fig. 26) and cylindrical elastic shells under strength and geometric constraints were solved. Optimal shell design taking into account geometrically nonlinear behavior under stability constraints was considered as well.

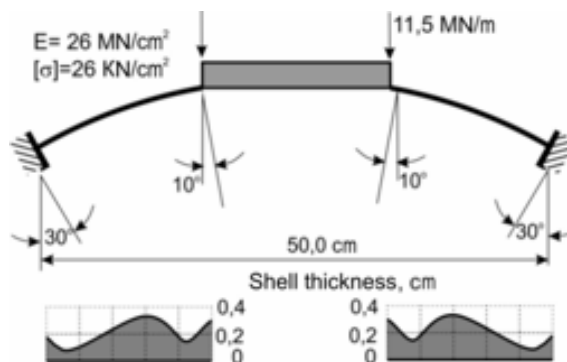


Fig. 26. A spherical shell with optimal mass distribution

Also the problem of choosing the optimal parameters of the passive vibration protection system was solved. Vibration dampers of simple systems subject to the harmonic loading of a changing frequency were considered. Later, the problem was generalized to the case of optimization of the pendulum damper during random loading [17], as well as to search for the optimal parameters of the pendulum damper of high-rise structures vibrations under the influence of wind pulsations [15].

In conclusion of this section, we once again mention the problem of determining the disadvantageous combination of forces acting on an elastic system. This problem, on the one hand, belongs to the classical directions of structural mechanics, such as, for example, the problem of loading lines of influence, and on the other hand, it turned out to be in a certain shadow. Interest



Georgii  
Vakhtangovich  
Isakhanov  
(1921–2012)

in it was revived during the transition to algorithmization of calculations, and here, the authors of [6] made a serious breakthrough in their time, consisting in using the idea of describing the logical relationship of individual load cases in the form of a directed graph. This idea was subsequently developed in [175] and found its completion in [163]. In the latter of them, a rather general concept of calculation is built, based on the need to search for a disadvantageous state, and this search does not complete the calculation, but organizes it from the very beginning. Here the work [131, 436], dedicated to this topic, also could be

mentioned.

Usually, the basis of calculations, searching for an unfavorable combination of loads, is the use of the superposition of the contributions of individual load cases, i.e. it is assumed that the calculation relates to a linear system. An attempt to solve the problem of finding unfavorable loading, based on the application of the theory of optimal control, was undertaken in [329], but it did not find wide application, since the complexity of the calculations turned out to be excessive. The problem is still waiting to be solved.

## 6. SCIENTIFIC SCHOOL OF THE STRUCTURAL MECHANICS OF THE KYIV NATIONAL UNIVERSITY OF CONSTRUCTION AND ARCHITECTURE

In 1961, at the Kyiv National University of Civil Engineering and Architecture (then the Kyiv Civil Engineering Institute, KCEI), on the initiative of Professor D.V. Vainberg the Research Laboratory of Thin-walled Spatial Structures was established. It was transformed in 1966 into the Problem Research Laboratory of Thin-walled Spatial Structures (PNDL TPK).



David Veniamynovich  
Vainberg  
(1905–1973)

Professor D.V. Vainberg headed the Department of Structural Mechanics and PNDL TPK until his death in 1973. From 1974 to 1989 the head of the department and the supervisor of the laboratory was Professor G.V. Isakhanov. From 1989 to the present, the Department of Structural Mechanics and the laboratory, subsequently transformed into a research institute, is headed by a full member of the National Academy of Pedagogical Sciences of Ukraine, Professor V.A. Bazhenov.

The Scientific and Research Institute of Structural Mechanics (Naukovo-doslidny institute budivel'noi mekhaniky, NDIBM), established on the basis of PNDL TPK in 1991, conducts fundamental and applied research on the theory and methods of calculating the strength, stability, and vibrations of complex spatial structures under the external influences of a different physical nature and developing on this basis a

problem and object oriented software. The results of the work have practical application for solving the problems of strength, stiffness, stability, modeling of vibration processes and determining the bearing capacity of building constructions and structures. Individual parts of the constructions, as well as critical structural elements of machines operating in various industries, including gas and steam blades turbines, turbine rotors, nuclear reactor shells, shallow shells of roofs of underground and ground structures, bearing elements of towers and masts, bar systems, pressure vessels, elements of valves, damper devices and a number of other important objects also subject to close attention and calculation.

The result of the activities of the Scientific School was the training of a significant number of highly qualified specialists in the field of structural mechanics and mechanics of a deformable solid. In particular, during the existence of the Scientific School, following researchers defended their candidate dissertations:

1964	V.M. Rakivnenko, G.B. Kovnerystov	1965	V.Z. Zhdan, I.A. Kolomiets
1966	F.O. Romanenko, O.L. Syniavsky, V.I. Guliaiev	1967	Yu.K. Chekushkin, A.V. Odynets
1968	O.S. Sakharov, V.P. Stukalov	1969	V.A. Bazhenov, V.M. Kyslooky, I.A. Bazylevych
1970	Jo.Z. Roitfarb, P.P. Voroshko S.M. Liubchenko	1971	Yu.V. Veriuzhsky
1972	V.M. Gerashchenko, R.K. Demianiuk	1973	V.K. Chybiriakov
1974	O.V. Shishov, D.R. Kolev, V.V. Kyrychovsky, Ye.O. Gotsuliak	1976	V.G. Kobiev, G.Yo. Melnichenko, S.Ya. Granat, P.P. Lizunov
1977	V.V. Gaidaichuk, G.G. Zavialov, Chu Viet Kyong	1978	O.I. Guliar, R.K. Bobrov, S.M. Chorny
1979	A.I. Vusatiuk, Spartak Mohammed Salem, A.S. Svystov	1980	M.O. Solovei, I.Ye. Goncharenko, O.A. Kyrychuk, Adnan Ali, Pemsing Krishna, Samsur Abdullah, Hoang Xuan Liong
1981	O.N. Beskov, V.Ye. Veryzhenko, O.I. Vynnyk, L.A. Vriukalo, O.L. Kozak, A.D. Legostaiev, B.M. Marzysyn, O.I. Ogloblia, O.O. Kholodenko	1982	N.T. Zhadrasinov, O.Ya. Petrenko, V.V. Savytsky, V.V. Khymentko, V.K. Tsykhanovsky, O.V. Shymanovsky
1983	G.S. Kondakov, S.L. Popov, O.V. Savchuk, V.M. Chaban	1984	V.M. Karkhaliov, P.G. Melnyk-Melnykov

- 1985 O.V. Gondliakh, Dan Khyu Kun, O.A. Zverev, O.I. Korzh, T.I. Matchenko, T.L. Savchenko, M.K. Sysengaliiev, Fan Din Ba
- 1986 O.V. Glimbovsky, V.M. Yermishev, S.V. Zablotsky, Saudi Khasen, V.V. Chemlaiev
- 1987 K.Ye. Boyko, L.S. Ivanova, V.B. Kovtunov, V.L. Koshkin, S.G. Kravchenko, T.A. Kushnirenko, Kyonh Le Chunh, Ye.D. Lumelsky, Nhuen Shy Chan, O.I. Pylypenko, I.V. Polovets, S.V. Potapov, A.G. Topor, Khettal Takhar, V.M. Chyrva
- 1988 Yu.M. Appanovych, G.G. Burtsev, A.A. Grom, O.O. Odynets
- 1989 V.G. Borysenko, O.V. Bratko, O.B. Vasyliiev, G.Ye. Zakharov, Ignas Aloyis Rubaratuka, V.Ye. Kravtsov, V.V. Lazhechnikov, O.Ye. Mayboroda, O.Yu. Mozharovsky, Yu.Z. Totoyev, O.B. Ushak
- 1990 V.S. Boyandin, G.L. Vasilieva, Yu.L. Dinkevich, O.V. Mirchevsky, Sadik Obanishola Mufutou, O.A. Fesenko, N.L. Filippova
- 1991 O.A. Bogutsky, O.V. Gerashchenko, A.M. Katsapchuk, Ye.E. Kotenko, O.P. Koshovy, O.B. Krytsky, Yu.S. Petryna, Said Ahmed Shah, Eneramadu Kelechi Obinna
- 1992 O.V. Belolipetska, Yu.V. Vorona, K.Ya. Golovatiuk, Yu.M. Dyadenchuk, T.G. Zakharchenko, I.O. Klimko, Obanishal Sadni, I.O. Serpak, Vazir Pad Shah, V.O. Yasinsky
- 1993 Gbenu Atiglo Raphael, V.O. Pokolenko, Xia I Puygen, P.P. Cheverda, Shu Ming
- 1994 V.K. Bondar, N.A. Valeieva, Jaber Chord, G.M. Ivanchenko, M.G. Kushnirenko, O.O. Lukianchenko, Temor Shah, Yu.O. Shinkar
- 1995 O.G. Kovalevska, V.O. Rutkovsky, Juan Carlos Inchausti
- 1996 Hablos Abd Razzak, G.L. Dmytriiev, Ayat Nouari
- 1997 N.A. Snizhko
- 1999 D.E. Prusov, Labu Mezian, Saidi Amin
- 2001 Yu.D. Geraimovych, S.O. Pyskunov, I.I. Solodei
- 2002 Genduzen Abdenur, Busettta Mubarek
- 2003 O.V. Kostina
- 2005 O.P. Kryvenko

2006	M.V. Goncharenko	2007	O.O. Shkryl
2008	M.S. Barabash	2009	V.P. Andryievsky
2011	S.V. Mytsiuk	2012	Yu.V. Maksymiuk
2013	M.O. Vabischevych, D.V. Bogdan	2019	A.V. Pikul, R.L. Strygun

A considerable number of representatives of the Scientific School defended doctoral dissertations:

1975	O.L. Syniavsky	1978	O.S. Sakharov
1979	V.I. Guliaiev	1981	Yu.V. Veriuzhsky
1984	V.A. Bazhenov	1987	V.Ye. Verizhenko
1989	V.K. Chybirikov, P.P. Lizunov, O.I. Guliar, V.V. Kyrychevsky, P.P. Voroshko	1990	Ye.O. Gotsuliak, Ye.S. Dekhtiariuk
1992	V.V. Gaidaichuk, G.B. Kovnerystov	1993	O.A. Kyrychuk
1994	O.V. Gondliakh	1995	O.L. Kozak, O.I. Ogloblia
1999	V.K. Tsykhanovsky	2004	S.Yu. Fialco, Tran Duc Chinh
2006	Ya.O. Slobodian	2008	M.O. Solovei
2009	V.M. Trach	2011	S.O. Pyskunov
2012	G.M. Ivanchenko	2013	I.I. Solodei
2018	O.O. Shkryl	2019	Yu.V. Maksymiuk, Yu.G. Kozub
2020	M.O. Vabischevych		

In total, during the existence of the scientific school, 31 doctoral and 173 candidate dissertations have been defended.

The achievement of such a level of training of scientific and technical personnel was greatly facilitated by the corresponding publications of applicants in the collection of "Opir Materialiv i Teoriia Sporud".

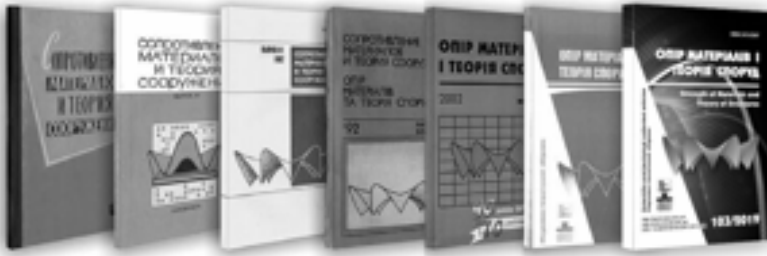
An interdepartmental collection of scientific articles "Soprotivlenie Materialov i Teoriya Sooruzheniy" was created in 1965. Since 1998, the collection has been published under the title "Opir Materialiv i Teoriia Sporud" (Strength of Materials and Theory of Structures: Collection of scientific articles) ISSN 2410-2547. The editorial board of the collection includes professors from universities in Poland, Vietnam, and the USA. 103 issues were published for the period 1965-2019. The collection publishes scientific articles that are prepared in Ukrainian, English and other languages and contain the results of basic research on topical problems of strength of material, structural mechanics, mechanics of a deformable solid, theory of structures, related applied problems of strength and reliability in mechanical engineering, construction and other industries of modern technology. It also highlights the issues of teaching structural mechanics, and provides information on new educational and scientific publications on the subject of the collection. The archive of collection issues is available on the *opir.knuba.edu.ua* website in

compliance with the open access policy in the sense of the Budapest Open Access Initiative, which makes it possible to disseminate new research results in each industry and in each country.

The collection is indexed in the scientific and metric databases Web of Science (<https://openscience.in.ua/ua-journals>), Index Copernicus (<https://journals.indexcopernicus.com/search/details?id=32331>), DOAJ (<https://doaj.org/toc/2410-2547>), has an estimate using the Journal International Compliance Index criterion. JIC index = 0.173 (<https://jicindex.com/journals/42-64>). The collection is also presented in the Ukrainian abstract journal "Source" and in the abstract database "Ukrainian Science".

Since 2017, according to the decision of the Scientific Council of the Ministry of Education and Science of Ukraine, the collection has a special status of a professional publication for publishing the results of research carried out in scientific institutions of Ukraine due to state budget funding in the direction of "Mechanics". The collection received category A in accordance with the Procedure for the Formation of the List of Scientific Professional Publications of Ukraine.

The collection goes to the leading libraries of Ukraine, in particular, goes to Vernadsky National Library of Ukraine (the full text of the collection is also available on the website of this library), the National Parliamentary Library of Ukraine, Vasyl Stefanyk National Scientific Library of Ukraine in Lviv and others.



From 2005 to 2012, a 6-volume edition of "Successes in Mechanics", dedicated to the beginning of the 3rd millennium and edited by Academician O.M. Guz was being published in Kyiv. The edition was intended to familiarize the world scientific community with the latest achievements of Ukrainian science in the field of mechanics. The publication includes generalized review articles published in the journal Applied Mechanics by leading scientists who took an active part in the development of the corresponding areas of mechanics. Among others, the edition presented reviews prepared by representatives of the Scientific School of Structural Mechanics of KNUCA dedicated to solving problems of nonlinear continuum mechanics with the help of MSFE and SFEM [21, 38], as well as the study of nonlinear deformation loads and stability of elastic inhomogeneous shells under thermomechanical loads [42].



NDIBM collaborates with scientific institutions of the National Academy of Sciences of Ukraine - S.P. Timoshenko Institute of Mechanics, G.S. Pisarenko Institute for Problems of Strength, E.O. Paton Electric Welding Institute, with research institutions in the field of construction - State Research Institute of Building Constructions (NDIBK), the Research Institute of Building Production (NIISP), OJSC "V. Shimanovsky UkrRDSteelconstruction", with industrial enterprises in the field of mechanical engineering - State Enterprise "Gas Turbine Research and Production Complex "Zorya-Mashproekt", State Enterprise Zaporozhye Machine-Building Design Bureau "Progress", Motor Sich JSC.

The results of research carried out at NDIBM were awarded the State Prizes of Ukraine in the field of science and technology:

1991 "Theory, methods of mathematical modeling and numerical analysis of the complex spatial structures processes of deformation";

2003 "Scientific research, development and implementation of low energy-intensive technologies and equipment in construction";

2013 "Development of the innovative model and terms for preparation of building industry specialists taking into account possibilities of modern materials and technologies".

## **7. JUSTIFICATION OF CALCULATION STRUCTURAL MODELS, RELIABILITY ANALYSIS**

A variety of modern space-planning solutions and new constructive forms, which are realized at the same time, have created a new design situation, when it is often impossible to focus on traditional and well-tested structural models. At the same time, modern computer technology creates practical possibilities for using calculation structural schemes of a new type. In this regard, the theory of structures has encountered numerous problems associated with the verification and justification of structural models that were not previously studied in detail. A typical example is the structural scheme of a modern high-rise building with bearing structures in the form of a set of plate-shell components.

Applied methods of calculating multi-storey buildings, including those that use the structure-foundation-soil interaction model was developed at KNUCA, NDIBK, KyivZNDIEP (see, for example, [54, 265, 308, 386]). More general problems of the analysis of structural schemes are presented in detail in the monograph [345].

Naturally, the formulation of new complicated problems led to the need for experimental confirmation of the main theoretical provisions. For example, the cycle of experimental studies conducted at the Institute of Mechanics on models of finned cylindrical shells made it possible to obtain data on the effect of geometric imperfections [1, 528]. An experimental estimate of the bearing capacity of spatial roofs was performed by NDIBK [399, 504]. Here the study of a large model of the roof in Kyiv can be mentioned (Fig. 27).

The formation and development of welding in construction required the implementation of a large amount of experimental work to confirm the

strength of welded structures. Such work was carried out at the E.O. Paton Electric Welding Institute by V.V. Shevernitsky, V.I. Trufyakov, V.I. Makhnenko, L.M. Lobanov et al. [299, 301, 392, 465].

In connection with the evaluation of experimental data, studies were carried out to assess the reliability of calculation models and their adequacy to the problem being solved [105, 106, 107].

One of the mass construction objects is high buildings therefore the justification of their design models is an urgent problem. A series of works in this direction was done by D.M. Podolsky in KyivZNDIEP [361, 362, 365]. Among them, it is useful to note the work [365], in which the important problem of taking into account the incompleteness of the available information was formulated. The fact is that a decision on the adequacy of the structural model is made on the basis of this incomplete information. In addition, it is important to evaluate the influence of how the non-ideality of the structural scheme affects the calculation results and how one can predict the expected deviations from the ideal computational model [193, 312].



Fig. 27. Roof of bus fleet

Some of the problems, dictated by the demands of practice, put forward a number of fundamentally new challenges. They had a noticeable influence on the direction of the research carried out in the field of the theory of structures. As an example, we can point to the Chernobyl catastrophe, which caused and raised a number of issues unconventional for the theory of structures.

In particular, it became necessary to retrospectively assess the loads on structures, as well as to predict the behavior of damaged structures, often based on inaccurate data on the degree of their destruction [236, 414, 415, 516, 520, 521]. The purpose of these studies was to get an idea of the state of the structural elements that had become inaccessible for explicit observation. Much work in this direction was carried out by the NDIBK team with the involvement of a number of specialists from other organizations [124].

It was necessary to estimate the risk of possible collapse of structures [266, 309, 394, 522], for which calculations of damaged structures of the 4th power unit and elements of the Shelter Structure were performed and estimates of residual resource were made.

Many of the very diverse problems, including those in the field of the theory of structures, arose in the process of designing a new safe confinement (NSC), which was erected over the Shelter Structure (Fig. 28).

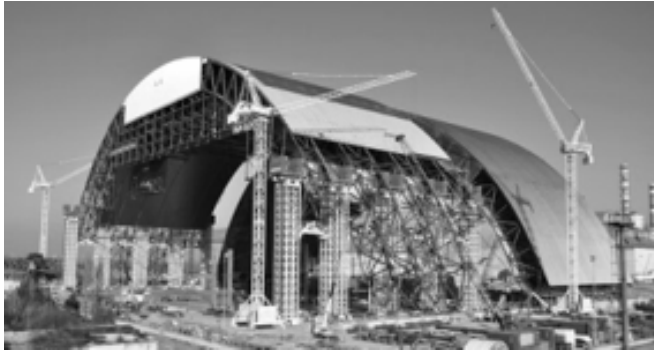


Fig. 28. Installation of confinement structures

Additional studies of possible extreme loads, such as tornadoes, clarification of the seismological situation, development of methods for calculating the effects of an avalanche that could occur when snow slides from an arch system with a span of 256 meters and a height of 108 meters were required (Fig. 29).

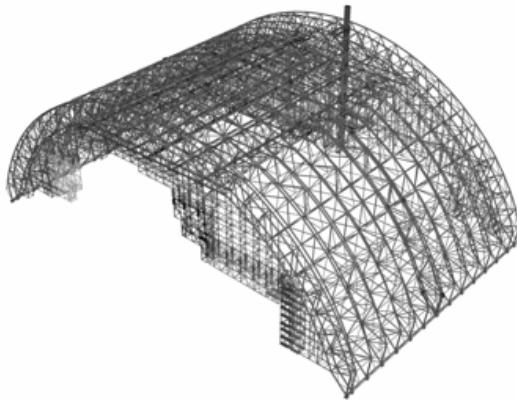


Fig. 29. Bearing frame of confinement (calculation model)

All this issues should be taken into account when designing the NSC [237, 238, 279, 313, 416]. The very problem of designing this grandiose and very responsible structure was also associated with a number of studies, in particular, the question of optimal shaping was studied [44], and the problem of the behavior of a large-span structure during an earthquake was analyzed taking into account the influence of asynchronous seismic excitations of supports on the dynamic response [31, 332].

Naturally, most of the mentioned studies were carried out on the basis of numerical methods that were implemented in the software, both of industrial type and specially designed for the analysis of the behavior of the NSC. Many of the techniques used in these calculations have a wider range of possible applications than just analysis of the strength of NSC structures [332].

In addition, this disaster has allowed realizing the existence of restrictions for the tendency to increase the unit capacity of objects. Explicitly, the concept of unit capacity growth was called into question, apparently for the first time by academician B.Ye. Paton [317] who noted that the "...growth of unit capacity of machines, construction systems, structures and installations is often not accompanied by the same increase in their reliability, and this can lead to large-scale losses, such as, for example, during the Chernobyl disaster". In the theory of structures, an analogue of this concept is the principle of material concentration. The limitations arising from this thesis of B.Ye. Paton were analyzed in [339].

### CONCLUDING REMARKS

Above was a brief presentation of the 120-year history of studies on the theory of structures performed by scientists of the Kyiv school. It not only testifies to the serious contribution to the theory and practice of computational analysis of building structures, but also makes it possible to talk about some trends that determine the future development of this applied science.

Perhaps the main thing here is the conversion of all the tools of the theory of structures into a numerical form of analysis, based on the development of modern computing tools, the algorithmization of known and new approaches to solving problems and the rigorous justification of these algorithms. The development of effective implementations of the methods of numerical analysis, especially when solving nonlinear problems of statics and dynamics, is of great importance as well.

The entire history of the development of structural mechanics [11, 34, 45] shows that one of the main paradigms is the desire for an increasingly detailed analysis of the behavior of structures and the use of detailed calculation schemes. The calculation models of modern structures contain thousands or even tens of thousands of elements, and this fact does not serve as an obstacle to their analysis when calculations are performed using computers. The problem is not the ability to perform the calculation, but the ability to analyze its results. Orientation to a detailed stress analysis leads to a paradoxical situation when the description of the analysis results becomes more difficult to comprehend. And here an important scientific problem emerges, consisting in the need to develop generalized characteristics of the stress state, allowing us to consider the features of the "behavior of the system as a whole."

The logic of the theory of structures development was aimed at taking into account factors that more accurately determine the stress and strain state. And as one of the urgent areas of research, the problem of taking into account the nonlinear behavior of the structural complex has been advanced. The complexity of non-linear analysis is due to the fact that one has to abandon a number of assumptions of classical structural mechanics and cannot use many

familiar principles and theorems (the principle of independence of the action of forces, the theorem on reciprocity of displacements, etc.). These difficulties are not completely overcome at present. We can only say with certainty that the implementation of non-linear analysis with sufficient completeness and accuracy for practice is impossible without the use of computers, so methods adapted for computers are of paramount importance.

"The challenge of computerization" was adequately received by the Kyiv school of the theory of structures. Here was the computer capital of the construction industry, and the LIRA and SCAD software systems created and developed in Kyiv were and remain the main toolkit of construction designers in all CIS countries.

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#### **KYIV SCHOOL OF THE THEORY OF STRUCTURES**

The paper presents a review of more than a century-long history of Kyiv school of the theory of structures. Particular attention is paid to the fundamentally new opportunities for the development of the theory of structures in the era of numerical analysis. The publication contains a wide bibliography.

**Keywords:** bar systems, stability, shells, structural mechanics, finite difference method, finite element method, calculation model.

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#### **КИЇВСЬКА ШКОЛА ТЕОРІЇ СПОРУД**

Стаття присвячена аналізу більш ніж вікової історії Київської школи теорії споруд. Особлива увага приділена принципово новим можливостям розвитку теорії споруд в епоху чисельного аналізу. Публікація містить широку бібліографію.

**Ключові слова:** стержневі системи, стійкість, оболонки, будівельна механіка, метод скінченних різниць, метод скінченних елементів, розрахункова модель.

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*Розглянута історія виникнення і становлення Київської школи теорії споруд. Особлива увага приділена принципово новим можливостям розвитку теорії споруд в епоху чисельного аналізу.*

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*The history of Kyiv school of the theory of structure birth and formation was considered. Particular attention was paid to the fundamentally new opportunities for the development of the theory of structures in the era of numerical analysis.*

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*Рассмотрена история зарождения и становления Киевской школы теории сооружений. Особое внимание уделено принципиально новым возможностям развития теории сооружений в эпоху численного анализа.*

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